AN INTELLIGENT SOLUTION APPROACH TO VEHICLE ROUTING PROBLEM

Minfang Huang
School of Management, Dalian University of Technology, P.R. China, 116023
E-mail: huangminfang03@hotmail.com, Phone: 86-411-8470-7893

Xiangpei Hu
School of Management, Dalian University of Technology, P.R. China, 116023
E-mail: drhxp@dlut.edu.cn, Phone: 86-411-8470-7893

Amy Z. Zeng
Dept. of Management, Worcester Polytechnic Institute, Worcester, MA 01609, USA
E-mail: azeng@wpi.edu, Phone: 508-831-6117

Qiao Cheng
School of Management, Dalian University of Technology, P.R. China, 116023
E-mail: sechengqiao@163.com, Phone: 86-411-8470-7893

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ABSTRACT. In this paper, we present a three-stage solution approach to the Vehicle Routing Problem (VRP). In the first stage, the distribution area is divided into several areas based on drivers’ experiences, and the customers in each area are further segmented into groups by Fuzzy clustering. In the second stage, a depth-first search algorithm is presented to generate feasible routing schemes. In the last stage, an integer programming model is constructed to identify the optimal routing schemes. The above approach is a combination of quantitative computing process and qualitative reasoning process.

Keywords: Vehicle Routing Problem (VRP), Fuzzy clustering, State space search
1 INTRODUCTION

Vehicle Routing Problem (VRP) is one of the most challenging issues in the area of combinational optimization. It has been verified to be a NP-hard problem. Since it was presented in 1959, a large amount of research results have been achieved. According to the solution processes of problems, a majority of the results can be classified into following two types of solution approaches [1].

(1) Exact solution approach. Here mathematical models, which may be integer programming models and graph models, are firstly constructed based on the kind of Vehicle Routing Problem, such as Capacitated VRP (CVRP), Multiple Depot VRP (MDVRP), VRP with Time Windows (VRPTW), etc. Then classical branch and bound algorithm, Gomory cuts and dynamic programming algorithm are used to find the optimal solutions. We can refer to [2] to find a good review on exact algorithms based on the branch and bound approach. The exact solution approaches have a serious drawback, that is, they are only suitable for solving problems with fairly small sizes as the computational time will increase exponentially with the increase of the size of VRP.

(2) Heuristics solution approach. Here efficient heuristic algorithms are constructed for all kinds of VRP. Then the corresponding computer programs are developed to find acceptable solutions. Several representative heuristic algorithms are C-W Algorithm, $\lambda$-opt Algorithm, Sweep Algorithm, two-phase heuristic, Tabu Search, biological simulation algorithms (including Ant Algorithm, Genetic Algorithm, Artificial Neural Network, and Simulated Annealing Algorithm). These solution
methods are suitable to bigger size VRP. More detailed reviews on heuristics can be found in [3].

Among all the heuristic results, one kind of two-stage heuristics, which is called Cluster first – Route second, is mostly related with what will be studied in this paper. The first stage aims at clustering, where all customers are classified into a number of groups. The second stage identifies routings, where each group is simply regarded as a TSP. The representative results are Sweep Algorithm [4] and Dispatch Algorithm [5].

Recently several new customer classification methods have been presented. For example, customers are classified based on Grid Algorithm and the concept generalized workload is introduced to balance the different routings in reference [6]. The mixed Genetic Algorithm is used to specify each single routing. In literatures [7], [8] and [9], Fuzzy Clustering Analysis is used to classify customers. Rodolfo Dondo and Jaime Cerda [10] classify the customers by Time Window, and sort vehicles by decreasing values of the ratio between a vehicle’s capacity and its fixed cost. Then they assign the vehicles to the groups. In most results, the two stages clustering and routing are entirely independent. And VRP with multiple vehicle capacities hasn’t been solved in these results. Although the result in [10] can deal with VRP with multiple vehicles, it also has the common drawback of local optimization due to the independence between the processes of clustering and the routing.

