Abstract

This paper studies a competitive Hotelling-style market with two symmetric banks that decide the pricing and location of their automated teller machines (ATMs). Two different systems are considered in this paper: an unregulated model wherein banks are allowed to set surcharges, and a regulated model in which surcharges are banned. We derive equilibrium outcomes and compare them in the two systems, and find that banks always maintain a certain distance between ATMs, but that distance is larger, indeed maximized, under the regulatory scheme. It has also been shown, surprisingly, that banks would actually always perform better in the regulated model when surcharges are banned, while consumers may be worse off.

Keywords: automated teller machine, surcharges, foreign fees, pricing, location
1. Introduction

Automated teller machine (ATM) fees are a point of contention between consumers and banks. When a non-customer uses an ATM, the bank that owns the ATM charges a fee known as a “surcharge,” while the customer’s own bank charges another fee called the “foreign fee.” The customer’s bank also sends the bank that owns the ATM a transfer payment called an “interchange fee.” Consumer groups claim that surcharging represents “double-dipping” or excessive charging and inappropriately increases consumer costs (PIRG, 2000). Thus, they seek to establish regulations banning surcharges, while banks reserve the right to charge for services provided to non-customers. It is possible that banks advocate surcharging in view of it being a source of revenue, or a way to cover costs. Banks also point out that customers don’t have to pay ATM fees, and they can simply go to their own bank’s ATMs. In fact, 52 percent of all bank customers never pay ATM fees (McBride, 2007). On the other hand, consumers think of surcharges as an additional charge that enriches banks at consumers’ expense and point to the estimated $4.4 billion in ATM fees to be collected in 2007. They also note that ATM surcharge fees only keep increasing even though technology costs, including ATMs, are known to decrease over time. Motivated by the controversy, we seek to examine how banks locate ATMs relative to one another and how they set the related fees in equilibrium, in the presence or absence of surcharges.

We model a competitive Hotelling-style market with two symmetric banks that decide upon the location and pricing of their own ATMs. We assume that each bank has one ATM. Two different systems are considered in this paper: (1) an “unregulated” model in which surcharges are allowed, and (2) a “regulated” model in which surcharges are banned. In both models, we employ a two-stage non-cooperative game in which banks first choose locations for their ATMs and then set prices, including surcharges, if applicable, and foreign fees. Banks assume that customers, both their own and their competitor’s, will rationally choose to transact at the location offering the lowest total cost of travel and fees. Our main objectives of this paper are to understand the prices and ATM locations in equilibrium for banks in both the regulated and unregulated regimes and to examine the effect of surcharge bans on the location decisions, fee structures, bank profits and consumer total cost.
We generate and analyze closed-form expressions for the banks’ optimal prices and locations for both regulated and unregulated models. Our analysis of the equilibrium ATM locations in both regulated and unregulated models indicates that banks would always maintain a certain distance between ATMs to generate revenues from fees; surcharges, if allowed, and/or foreign fees. When surcharges are banned our model shows that banks locate their ATMs at the endpoints of the linear market. This configuration maximizes the total travel distance and leads to the highest total travel cost of consumers, i.e., the least convenient location arrangement. When surcharges are allowed, banks in equilibrium maintain a smaller distance between the ATMs. In view of the equilibrium locations in both regulated and unregulated models, banks tend to locate their ATMs further apart and in less convenient locations for their consumers in the regulated model equilibrium. Further, in the unregulated model, it is possible that symmetric players (i.e., banks) with complete information could generate asymmetric ATM locations in equilibrium.

Our results on the equilibrium profits of banks in both regulated and unregulated models reveal that banks would actually always perform better in the regulated case, relative to the unregulated case. This result suggests that banks should prefer to have surcharges banned. This seemingly counter-intuitive result most likely stems from the fact that allowing banks to surcharge creates another source of price competition between banks in addition to the competition on foreign fees. However, we show that consumers may be worse off when surcharges are banned in the regulated model, which is in stark contrast with the consumer groups’ expectation. The reason for this finding is as follows: In the regulated model, the consumers’ total cost is decreasing in the marginal transaction cost at each bank. This occurs because as the transaction cost increases, a bank lowers its foreign fee to encourage its consumers to use the competitor’s ATM to avoid the transaction cost. In the unregulated model, the consumers’ total cost is independent of the transaction cost due to the offsetting effect of the transaction cost on surcharges and foreign fees. Thus, when the transaction cost is sufficiently high (low), consumers could be better (worse) off in the regulated model, as compared to the unregulated model.

Other work has addressed similar issues by using different methods. Of particular relevance for this paper are a few papers that use modeling to address ATM pricing and location. There
are several papers using spatial models of ATM competition to examine the setting of ATM fees. McAndrews (2001) uses a single-stage model that only considers banks’ price setting. In particular, \( N \) banks, each of which has multiple ATMs uniformly distributed around a circle, decide upon the ATM surcharge and foreign fee. He examines the factors that may affect banks’ pricing behavior. Massoud and Bernhardt (2002) develop a theoretical model with two banks located at each end of the diameter of a circle and the bank customers uniformly distributed on the perimeter. In their paper, surcharges can be used not only as a revenue source, but also as a tool to attract newcomers to the market to establish deposit accounts. The authors consider the pricing issue of differentiated bank services by banks of possibly different sizes. They show that surcharge bans would result in greater bank profits, and possibly lower consumer welfare relative to the case when surcharges are in effect. This result is consistent with the main results in our model, which has a different setting and includes the location decision that they do not consider. In another paper, Bernhardt and Massoud (2005) endogenize the number of ATMs a bank can install, in addition to their pricing decisions. They allow banks to choose the size of their own network and locate their ATMs on a spatial line. But a bank’s choice of ATM locations depends only on its own network. In our model, each bank is allowed to operate one ATM only. However, we allow a bank’s choice of its ATM location to be sensitive to its competitor’s ATM location.

