Responsive Pricing under Supply Uncertainty

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1 Introduction

Due to long supply lead time and short selling season, retailers usually can place their orders only once before the start of the selling season. For example, in the fashion industry, it is common for retailers to place their orders many months before the selling season. The reader is referred to Fisher and Raman (1996) for an excellent description of the ordering process in the fashion industry. Since accurate supply or demand information is rarely available in advance, it is difficult for retailers to determine cost effective order quantities. For instance, due to uncertain supply yields, transportation delays, shrinkage during shipment, retailers usually do not know for sure if they would receive the exact quantity being ordered prior to the selling season. Besides uncertain supply, retailers often face uncertain demand as well. As such, many retailers have to struggle with the overstocking and understocking issues.

To reduce the overstocking and understocking costs, various researchers have examined different approaches for helping retailers to meet uncertain demand. First, Fisher and Raman (1996) consider a situation in which a retailer can place a second order in a later period after the first order when more accurate demand information is available. Next, Tang et al. (2004) consider a situation in which a retailer offers each customer two options: pre-commit an order at a reduced price before the selling season, or buy the product at the regular price during the selling season. These two ideas would certainly improve supply chain performance; however, some suppliers and retailers may have specific concerns about the requirements associated with these two ideas. For example, to implement the accurate response concept in Fisher and Raman (1996), the supplier needs to have sufficient capacity to handle the second order on short notice. Also, to implement the early commitment program in Tang et al. (2004), a retailer need to show his customers a sample of the seasonal products before the selling season, which could make the retailer or the manufacturer more vulnerable to copycats. In this paper, we consider a situation in which the accurate response and the early commitment program are impractical due to the aforementioned concerns. Instead of having the flexibility
to place two separate orders or to sell the product at two different retail prices, we consider a situation in which the retailer can place exactly one order and select only one retail price before the selling season. This concept is known as ‘responsive pricing.’ Van Mieghem and Dada (1999) is the first to analyze the responsive pricing concept as a mechanism for a retailer to manage uncertain demand.

While Van Mieghem and Dada (1999) and Chod and Rudi (2005) focus on the issue of demand uncertainty, we examine the benefits of responsive pricing under supply uncertainty. In the context of supply uncertainty, the reader is referred to Yano and Lee (1993) for a comprehensive review of lot sizing models with uncertain supply yield. To our knowledge, our model is the first that examines the joint decisions of order quantity and retail pricing under supply uncertainty. As an initial attempt to analyze the issue of responsive pricing under supply uncertainty, we consider a situation in which the demand function is known but the supply yield is uncertain.

2 The Base Model

Consider a situation in which a retailer orders a seasonal product from a supplier and sells the product over a selling season. We assume that the retailer knows that the product demand function is given by: \( D = \alpha - \beta p \), where \( \alpha > 0 \) represents the potential market size, \( \beta > 0 \) represents the price sensitivity, and \( p \) represents the retail price. While the retailer has perfect information about the demand function, he has to deal with supply uncertainty in the following manner. First, the supply lead time is equal to one period, and hence, the retailer needs to decide on the order quantity \( Q \) at the beginning of period 1 so that the retailer will receive the order at the end of period 1 prior to the selling season that starts at the beginning of period 2 and ends at the end of period 2. Second, for any order quantity \( Q \), the retailer will receive only \( yQ \) non-defective units, where \( y \) represents the supply yield. In our model, we assume that \( y \) is a random variable that takes on \( N \) different discrete values, say, \( y_n \) for \( n = 1, 2, \ldots, N \), where \( 0 < y_1 < y_2 < \cdots < y_{N-1} < y_N \leq 1 \) and \( \text{Prob}\{y = y_n\} = \lambda_n \). Third, the retailer pays the supplier \( c \) per unit at the beginning of period 1, and the retailer disposes the unsold units at \( s \) per unit at the end of period 2. Without loss of generality, we assume \( s = 0 \).
2.1 Problem Formulation