Decreasing the problem’s state space will be a main objective while solving VRP NP-hard problem. Focusing on decreasing the state space of feasible vehicle routings,
regarding a smaller solution cost as an objective, according to the drivers’ delivery experience, we present a three-stage solution approach (namely CGSM approach) which incorporates qualitative reasoning process and quantitative computing process. The remainder of this paper is organized as follows. Section 2 defines the Capacitated VRP and its parameters. In section 3, we present the theory of the CGSM approach, including customer classification, the generation of the vehicle routing schemes, and construction of an integer programming model. Finally, we present our concluding remarks and summarize future research directions in Section 4.

2 PROBLEM DESCRIPTION AND ITS PARAMETERS

Capacity VRP (CVRP) is one of the most popular kinds of VRP. Therefore we choose CVRP with one depot as our research object. All the initial parameters are summarized and explained below.

\( N \) -- the total number of customers;
\( z_i \) -- the \( i \)-th customer;
\( Z \) -- the set of customers, \( Z = \{ z_1, z_2, \cdots, z_N \} \);
\( x_i, y_i \) -- the location of \( i \)-th customer, \( i = 1, \cdots, N \);
\( q_i \) -- the demand of the \( i \)-th customer, \( i = 1, \cdots, N \);
\( K \) -- the number of vehicle types;
\( p_i \) -- operating cost of the \( i \)-th vehicle type, \( i = 1, \cdots, K \);
\( v_i \) -- load capacity of the \( i \)-th vehicle type, \( i = 1, \cdots, K \);
\( U_t \) -- average unloading time;
\( T \) -- the longest travel time;
Several parameters used to describe the information of classified customer are explained below.

\( M \) -- the number of groups of classified customers;

\( C_i \) -- the \( i \)-th group, \( i = 1, \cdots, M \);

\( nC_i \) -- the number of customers in the \( i \)-th group, \( i = 1, \cdots, M \);

\( p_{ij} \) -- the \( j \)-th customer in the \( i \)-th group, \( i = 1, \cdots, M, j = 1, \cdots, nC_i \);

\( q_{ij} \) -- demand of the \( j \)-th customer in the \( i \)-th group, \( i = 1, \cdots, M, j = 1, \cdots, nC_i \). Here we assume each customer’s demand is a mean value \( Q \);

\( D_{ij} \) -- the travel time from the central depot to the \( j \)-th customer in the \( i \)-th group, \( i = 1, \cdots, M, j = 1, \cdots, nC_i \);

\( I_{tk} \) -- the travel time from the \( k \)-th customer in the \( i \)-th group to its candidate customers, \( i = 1, \cdots, M, j = 1, \cdots, nC_i \);

\( E_{ijkl} \) -- the travel time from the \( j \)-th customer in the \( i \)-th group to the \( l \)-th customer in the \( k \)-th group, \( i, k=1, \cdots, M, j=1, \cdots, nC_i, l=1, \cdots, nC_k \).

The objective is to minimize the number of vehicles and achieve each vehicle’s corresponding optimal route within the travel time and load constraints.

3 THE PROCESS OF CGSM APPROACH

The CGSM approach will include three stages. The first stage is Clustering Customers (C). According to the drivers’ experience, the distribution area is divided into smaller areas. Then using Fuzzy Clustering we subdivide the customers in the smaller areas into groups. The customers in the same group have several similar attributes. In the second stage, Generating Routing Schemes (GS), we adopt the
depth-first search algorithm to generate better feasible routing schemes, and use the travel time and capacity to control the search depth. The last stage is Modeling and Solving (M). Based on the distribution area division, an integer programming model is constructed to achieve the optimal routing schemes.

The flow chart of the CGSM approach is shown in Figure 1. This paper mainly discusses the research work on customer clustering. In reference [12], we can find the detailed research results on the theory of routing scheme generation and modeling and solving.