Croft and Spencer (2004) study the implications of ATM fees (surcharges, foreign fees, and interchange fees) and number of depositors for banks and non-banks. They employ a circular market, where banks deploy fixed numbers of ATMs that are uniformly distributed and “interlaced” around the circle, and a branch in the center. They find that surcharging raises the price of an ATM transaction above the joint profit-maximizing level. Moreover, larger banks (with more depositors) earn less revenue from surcharging. The Croft and Spencer paper is probably the most complete of those listed above in the sense that the authors employ surcharges, foreign fees, and interchange fees in the modeling. They use a four-stage model that includes joint determination of interchange fees in the zeroth stage, followed by account fee determination, then consumer choice, and setting of surcharges and foreign fees in the final stage. As part of their analysis they vary the number of locations, but always consider uniform distances between ATMs. Our paper extends theirs
by explicitly modeling the strategic implications of location decisions and their interaction with pricing. In that sense, we are adding an additional stage, for ATM location, to the model that occurs between the consumer choice and the ATM pricing stages. We then focus our analysis on the final two stages, ATM location followed by ATM fee determination.

Note that all of the papers cited above, except Bernhardt and Massoud (2005), use a circle to model the market for ATM services and assume that banks are uniformly placed around the circle with an equal distance in between. In our paper, as one of our main objectives is to specifically model the location decision of ATMs, we adopt a linear market, used in Bernhardt and Massoud (2005), to facilitate the analysis of the ATM location issue. In a circular market, the location decision is largely irrelevant, as the two banks will split the market symmetrically regardless of the locations of the ATMs unless they are located at the exactly same spot.

Although our paper and those in the literature all examine the issue of ATM price setting analytically, our work differs in the following aspects. First, we employ a Hotelling-style model rather than a circular market. Second, our model does not consider consumers’ depository decisions. This is appropriate when customers face significant switching costs. Croft and Spencer (2004) point out several empirical studies, most of which demonstrate that such costs exist in practice. Furthermore, we presume that a depository relationship choice is a long-term decision while ATM use is a very short-term decision best modeled without the larger depository relationship. Therefore, we exogenize consumers’ deposit decisions. Finally, since our interest is mainly on banks’ pricing and location behavior and in particular the impact of surcharge regulation on banks’ behavior, their profits and consumer welfare, rather than competition between banks of different sizes, we assume banks have the same number of ATMs.

There is a large number of empirical papers looking at the issue of ATM pricing and its impact on the network. Much of the literature is concerned with the competitive “fairness” effects of ATM pricing, e.g., whether large banks can inhibit the profitability and competitiveness of smaller banks through the use of ATM surcharges, or with the consumer-level welfare effects of pricing decisions. For example, Prager (2001) empirically examines the relative performance of small banks during a time frame in which the ban on surcharges is relaxed. The author shows that small banks in
the states where surcharging was permitted did not appear to suffer in regards to market share and profitability in comparison with those in states where surcharging was prohibited. However, Hannan et al. (2003) empirically show that profitability of surcharging increases with the bank’s ATM market share. Their results imply that larger banks may use surcharging as a strategic tool to increase the cost for customers of the rival banks to access cash. Hannan (2005) studies the implications of lifting surcharge bans for banks’ market share and for the market structure in the case of Iowa and its neighbor states. Gilbert (1991) and Salop (1990, 1991) examine the option of assigning the responsibility for fee-setting to the network, a centralized party to the competition. See Hannan et al. (2003) for a broad review of the relevant empirical and analytical literature.

McAndrews (1998) documents the widespread debate over the networks’ decision to allow surcharges and examines the effect of surcharging on the economics of ATMs deployment, customer convenience, and banking competition. He finds that surcharges would reduce the customers’ usage of the services provided by the foreign banks. There are several recent papers studying the effect of surcharges on the ATM network and on consumer welfare. Both Ishii (2004) and Knittel and Stango (2004) show that surcharges would increase the costs of the banks’ competitors by introducing incompatibility into the ATM network. Further, Ishii (2004) indicates that a move to compatibility by removing surcharges would substantially increase consumer surplus. However, Knittel and Stango (2004) argue that the increase in consumer welfare due to the increased availability of ATMs resulting from surcharges is likely to more than offset the decrease in consumer welfare due to a detrimental effect of network incompatibility stemming from surcharges. In terms of total welfare including consumers and producers, Gowrisankaran and Krainer (2005a, 2005b) show that the total welfare would be relatively unchanged with a lifting of a surcharge ban, and that an outright ban of surcharges would not significantly deter the employment of ATMs.

Our paper departs from these empirical papers in that it employs a modeling approach that allows us to capture the essential features of the ATM deployment and derive fundamental insights about ATM pricing and location problems. We use a simple two-stage model with two symmetric banks to explicitly consider the fee and location decisions between banks. We assume that the two banks have the same fixed density of consumers in the local market, and that there will be
no potential newcomers to the market. Thus, surcharges are not considered as a means to attract new consumers or a factor to influence consumers’ choice of the deposit bank. Surcharges are a source of revenue only. Since banks are of a same size and have one ATM each, there is no issue of conflict between large banks and smaller banks. Assuming symmetric banks, we are interested in their endogenous location decisions.

The remainder of the paper is structured as follows. In the next section, we introduce the model setup. §3 studies the unregulated model wherein banks can charge surcharges and foreign fees freely. §4 considers the regulated model which precludes surcharges. We compare the regulated and unregulated models in §5 and conclude in §6. All proofs are presented in the appendix.

2. The Model

We consider a model with two banks, A and B, each operating an ATM and competing for ATM transactions in a stylized geographic market. We use a line segment to represent the geographic market for ATM transactions. In practical terms, a linear market represents Hotelling’s (1929) “Main Street” description of firms distributed in a high-intensity region of a city. Such arrangements are often illustrated in layman’s language references to “restaurant row” or “auto row” or “the strip,” referring to the common occurrence of similar businesses being grouped along a major road. We denote the length of the line segment as \( L \). Customers of each bank are assumed to be uniformly distributed along the line. Without loss of generality, we normalize the density of the distribution to 1.