First, under the No Responsive Pricing (NRP) policy, the retailer has to determine the optimal order quantity $Q'$ and optimal retail price $p'$ at the beginning of period 1 so as to maximize the retailer’s expected profit. In this case, the retailer’s problem $P(NRP)$ can be formulated as follows:

$$\Pi' = \max_{Q,p} E_y \{ -cQ + p \cdot \min\{yQ, D\} \}.$$  (2.1)

Next, under the Responsive Pricing (RP) policy, the retailer would first determine the order quantity $Q^*$ at the beginning of period 1. Then the retailer would determine the retail price $p^*$ at the beginning of period 2 after observing the actual yield realized at the end of period 1. Then the retailer’s problem $P(RP)$ is as follows:

$$\Pi^* = \max_Q -cQ + E_y \{ \max_p \{ p \cdot \min\{yQ, D\} \} \}.$$  (2.2)

Suppose we implement the optimal order quantity $Q'$ and the optimal retail price $p'$ under the NRP policy. Then it is easy to check from (2.2) and (2.1) that

$$\Pi^* \geq -cQ' + E_y \{ p' \min\{yQ', (\alpha - \beta p')\} \} = \Pi'.$$

2.2 Analysis of the No Responsive Pricing policy

Since $D = \alpha - \beta p$, the No Responsive Pricing problem $P(NRP)$ given in (2.1) can be rewritten as:

$$\Pi' = \max_{Q,p} E_y \{ \Pi'(Q,p|y) \}, \quad \text{where}$$

$$\Pi'(Q,p|y) = \begin{cases} 
-cQ + pyQ & \text{if } y \leq \frac{\alpha - \beta p}{Q}, \\
-cQ + p(\alpha - \beta p) & \text{if } y > \frac{\alpha - \beta p}{Q}.
\end{cases}$$

Suppose we set $y_0 = 0$ and $y_{N+1} = Y$, where $Y$ is a sufficiently large number. For any given $Q$ and $p$, there must exist a $k$, $k = 0, 1, \ldots, N$, so that $y_k \leq \frac{\alpha - \beta p}{Q} < y_{k+1}$ and

$$E_y \{ \Pi'(Q,p|y) \} = \sum_{m=1}^k \lambda_m (-cQ + py_m Q) + \sum_{m=k+1}^N \lambda_m (-cQ + p(\alpha - \beta p)) \quad \text{if } y_k \leq \frac{\alpha - \beta p}{Q} < y_{k+1}.$$
By denoting the terms \( \sum_{m=0}^{0} = 0 \) and \( \sum_{N+1}^{N} = 0 \), we can decompose problem \( P(NRP) \) into \( N+1 \) subproblems \( P_k(NRP) \), where \( k = 0, 1, \cdots, N \) and

\[
\Pi'_k = \max_{Q, p} \left\{ \sum_{m=1}^{k} \lambda_m(-cQ + py_mQ) + \sum_{m=k+1}^{N} \lambda_m(-cQ + p(\alpha - \beta p)) \right\} \tag{2.3}
\]

subject to \( y_k \leq \frac{\alpha - \beta p}{Q} < y_{k+1} \).

Let \( Q'_k \) and \( p'_k \) be the optimal order quantity and retail price for subproblem \( P_k(NRP) \), respectively. Also, let \( k' \in \arg\max\{\Pi'_k : k = 0, 1, 2, \cdots, N\} \). In this case, one can utilize the solutions of these \( N+1 \) subproblems \( P_k(NRP) \) to determine the optimal expected profit and the optimal solutions to the original problem \( P(NRP) \) as follows:

\[
\Pi' = \Pi'_{k'}, \ Q' = Q'_{k'}, \ p' = p'_{k'}. \tag{2.4}
\]

**Proposition 1** Let \( u_k = \sum_{m=1}^{k} \lambda_m y_m \) and \( v_k = \sum_{m=k+1}^{N} \lambda_m \). The solutions for subproblem \( P_k(NRP) \) can be expressed as follows:

1. Suppose \( \frac{\beta c}{\alpha} < \frac{u_k(u_k + v_k y_{k+1})}{u_k + 2v_k y_{k+1}} \). Then \( p'_k = \frac{c}{2(u_k + y_k v_k)} + \frac{\alpha}{2\beta} \), \( Q'_k = \frac{\alpha}{2y_k} - \frac{\beta c}{2(u_k + v_k y_{k+1})y_k} \) and \( \Pi'_k = \).