![Flow chart of CGSM approach](image)

**Figure 1 The flow chart of CGSM approach**

### 3.1 The first stage—customer clustering

Dividing customers by a number of attributes is an efficient way to decrease the VRP’s state space, especially for those large-scale VRP. There have been research results about customer clustering. However, the current methods merely classify customers according to either qualitative factors or quantitative factors. Although the traditional methods for distribution are lack of efficiency, they could deal with the qualitative factors in the distribution process, for example, special geographical environment (river or one-way street), and traffic conditions (traffic jam in different
periods). If we can represent the traditional distribution methods in computer, we will incorporate these valuable qualitative factors into the quantitative computing process. Therefore we can widen the solution approach and improve its practical value.

3.1.1 Knowledge-based processing of the drivers’ experience

Customers will be classified initially according to the drivers’ delivery experience in which customer information, traffic information and geographic information will be considered. Based on the characteristics and structure of the human’s delivery experience, we choose tree-like knowledge representation method [11] to represent it, which is shown in Figure 2.

Figure 2 Tree-like-based knowledge representation of the drivers’ delivery experience

The knowledge-based processing flow of the drivers’ delivery experience is shown in figure 3. The knowledge base of delivery experience is used to dynamically acquire the knowledge of delivery experience, and provides the data and knowledge foundation for qualitative reasoning process for VRP solution. Distribution information, which is processed by knowledge-based delivery experience, will
generate the initial information of the dividing results of the distribution area.

![Figure 3 The knowledge processing flow of the drivers’ experience](image)

3.1.2 Fuzzy Clustering-based customer subdivision

Unlike the hard clustering techniques, which assign each data sample to one and only one cluster, Fuzzy Clustering utilizes fuzzy partitioning to group data such that any given data sample is allowed to belong to several groups with different degrees of similarity bounded within the range of 0 and 1. The architecture of the proposed Fuzzy Clustering-based customer subdivision algorithm is composed of 3 major mechanisms including [13] (1) evaluation indexes choosing and data standardization, (2) generating fuzzy correlation matrix, and (3) customer grouping. These steps are detailed in the following.

(1) Evaluation indexes choosing and data standardization

We can choose the customer’s location \((x_i, y_i)\) and traffic information \((Tr_i)\) as the evaluation indexes. \(r_i\) can be achieved by the experts bounded within the range of 1 and 5. The following is the initial data matrix.
In order to analyze and compare the data in the matrix $A$ conveniently, the procedure of standardize with respect to $u_{ij}$ is conducted, and herein, the standardization value of $u_{ij}$ is given by

$$u_i^\prime = \frac{u_i - \min_{j=1,2,3} u_{ij}}{ \max_{j=1,2,3} u_{ij} - \min_{j=1,2,3} u_{ij}} \quad (j=1,2,3).$$

Where $u_i = \frac{u_i - \bar{u}_j}{s_j}$ ($i=1,2,...,n; \ k=1,2,...,m$). $\bar{u}_j$ and $s_j$ correspond to the values of mean and standard deviation with respect to $u_{ij}$, respectively, as denoted by

$$\bar{u}_j = \frac{1}{N} \sum_{i=1}^{n} u_{ij},$$
$$s_j = \sqrt{\frac{1}{N} \sum_{i=1}^{n} (u_{ij} - \bar{u}_j)^2}$$

(2) Generation of fuzzy correlation matrix

A fuzzy correlation matrix $R$ is constructed in which each element $r_{ij}$ represents the correlation between a given pair of customers $i$ and $j$. $R$ is expressed as

$$R = \begin{bmatrix}
  r_{11} & r_{12} & \cdots & r_{1n} \\
  r_{21} & r_{22} & \cdots & r_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  r_{n1} & r_{n2} & \cdots & r_{nn}
\end{bmatrix}, \quad r_{ij} = \frac{\sum_{k=1}^{3} u_{ik}^2 \cdot u_{jk}^\prime}{\sqrt{\sum_{k=1}^{3} u_{ik}^2} \cdot \sqrt{\sum_{k=1}^{3} u_{jk}^2}}$$

$r_{ij}$ is more close to 1, the given pair of two customers are closer.