In this paper, we model the current industry practice of charging both “surcharges” and “foreign fees.” Surcharges are fees paid by non-customers to the bank that operates the ATM. Foreign fees are paid to the bank by their own customers when those customers use a different bank’s ATM. Customers are sensitive to the total cost of a transaction, which includes the fees paid to the home bank and/or foreign bank, and the traveling cost to complete the transaction. As a result, a bank must consider the competing bank’s surcharge and foreign fee in determining consumer behavior and resulting market share and profit. In addition to the pricing game between two banks, we also allow them to freely locate their ATMs along the line segment before they set their surcharges and
foreign fees for the ATM service. Therefore, we consider a two-stage model to study the pricing and location games between the two banks. The sequence of events is as follows (Figure 1).

Figure 1: The decision variables on the line segment of length $L$

$$
(S_A, F_A) \quad (S_B, F_B)
$$

| 0 | $X_A$ | $X_B$ | $L$ |

- Stage 1 (Location Game): Banks $A$ and $B$ simultaneously locate their ATMs at $X_A$ and $X_B$, respectively. Without loss of generality, we assume that $0 \leq X_A \leq X_B \leq L$.

- Stage 2 (Pricing Game): Given $X_A$ and $X_B$, bank $A$ sets a surcharge $S_A$ and a foreign fee $F_A$ for its ATM at $X_A$, and bank $B$ determines a surcharge $S_B$ and a foreign fee $F_B$ for its ATM at $X_B$.

Backward induction will be used to solve the two-stage model. We will first examine the pricing game in Stage 2, and then, taking into account the equilibrium prices in Stage 2, we carry out a location analysis in Stage 1. Note that in only focusing on these two stages of the competition between bank ATMs, we follow the primary modeling method of Croft and Spencer (2004) in that pricing ATM services comes after consumers have chosen their account relationships. That is, we do not specifically model competition for deposits.

Table 1 delineates the relevant parameters in the model. Let $c$ represent the marginal cost to the bank of processing an ATM transaction. When a consumer uses the ATM of a foreign bank, that transaction must be processed through a third party network and a fee needs to be paid to the third party by the consumer’s home bank. In industry parlance this is called a “switch fee” which is denoted by $n$ in our paper. Furthermore, the customer’s home bank also sends the foreign bank who owns the ATM a transfer payment, i.e., the “interchange fee,” which is denoted by $I$. To give a bank, say bank $A$, an incentive to push its own consumers to the foreign bank, say bank $B$, we assume throughout this paper that $c > I + n$, where $c$ is the cost to bank $A$ if its consumers transact at bank $A$, and $I + n$ is the cost to bank $A$ if its consumers use bank $B$’s ATM.
Table 1: Model parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>Length of the linear market</td>
</tr>
<tr>
<td>$c$</td>
<td>Variable transaction cost</td>
</tr>
<tr>
<td>$n$</td>
<td>Network switch fee</td>
</tr>
<tr>
<td>$I$</td>
<td>Interchange fee</td>
</tr>
<tr>
<td>$t$</td>
<td>Customer travel cost, in $$/unit of distance</td>
</tr>
<tr>
<td>$M$</td>
<td>Customer’s reservation price for transactions</td>
</tr>
</tbody>
</table>

Furthermore, we assume that a customer has a reservation price for ATM transactions, denoted by $M$, and $L \cdot t \leq M$. We make this assumption to allow all customers to have the option to travel to their home banks to do the transaction. In other words, we rule out the unrealistic situation in which two banks choose ATM locations on an extremely long “street.” We also assume that $L \cdot t > n$. We make this assumption so that the “street” is not too short and thus competition exists between the two banks and there is some room for banks to charge fees when serving their competitor’s customers.

For convenience, we summarize the decision variables in Table 2.

Table 2: Decision variables

<table>
<thead>
<tr>
<th>Decision Variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_A, X_B$</td>
<td>Locations of banks $A$ and $B$, respectively</td>
</tr>
<tr>
<td>$S_A, S_B$</td>
<td>Surcharges by banks $A$ and $B$, respectively</td>
</tr>
<tr>
<td>$F_A, F_B$</td>
<td>Foreign fees by banks $A$ and $B$, respectively</td>
</tr>
</tbody>
</table>

2.1 Consumers’ demand

In this subsection, we derive the demand functions for banks $A$ and $B$. We assume that customers are at fixed locations and each demands a single and homogenous transaction. To avoid the
previously discussed vagaries of modeling deposit competition in relation to ATM fees, we assume that customers have an existing relationship with one bank, and that customers of either bank A or B are uniformly distributed on the line segment, L, with density one. Customers will choose to transact at the location that minimizes their total cost as long as it doesn’t exceed the reservation price. A consumer’s total cost consists of travel cost and prices paid, if applicable. We further assume that if a customer goes to his own deposit bank’s ATM, there is no extra fee charged to the customer except the traveling cost. This assumption is supported by the current retail banking practice. As previously mentioned, a bank charges foreign fees when their own customers use another bank’s ATM and surcharges to other institution’s customers using the bank’s ATM. A bank customer at a particular location will make a decision regarding which bank’s ATM to use. Intuitively, if all banks eliminate all fees, then customers will simply go to the closest ATM, regardless of bank affiliation. However, the fee structure gives rise to a competitive set of decisions for the banks. It is this dynamic that we model.

Based on Figure 1, we next derive the demand functions for both banks assuming given locations of two ATMs, $X_A$ and $X_B$, where $0 \leq X_A \leq X_B \leq L$. We take bank B’s customers as an example. Clearly, B’s customers to the right of $X_B$ will all go to B for the ATM service. Now, let us consider the following two cases for B’s customers: (i) B’s customers to the left of $X_A$ and (ii) B’s customers in the middle of $X_A$ and $X_B$. Denote by $x$ the distance of B’s customers to location $X_A$.