2. Suppose \( \frac{u_k(u_k + v_k y_{k+1})}{u_k + 2v_k y_{k+1}} \leq \frac{\beta c}{\alpha} < u_k + v_k y_{k+1} \). Then

\[
\begin{align*}
p'_k &= \begin{cases} 
\frac{c}{2(u_k + y_k v_k)} + \frac{\alpha}{2\beta} & \text{if } \frac{\alpha(u_k + v_k y_{k+1}) - \beta c}{y_k(u_k + v_k y_{k+1})} - \frac{\beta c}{y_k(u_k + v_k y_{k+1})} < 0, \\
\frac{\alpha}{2y_k} - \frac{\beta c}{2(u_k + v_k y_{k+1})y_k} & \text{if } \frac{\alpha(u_k + v_k y_{k+1}) - \beta c}{y_k(u_k + v_k y_{k+1})} - \frac{\beta c}{y_k(u_k + v_k y_{k+1})} \geq 0.
\end{cases}
\end{align*}
\]

\[
\begin{align*}
Q'_k &= \begin{cases} 
\frac{\alpha}{2y_k} - \frac{\beta c}{2(u_k + v_k y_{k+1})y_k} & \text{if } \frac{\alpha(u_k + v_k y_{k+1}) - \beta c}{y_k(u_k + v_k y_{k+1})} - \frac{\beta c}{y_k(u_k + v_k y_{k+1})} < 0, \\
\frac{\alpha}{2y_k} - \frac{\beta c}{2(u_k + v_k y_{k+1})y_k} & \text{if } \frac{\alpha(u_k + v_k y_{k+1}) - \beta c}{y_k(u_k + v_k y_{k+1})} - \frac{\beta c}{y_k(u_k + v_k y_{k+1})} \geq 0.
\end{cases}
\end{align*}
\]

\[
\Pi'_k = \begin{cases} 
\frac{\alpha(u_k + v_k y_{k+1}) - \beta c}{4y_k(u_k + v_k y_{k+1})} & \text{if } \frac{\alpha(u_k + v_k y_{k+1}) - \beta c}{y_k(u_k + v_k y_{k+1})} - \frac{\beta c}{y_k(u_k + v_k y_{k+1})} < 0, \\
\frac{\alpha(u_k + v_k y_{k+1}) - \beta c}{4y_k(u_k + v_k y_{k+1})} & \text{if } \frac{\alpha(u_k + v_k y_{k+1}) - \beta c}{y_k(u_k + v_k y_{k+1})} - \frac{\beta c}{y_k(u_k + v_k y_{k+1})} \geq 0.
\end{cases}
\]

3. Suppose \( \frac{u_k(u_k + v_k y_{k+1})}{u_k + 2v_k y_{k+1}} \leq \frac{\beta c}{\alpha} \leq u_k + v_k y_{k+1} \). Then \( p'_k = \frac{c}{2(u_k + y_k v_k)} + \frac{\alpha}{2\beta} \), \( Q'_k = \frac{\alpha}{2y_k} - \frac{\beta c}{2(u_k + v_k y_{k+1})y_k} \) and \( \Pi'_k = \).

4. Suppose \( \frac{\beta c}{\alpha} \geq u_k + v_k y_{k+1} \). Then \( Q'_k = 0 \), \( p'_k \) is arbitrary and \( \Pi'_k = 0 \).