(3) Customer grouping

After the above process, $t(R)$ is achieved according to the process of squaring in which $R^2$, $R^4$, $R^6$, etc., are calculated. When the value of $R^k \ast R^k$ is firstly equals the value of $R^k$, the matrix $R^k$ is $t(R)$. $t(R)$ is a fuzzy equivalence matrix. $\lambda$ is a threshold
for identifying the relative similarity between a given pair of customers. We can achieve dynamic clustering results according to the different values of $\lambda$.

3.1.3 Candidate list

In order to simplify the travel time between two adjacent customers in the same group, *Candidate list* is constructed.

**Definition**

*Candidate list* is defined as follows. $P_{ij}$ and $P_{ik}$ are two customers in $i$-th group. The travel time between $P_{ij}$ and $P_{ik}$ is $Val(t_{ijk})$. $Val(t_{ijk})$ is sorted increasingly into the set $List(P_{ij})$. $n_i$ is used to represent the size of the candidate list, and $List(P_{ij})_n$ to represent the former $n$ elements of the set $List(P_{ij})$. $List(P_{ij})_m$ is the candidate list of $P_{ij}$. Furthermore, the travel time between $P_{ij}$ and its candidate customers is $It_{ij} = \frac{\sum Val(t_{ijk})}{n_i}$. $It_{ij}$ is simplified to a constant $It$, $It = \frac{\sum It_j}{n_i}$. That is, the travel time between two customers in the same group is identical. And the travel time from the $j$-th customer in the $i$-th group and the $l$-th customer in the $k$-th group is simplified to $Et_{lk}$, where $Et_{lk} = It_i + It_k$.

3.2 The second stage -- Vehicle routing scheme generation

A feasible vehicle routing scheme refers to a vehicle’s routing plan that starts with the depot and ends with the last customer that either uses up the full vehicle load or satisfies the given time window. On the base of the first stage work, customer clustering, we achieve several groups of customer. Therefore, a routing scheme forms as follows. One vehicle starts with the depot. Then it chooses a custom from its candidate list and all the other groups of customers. After it serves this customer, it
will choose the next customer from its candidate list and other groups. And so on, until one constraint is satisfied. We observe that the set of routing schemes forms a state-space, in which a customer is a search node. A routing scheme is a path spanning through the state-space from an initial state to a goal state. In this case, the central depot is a search node corresponding to the initial state, and the last customer a vehicle will serve within the travel time or the load constraint is the goal state. Thus, the generation of vehicle routing schemes is turned into the path searching through the state-space. Detailed research results for intelligent vehicle routing scheme generation can be found in reference [12].

3.3 The third stage-- modeling and solving

Let \( x_j \) be the number of vehicles serving customers according to the \( j \)-th routing scheme. Let \( m_k \) be the number of feasible routing schemes that \( k \)-th type vehicle serves. Then the total number of routing schemes that all types of vehicles can form within their travel time and capacity constraints is \( S = \sum_{i=1}^{K} m_i \). Thus, the optimized model of VRP is below.

\[
\begin{align*}
\min z &= \sum_{i=1}^{K} (p_i \sum_{j=1}^{m_i} x_j) \\
\sum_{j=1}^{S} a_{ij} x_j &\geq nC_i, \quad i = 1, \ldots, M \\
x_j &\geq 0, x_j \in \text{int}
\end{align*}
\]

Where \( p_i \) is operating cost of the \( i \)-th vehicle type, Eq. (1) imply the total cost of vehicle distribution is the least, Eq. (2) imply the demand of customers’ group that should be satisfied in each distribution area. This integer programming model can be solved easily by the module of Integer Linear Programming in OR solution software.
such as the LINDO.

4 CONCLUSIONS

This paper presents a new solution approach to VRP, in which quantitative computing process and qualitative reasoning process are combined. This approach aims at minimizing the state space of problem and increasing efficiency and intelligence of problem solution.

Our future directions include improving the generalization and practicability of CGSM approach for all kinds of VRP problem, such as Multiple Depot VRP, Stochastic VRP, VRP with Backhauls, VRP with Pick-Up and Delivering, VRP with Time Windows, and so on. Furthermore, knowledge representation structure of the drivers’ experience should be extended and dynamic candidate list should be constructed.

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6 REFERENCES