Case (i): A customer of bank B located to the left of $X_A$ and having a distance of $x$ to $X_A$ will go to A’s ATM for service if and only if:

$$S_A + F_B + x \cdot t \leq (X_B - X_A) \cdot t + x \cdot t. \hspace{1cm} (1)$$

Note that the left hand side of (1) is the total price that the B’s customer needs to pay if he uses the other bank’s ATM, and the right hand side is the traveling cost he needs to pay if he goes to his own bank. Inequality (1) is equivalent to $S_A + F_B \leq (X_B - X_A) \cdot t$. Thus, we conclude that all of B’s customers to the left of $X_A$ will go to A if and only if\footnote{Here, we implicitly assume that when customers are indifferent between transacting at their own deposit bank and at the other bank, they will complete transactions at the other bank. This assumption is made for computational} $S_A + F_B \leq (X_B - X_A) \cdot t$. Otherwise, they would all go to B.
Case (ii): Similarly, a customer of bank $B$ located in the middle of $X_A$ and $X_B$ and having a distance of $x$ to $X_A$ will go to $A$’s ATM if and only if:

$$S_A + F_B + x \cdot t \leq (X_B - X_A - x) \cdot t,$$

which is equivalent to $x \leq \frac{X_B - X_A}{2} - \frac{S_A + F_B}{2t}$. All the remaining customers visit their own bank $B$.

Thus, based on the analysis above, we have the following:

$$q_A^b = \begin{cases} 0 & \text{if } S_A + F_B \geq (X_B - X_A) \cdot t + \epsilon, \\ \frac{(X_A + X_B)}{2} - \frac{S_A + F_B}{2t} & \text{if } S_A + F_B \leq (X_B - X_A) \cdot t, \end{cases}$$

where $q_A^b$ stands for the number of $B$’s customers going to $A$’s ATM for transactions, and $\epsilon$ is an arbitrarily small and strictly positive number. The notation $\epsilon$ is used to transform inequality $a > b$ to $a \geq b + \epsilon$, and it is introduced to avoid the occurrence of the strict inequality in the computation.

Accordingly, we have the number of $B$’s customers going to their own bank $B$: $q_B^b = L - q_A^b$.

Similarly, we can derive the number of $A$’s customers going to $B$’s ATM for transactions:

$$q_B^a = \begin{cases} 0 & \text{if } S_B + F_A \geq (X_B - X_A) \cdot t + \epsilon, \\ \frac{2L - (X_A + X_B)}{2} - \frac{S_B + F_A}{2t} & \text{if } S_B + F_A \leq (X_B - X_A) \cdot t, \end{cases}$$

Accordingly, the number of $A$’s customers going to their own bank $A$ for transactions is $q_A^a = L - q_B^a$.

### 2.2 Banks’ profit functions

A bank’s operating profit consists of three parts:

1. Serving other banks’ customers provides surcharge income and the income of the interchange fee, but incurs the marginal cost.

2. When a bank’s customers use the other bank’s ATM services, the bank receives revenue from the foreign fee but incurs the network switch fee and the cost of the interchange fee. Note that since the bank does not process the initial transaction, but simply processes data from the network, the bank does not incur the marginal cost $c$.

purposes only. That is to avoid the occurrence of the strict inequality $S_A + F_B < (X_B - X_A) \cdot t$, which will create unnecessary complexity in computation of the equilibrium prices and locations. This assumption will not affect any of the results derived in the sequel.
(3) Finally, when the bank’s own customers use its ATM, the bank simply incurs the marginal cost $c$.

These terms combine to form banks’ profit functions.

$$
\Pi_A = (S_A - c + I) q^b_A + (F_A - n - I) q^b_B - c \cdot (L - q^b_B),
$$
$$
\Pi_B = (S_B - c + I) q^a_B + (F_B - n - I) q^a_A - c \cdot (L - q^a_A).
$$

The banks’ profit functions can be rewritten as:

$$
\Pi_A = (S_A - c + I) q^b_A + (F_A - n + c - I) q^b_B - c \cdot L, \quad (5)
$$
$$
\Pi_B = (S_B - c + I) q^a_B + (F_B - n + c - I) q^a_A - c \cdot L. \quad (6)
$$

In this paper, we consider two different business scenarios: an unrestricted (or unregulated) model and a regulated model. In the unregulated case, banks are free to charge both surcharges and foreign fees. In the regulated case, banks are not allowed to charge surcharges (i.e., $S_A = S_B = 0$) and thus only set foreign fees. We consider the unregulated model first.

3. Unregulated Model

As described in Section 2, we consider a two-stage game: the location game in Stage 1 and the pricing game in Stage 2. We first study the pricing game in Stage 2 for a given set of locations of two ATMs, $X_A$ and $X_B$, where $0 \leq X_A \leq X_B \leq L$.

3.1 Pricing game in Stage 2

Given locations $X_A$ and $X_B$, each bank optimally sets surcharges and foreign fees in response to the other bank’s optimal choices. Prices are set simultaneously to maximize their own corresponding profit functions. From the demand functions, given by (3) and (4), and the profit functions, given by (5) and (6), one can easily verify that, in equilibrium, a bank’s surcharge is related to the other bank’s foreign fee only. The reason is that the number of a bank’s customers switching to the other bank’s ATM depends only on the sum of this bank’s foreign fee and the other bank’s surcharge. The equilibrium surcharges and foreign fees are presented in the following proposition. All proofs are presented in the appendix.
**Proposition 1** *(Pricing Equilibrium under Given Locations)* Given banks’ ATM locations $X_A$ and $X_B$, the equilibrium fee structures, $(S^*_A, F^*_A)$ and $(S^*_B, F^*_B)$, can be characterized in five different regions in Figure 2. In the figure, the $x$-axis represents the location of bank A’s ATM, and $y$-axis is bank B’s ATM. Based on Figure 2, we summarize the equilibrium prices in Table 3, and explain each region in details therein.