2.3 Analysis of the Responsive Pricing Policy

We now analyze the retailer’s optimal expected profit under the Responsive Pricing policy. Observe from (2.2) that problem $P(RP)$ can be rewritten as:

$$\Pi^* = \max_Q \{E_y \max_p \{\Pi(Q, p|y)\}\}, \quad \text{where}$$

$$\Pi(Q, p|y) = \begin{cases} 
-cQ + pyQ & \text{if } p \leq \frac{\alpha - yQ}{\beta} , \\
-cQ + p(\alpha - \beta p) & \text{if } p > \frac{\alpha - yQ}{\beta} .
\end{cases}$$

After determining the optimal retail price $p^*(Q|y)$ and to take the expectation of $\Pi(Q, p^*|y)$ with respect to $y$, we define $w_k = \sum_{m=1}^{k} \lambda_m y_m^2$, and $w_0 = 0$. Let us also define the following break points: $x_{N+1} = 0, x_N = \frac{\alpha}{2y_N}, \ldots, x_k = \frac{\alpha}{2y_k}$, for $k = N, N - 1, \cdots, 1$, and $x_0 = X$, where $X$ is a large number. Then $x_k$ is decreasing in $k$; i.e., $x_{N+1} < x_N < x_{N-1} < \cdots < x_1 < x_0$. In this case, there must exist a $k$ so that $x_{k+1} < Q \leq x_k$ and the retailer’s expected profit $E_y (\Pi(Q, p^*|y))$ can be expressed as follows:

$$E_y(\Pi(Q, p^*|y)) = \left(\frac{\alpha}{\beta} u_k - c\right)Q - \frac{w_k}{\beta}Q^2 + \frac{\alpha^2 v_k}{4\beta} \quad \text{if } x_{k+1} < Q \leq x_k, \quad (2.6)$$

where $k = N, N - 1, \cdots, 0$. Then it is easy to see that the objective function of problem $P(RP)$ given in (2.5) is a piece-wise concave function. By evaluating the derivative of this piece-wise concave function at various break points $x_k = \frac{\alpha}{2y_k}$, we can determine the unique optimal solution to the problem $P(RP)$ as follows:

\textbf{Proposition 2} Suppose $\frac{\beta c}{\alpha} < \mu$. Then

$$k^* = \text{argmax}\{u_k - \frac{w_k}{y_k} < \frac{\beta c}{\alpha} : k = N, N - 1, \cdots, 1\}. \quad (2.7)$$

Also, the optimal order quantity $Q^*$ and the retailer’s optimal expected profit $\Pi^*$ under the Responsive Pricing policy can be expressed as:

$$Q^* = \frac{\alpha u_{k^*} - \beta c}{2w_{k^*}}, \quad \text{and} \quad (2.8)$$

$$\Pi^* = \frac{(\alpha u_{k^*} - \beta c)^2}{4\beta w_{k^*}} + \frac{\alpha^2 v_{k^*}}{4\beta}. \quad (2.9)$$

Moreover, suppose $\frac{\beta c}{\alpha} \geq \mu$. Then $Q^* = 0$ and $\Pi^* = 0$. 

5
3 Conclusion

We have developed a two-stage stochastic model for determining the optimal order quantity and optimal retail price under supply uncertainty. Specifically, we show the Responsive Pricing policy dominates the No Responsive Pricing policy in terms of the retailer’s optimal expected profit. By examining the underlying structure of the problems associated with these two pricing policy, we have developed simple approaches for determining the optimal order quantity and retail price for any discrete supply yield distribution. By using these solution approaches, we examine the impact of yield distribution on the optimal order quantity, retail price, and retailer’s expected profit. Our numerical analysis indicates that the Responsive Pricing policy is more beneficial when the supply yield is highly uncertain (low mean or high variance) or when the unit cost is high. We have also shown how to extend our analysis of the Responsive Pricing policy to examine three issues including emergency order, supplier selection, and order allocation among multiple suppliers. Our model has certain limitations including deterministic demand and linear demand function. We plan to extend our model to examine the issue of responsive pricing under uncertain demand and uncertain supply in our future research.

4 References