**Figure 2:** The regions for equilibrium prices under given locations of ATMs

From the equilibrium surcharge and foreign fees given in Table 3, we can observe that the effect of the transaction cost $c$ and the interchange fee $I$ on surcharge offsets their effect on the foreign fee. The intuition behind this observation is simple. For example, if the unit transaction cost increases, a bank, say, $A$, should encourage its own consumers to go to the other bank $B$ for transactions by lowering its foreign fee. In the meantime, bank $A$ should also avoid as many consumers as possible from bank $B$ by increasing its surcharge in order to reduce its total transaction cost. Thus, the foreign fee (surcharge) should decrease (increase) in the transaction cost. On the other hand, if the interchange fee increases, a bank should discourage its own consumers to go to the other bank by increasing its foreign fee, and in the meantime, this bank should attract as many consumers as
Table 3: Equilibrium values of prices under given locations

<table>
<thead>
<tr>
<th>Region</th>
<th>$S^<em>_A$ and $F^</em>_B$</th>
<th>$S^<em>_B$ and $F^</em>_A$</th>
<th>$q^h_A$ and $q^h_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region 1</td>
<td>$S^<em>_A + F^</em>_B \geq (X_B - X_A) \cdot t + \epsilon$</td>
<td>$S^<em>_B + F^</em>_A \geq (X_B - X_A) t + \epsilon$</td>
<td>$q^h_A = q^h_B = 0$</td>
</tr>
<tr>
<td>Two ATMs are quite close to each other. Prices are set such that both banks retain their own customers.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Region 2</td>
<td>$S^<em>_A + F^</em>_B = (X_B - X_A) t$</td>
<td>$S^<em>_B + F^</em>_A = (X_B - X_A) t$</td>
<td>$q^h_A = X_A$ and $q^h_B = L - X_B$</td>
</tr>
<tr>
<td>Two ATMs are close but not too close. Prices are set such that $A$’s customers to the right of $B$ will visit $B$, and $B$’s customers to the left of $A$ will visit $A$. All other customers go to their own home bank.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Region 4</td>
<td>$S^*_A = \frac{(X_A + X_B) t - n + 3(c-1)}{3}$</td>
<td>$S^*_B = (X_B - X_A) t$</td>
<td>$q^h_A = \frac{(X_A + X_B) t - n}{6 \epsilon}$</td>
</tr>
<tr>
<td>$F^*_B = \frac{(X_A + X_B) t + 2n - 3(c-1)}{3}$</td>
<td>$q^h_B = L - X_B$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two ATMs are a bit far apart. Prices are set such that $A$’s customers to the right of $B$ visit $B$, and $B$’s customers to the left of $A$, and part of $B$’s customers in the segment, $AB$, visit $A$. All other customers go to their own home bank.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Region 5</td>
<td>$S^<em>_A + F^</em>_B = (X_B - X_A) t$</td>
<td>$S^*_B = \frac{2L - (X_A + X_B) t - n + 3(c-1)}{3}$</td>
<td>$q^h_A = X_A$</td>
</tr>
<tr>
<td>$F^*_A = \frac{2L - (X_A + X_B) t + 2n - 3(c-1)}{3}$</td>
<td>$q^h_B = \frac{2L - (X_A + X_B) t - n}{6 \epsilon}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two ATMs are a bit far apart. Prices are set such that $B$’s customers to the right of $A$ visit $A$, and $A$’s customers to the left of $B$, and part of $A$’s customers in the segment, $AB$, visit $B$. All other customers go to their own home bank.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Region 3</td>
<td>$S^*_A = \frac{(X_A + X_B) t - n + 3(c-1)}{3}$</td>
<td>$S^*_B = \frac{2L - (X_A + X_B) t - n + 3(c-1)}{3}$</td>
<td>$q^h_A = \frac{(X_A + X_B) t - n}{6 \epsilon}$</td>
</tr>
<tr>
<td>$F^*_B = \frac{(X_A + X_B) t + 2n - 3(c-1)}{3}$</td>
<td>$F^*_A = \frac{2L - (X_A + X_B) t + 2n - 3(c-1)}{3}$</td>
<td>$q^h_B = \frac{2L - (X_A + X_B) t - n}{6 \epsilon}$</td>
<td></td>
</tr>
<tr>
<td>Two ATMs are quite far apart. This is the region where prices are set such that $A$’s customers to the left of $B$, and part of $A$’s customers in the segment, $AB$, visit $B$, and $B$’s customers to the left of $A$, and part of $B$’s customers in the segment, $AB$, visit $A$. All other customers go to their own home bank.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
it can from its competitor by lowering its surcharge to earn more interchange fee. Therefore, the foreign fee (surcharge) should increase (decrease) in the interchange fee.

3.2 Location game in Stage 1

In Stage 1, taking into account banks’ equilibrium reaction functions of surcharges and foreign fees in Stage 2, banks simultaneously set the locations of their own ATMs. Note from Table 3 that when the closeness of the locations of ATMs falls in Regions 1, 2, 4 or 5, there exist multiple equilibria for prices. In Region 1, due to the fact that \(q_A^b = q_B^a = 0\), we always have \(\Pi_A = \Pi_B = -c \cdot L\), regardless of the equilibrium values of prices. Thus, the existence of multiple equilibria is not an issue in studying the location problem in Stage 1. In Region 2, or 4, or 5, the customers’ demand function, which is affected by the multiple price equilibria, is not zero. This implies that different price equilibria would result in different profit allocation between the two banks. Thus, we need to set a rule to single out a price equilibrium to continue the location problem. Note that in all of these three regions, the sum of banks A and B’s profits which are affected by the multiple price equilibria is always a fixed constant. For example, in Region 4, there are multiple equilibria for \((F_A^*, S_B^*)\) satisfying \(S_B^* + F_A^* = (X_B - X_A)t\). Bank A’s profit affected by \((F_A^*, S_B^*)\) is \((F_A^* - n + c - I)q_B^a\), and bank B’s profit affected by \((F_A^*, S_B^*)\) is \((S_B^* - c + I)q_B^a\), where from Table 3, \(q_B^a = L - X_B\). Thus, their sum is \(((X_B - X_A)t - n)(L - X_B)\), for any set of \((F_A^*, S_B^*)\) of all possible multiple equilibria. Therefore, we can conclude that the question on how to single out an equilibrium set of surcharges and foreign fees becomes a typical problem of profit splitting in a fixed-sum game with two players. In a system with two symmetric players, a half-half profit splitting is the most popular profit sharing rule in game theory and it can be justified by various solution concepts, e.g., the Nash bargaining solution (Nash, 1950), and the Rubinstein’s alternating offer solution (Rubinstein, 1982). Consistent with convention, we make the following assumption to identify a unique equilibrium set of surcharges and foreign fees in the case with multiple equilibria.

**Assumption 1** (Equal Split) When there are multiple equilibria for surcharges and foreign fees for the problem in Stage 2 under given locations of ATMs, we assume that banks A and B will choose a set of equilibrium prices which would split equally the total profit affected by the multiple equilibria.
Based on this assumption, we can immediately derive the following result.

**Corollary 1** If there are multiple equilibria for surcharges and foreign fees and they satisfy $S_i^* + F_j^* = (X_B - X_A)t$ for $i \neq j$ and $i, j \in \{A, B\}$, then there exists a unique equilibrium of prices, $(S_i^*, F_j^*)$, in order to result in an equal profit splitting between the two banks. The unique set of prices satisfies $S_i^* = \frac{(X_B - X_A)t + 2(c - I) - n}{2}$ and $F_j^* = \frac{(X_B - X_A)t - 2(c - I) + n}{2}$.

The set of prices given in Corollary 1 implies that when a consumer decides to transact at a non-deposit bank, it would generate the same amount profit margin to its own bank (through a foreign fee) and to the non-deposit bank (through a surcharge fee). That is, $S_i^* - c + I = F_j^* - n + c - I$.

Given the equilibrium prices presented in Table 3 and Corollary 1, we characterize the equilibrium locations in Stage 1 in the following proposition.

**Proposition 2** *(Location Equilibrium)* There exist multiple location equilibria in Stage 1, and they are in Region 2 and Region 3 only. More specifically:

1. **The location equilibria in Region 2 are characterized in the following three cases:**

   a. The location equilibrium, $(X_A^*, X_B^*)$, satisfies $X_B^*t = 5X_A^*t + n$ and $X_A^* \leq \frac{L - t - n}{8t}$. Among all of these location equilibria, $(X_A^* = \frac{L - t - n}{8t}, X_B^* = \frac{5L - t + 3n}{8t})$ maximizes both banks’ profits in equilibrium, and their maximum profits are $\Pi_A^* = \Pi_B^* = \frac{(L - t - n)^2}{8t} - c \cdot L$.

   b. The location equilibrium, $(X_A^*, X_B^*)$, satisfies $2X_B^*t = L \cdot t + 2X_A^*t + n$ and $X_A^* \in [\frac{L - t - n}{8t}, \frac{3L - t - 3n}{8t}]$. Further, both banks would realize the exactly same profit across all of these location equilibria, and their equilibrium profits are $\Pi_A^* = \Pi_B^* = \frac{(L - t - n)^2}{8t} - c \cdot L$.

   c. The location equilibrium, $(X_A^*, X_B^*)$, satisfies $5X_B^*t = 4L \cdot t + X_A^*t + n$ and $X_A^* \geq \frac{3L - t - 3n}{8t}$. Among all of these location equilibria, $(X_A^* = \frac{3L - t - 3n}{8t}, X_B^* = \frac{7L - t + n}{8t})$ maximizes both banks’ profits in equilibrium, and their maximum profits are $\Pi_A^* = \Pi_B^* = \frac{(L - t - n)^2}{8t} - c \cdot L$.

2. **The location equilibrium in Region 3 is either $(X_A^* = 0, X_B^* = \frac{4L - t + n}{5t})$ or $(X_A^* = \frac{L - t - n}{5t}, X_B^* = L)$. Both banks realize equal profits at these two locations, $\Pi_A^* = \Pi_B^* = \frac{26(L - t - n)^2}{225t} - c \cdot L$.

Proposition 2 indicates that for location equilibria in Regions 2 and 3, banks would always like to locate their ATMs with a certain distance in between. This is quite intuitive since if two ATMs
are located at the exactly same spot, then neither of the banks would be able to generate any revenues from surcharges and foreign fees because all customers would use their home banks for ATM transactions. However, it is quite surprising that it is possible for asymmetric locations to emerge in equilibrium in both Regions 2 and 3.

Given various multiple location equilibria for banks to choose, a natural question is which one of them is most likely to happen. Based on banks’ equilibrium profits in all scenarios presented in Proposition 2, we address this question in the next finding.

**Corollary 2 (Dominating Location Equilibrium)** From both banks’ point of view, in Region 2, the location equilibrium, \((X_A^*, X_B^*)\), satisfying
\[
2X_B^* t = L \cdot t + 2X_A^* t + n \quad \text{and} \quad X_A^* \in \left[\frac{L - n}{2t}, \frac{3L - 3n}{2t}\right],
\]
dominates all other equilibria in this region. Furthermore, this set of dominating location equilibria would also strictly dominate the two possible equilibria in Region 3.

By Corollary 2, we can conclude that the location equilibria, which satisfies
\[
2X_B^* t = L \cdot t + 2X_A^* t + n \quad \text{and} \quad X_A^* \in \left[\frac{L - n}{2t}, \frac{3L - 3n}{2t}\right],
\]
are the dominating equilibria and it is most likely to be adopted by both banks in equilibrium. Note that this set of location equilibria is in Region 2.

Finally, it is worth mentioning that the equilibrium locations of ATMs in both Regions 2 and 3 are independent of the transaction cost \(c\) and the interchange fee \(I\). The reason is due to the fact that the effects of \(c\) and \(I\) on surcharge and foreign fees offset each other (see the discussion in §3.1 for more details). It further leads to the conclusion that the equilibrium profits of both banks are independent of the interchange fee.

### 3.3 Consumers’ total cost

In this section, we examine consumers’ total cost at different equilibrium locations. Recall that customers are assumed to be uniformly distributed on the line segment \(L\), and that consumers’ surplus consists of the total traveling cost and possible fees that a customer pays when making ATM transactions at the foreign bank’s ATM. From Corollary 2, we only consider the dominating location equilibria for both banks, which satisfies
\[
2X_B^* t = L \cdot t + 2X_A^* t + n \quad \text{and} \quad X_A^* \in \left[\frac{L - n}{2t}, \frac{3L - 3n}{2t}\right].
\]
From Corollary 1, we have
\[
S_A^* + F_B^* = S_B^* + F_A^* = \frac{Lt + n}{2}.
\]
(1) Bank B’s customers residing at $[0, X_A^*]$ will go to bank A. Their total cost is

$$TC_1^b = (S_A^* + F_B^*) X_A^* + \int_0^{X_A^*} (X_A^* - x) t \cdot 1 dx = \frac{L \cdot t + n}{2} X_A^* + \frac{t}{2}(X_A^*)^2.$$ 

(2) Bank B’s customers residing at $[X_A^*, X_B^*]$ will go to bank B. Their total cost is

$$TC_2^b = \int_{X_A^*}^{X_B^*} (X_B^* - x) t \cdot 1 dx = \frac{t}{2}(X_B^* - X_A^*)^2.$$ 

(3) Bank B’s customers residing at $[X_B^*, L]$ will go to bank B. Their total cost is

$$TC_3^b = \int_{X_B^*}^{L} (x - X_B^*) t \cdot 1 dx = \frac{t}{2}(L - X_B^*)^2.$$ 

Therefore, bank B’s customers’ total cost is $TC_1^b + TC_2^b + TC_3^b$. Similarly, we can derive the three cost components for bank A’s customers, and they are $TC_1^a = \int_0^{X_A^*} (X_A^* - x) t \cdot 1 dx = \frac{t}{2}(X_A^*)^2$, $TC_2^a = \int_{X_A^*}^{X_B^*} (x - X_A^*) t \cdot 1 dx = \frac{t}{2}(X_B^* - X_A^*)^2$, and $TC_3^a = (S_B^* + F_A^*)(L - X_B^*) + \int_{X_B^*}^{L} (x - x_B^*) t \cdot 1 dx = \frac{L \cdot t + n}{2}(L - X_B^*) + \frac{t}{2}(L - X_B^*)^2$. Thus, by adding banks A and B’s customers’ total cost, we have the consumers’ total cost, which is presented in the following proposition.

**Proposition 3** (Consumers’ Total Cost) Given the dominating location equilibria in Region 2, i.e., $(X_A^*, X_B^*)$ satisfies $2X_B^* t = L \cdot t + 2X_A^* t + n$ and $X_A^* \in \left[\frac{L \cdot t - n}{4t}, \frac{3L \cdot t - 3n}{8t}\right]$, the consumers’ total cost is

$$TC = \frac{L \cdot t + n}{2}(L - X_B^* + X_A^*) + (X_A^*)^2 t + (X_B^* - X_A^*)^2 t + (L - X_B^*)^2 t.$$ 

Further, the consumers’ total cost function is convex in $X_A^*$ or $X_B^*$, and is minimized at a symmetric location, $(X_A^* = \frac{L \cdot t - n}{4t}, X_B^* = \frac{3L \cdot t - n}{4t})$, and the minimum consumers’ total cost is $\frac{5L^2 t + 2L t \cdot n + n^2}{8t}$. 

Note that if we use a similar approach to calculate the consumers’ total cost under the two possible location equilibria in Region 3, we can derive the consumers’ total cost, $TC = \frac{37L^2 t^2 + 7L t \cdot n + n^2}{45t}$. It can be verified that the total cost in Region 3 is even greater than the maximum consumers’ total cost in Region 2. Thus, consumers’ preference of Region 2 over Region 3 is consistent with the banks’ preference. That is, both banks and their customers would prefer the location equilibrium in Region 2. Further, there exists a unique symmetric location equilibrium in Region 2, $(X_A^* = \frac{L \cdot t - n}{4t}, X_B^* = \frac{3L \cdot t - n}{4t})$, which would maximize both banks’ profits, and at the same time,
minimize their consumers’ total cost. In view of this observation, we will use this symmetric location equilibrium and the corresponding values of other decisions and profits to make a comparison between the unregulated and regulated models in §5.

Finally, since the effects of \( c \) and \( I \) on the foreign fee and surcharge offset each other, it is not surprising to observe that the total fees paid by consumers, including their traveling cost, are independent of \( c \) and \( I \).

4. Regulated Model

In the regulated model, we assume that regulation precludes the use of surcharges. Thus, we set \( S_A \) and \( S_B \) to zero in this section. The profit functions of both banks, given in (5) and (6), reduce to:

\[
\Pi_A = (F_A - n + c - I)q_A^b - c \cdot L \quad \text{and} \quad \Pi_B = (F_B - n + c - I)q_B^a - c \cdot L,
\]

where \( q_A^b \) and \( q_B^a \) are given by (3) and (4) with \( S_A = S_B = 0 \), respectively.

Following the analysis of the pricing game in the unregulated model studied in §3.1, we present the equilibrium foreign fees in the following lemma.

**Lemma 1** *(Pricing Equilibrium under Given Locations and Regulation)* In the regulated model with precluded surcharges, for any given ATM locations, \( X_A \) and \( X_B \), where \( 0 \leq X_A \leq X_B \leq L \), the equilibrium foreign fees, \( F_A^* \) and \( F_B^* \), can be characterized as follows.

1. Bank A’s foreign fee is:

\[
F_A^* = \begin{cases} 
\frac{2L - (X_A + X_B)t + n - c + I}{2} & \text{if } 0 \leq A_1, \\
(X_B - X_A)t & \text{if } A_1 \leq 0 \leq A_2, \\
(X_B - X_A)t + \epsilon & \text{if } A_2 \leq 0,
\end{cases}
\]

where \( A_1 = (3X_B - X_A - 2L)t - n + c - I \) and \( A_2 = (X_B - X_A)t - n + c - I \).

2. Bank B’s foreign fee is:

\[
F_B^* = \begin{cases} 
\frac{(X_A + X_B)t + n - c + I}{2} & \text{if } 0 \leq B_1, \\
(X_B - X_A)t & \text{if } B_1 \leq 0 \leq B_2, \\
(X_B - X_A)t + \epsilon & \text{if } B_2 \leq 0,
\end{cases}
\]

where \( B_1 = (X_B - 3X_A)t - n + c - I \) and \( B_2 = (X_B - X_A)t - n + c - I \).
Taking into account the banks’ reaction functions of the foreign fees in Stage 2, both banks simultaneously choose the locations of their ATMs in Stage 1.

**Proposition 4 (Location Equilibrium under Regulation)** In the regulated model with precluded surcharges, there exists a unique set of locations in equilibrium, namely, \((X_A^* = 0, X_B^* = L)\). Accordingly, in equilibrium, we have:

1. the foreign fees: \(F_A^* = F_B^* = \frac{L \cdot t + n + c + I}{2} \),
2. the number of customers switching to the non-deposit ATM: \(q_A^* = q_B^* = \frac{L \cdot t - n + c - I}{4t} \),
3. both banks’ profits: \(\Pi_A^* = \Pi_B^* = \frac{(L \cdot t + n + c - I)^2}{8t} - c \cdot L \), and
4. the consumers’ total cost: \(TC = \frac{(3L \cdot t + n + c + I)(5L \cdot t - n + c - I)}{8t} - L^2 t \).

Proposition 4 clearly indicates that if surcharges are precluded in practice, banks would locate their ATMs at the two extreme endpoints of the line segment. The intuition is that banks do not have an incentive to serve their own customers through their ATMs, because they incur variable cost. When banks cannot charge surcharges, they have a weaker incentive to serve the other bank’s customers since they receive only the interchange fee. Therefore, the only way to compromise is to locate further apart in order that some of their own customers will use the other bank’s ATM, resulting in revenue from foreign fees for the home bank. Different from the unregulated model wherein the consumers’ total cost is independent of the transaction cost \(c\) and the interchange fee \(I\), in the regulated model, the consumers’ total cost is a function of \(c\) and \(I\).

**Corollary 3 (Sensitivity Analysis with Transaction Cost)** As the ATM variable cost increases, banks’ profit decreases and consumers’ total cost also decreases.

From the above corollary, we see that when the ATM variable cost is large, consumers are better off. This is because when \(c\) is large, banks have an incentive to lower foreign fee so that more customers will use the ATM of the non-affiliated bank. On the other hand, the banks are not able to increase surcharge to offset the increase in the variable cost. Therefore, as the variable cost \(c\) increases, banks are worse off while the consumers are better off.
5. Comparison between Unregulated and Regulated Models

In this section we compare the unregulated and regulated models. Recall from the analysis in the unregulated model in §3 that both the banks and their customers prefer the location equilibria in Region 2 over those in Region 3. Further, there exists a unique symmetric location equilibrium in Region 2, \((X_A^* = \frac{L \cdot t - n}{4}, X_B^* = \frac{3L + n}{4})\), which maximizes both banks’ profits and minimizes their consumers’ total cost (Corollary 2 and Proposition 3). Thus, in this section, we would compare the equilibrium values under this location equilibrium in the unregulated model with the values in the regulated model, unless otherwise noted. We consider the following three aspects: (i) the distance between the equilibrium locations of two ATMs, (ii) an individual bank’s equilibrium profit and the total profit of both banks, and (iii) consumers’ total cost.

(i) Distance of locations of ATMs. Let \(X_{AB}^U\) and \(X_{AB}^R\) be the distance between the equilibrium locations of two ATMs in the unregulated and regulated systems, respectively. Given \((X_A^* = \frac{L \cdot t - n}{4}, X_B^* = \frac{3L + n}{4})\) in the unregulated model, the distance between the locations of two ATMs\(^2\) is: \(X_{AB}^U = \frac{L \cdot t + n}{2t}\). Note from the location analysis of the regulated system in Section 4 (Proposition 4) that the distance between the unique equilibrium of locations of ATMs in the regulated system is: \(X_{AB}^R = L\). Immediately, we have the following result:

**Corollary 4** In equilibrium, \(X_{AB}^R > X_{AB}^U\).

Corollary 4 suggests that banks would place their ATMs closer together when they are allowed to levy surcharges and ATMs tend to be located further apart in regulation. The reason is as follows: In either case, banks do not have an incentive to place the ATMs next to each other. In the unregulated case, banks are better off serving foreign customers or having their own customers served by the other bank’s ATM. Under such circumstances, banks locate their ATMs in a “collusive” manner without actually colluding. They maintain a certain distance in between. In this way, they leave a large number of one bank’s customers nearer the other bank’s ATM, allowing this bank to benefit the most from foreign fees and the other bank to benefit the most from surcharges. Under regulation,

\(^2\)Note that the distance between any equilibrium location \(X_B^*\) and \(X_A^*\) in Region 2 when banks choose the locations to maximize their profits remains at \(\frac{L \cdot t + n}{2t}\), regardless of the consumers’ total cost.
banks do not have an incentive to serve either bank’s customers. Therefore, the only way to push more of its own customers to the other bank, and to serve fewer foreign customers is to locate at extreme points.

(ii) Banks’ profits. Recall that in both the regulated and unregulated models, under any equilibrium location of ATMs, both banks would always realize equal profits. Thus, the relationship between the individual banks’ profit in the regulated and unregulated models also represents that of the total profit of both banks. Denote by $\Pi^U$ and $\Pi^R$ an individual bank’s equilibrium profit in the unregulated and regulated models, respectively.

**Proposition 5 (Higher Profit under Regulation)** In equilibrium, $\Pi^R > \Pi^U$.

It is quite surprising to observe that regulations banning surcharges would always benefit both banks, compared to their profits under the unregulated system. We note that this result is consistent with a similar result in Massoud and Bernhardt (2002). A possible explanation is that allowing banks to surcharge creates another source of price competition between both banks, in addition to foreign fee competition. The resulting excess of competition results in lower total bank profits.

(iii) Customers’ total cost. Denote by $TC^U$ and $TC^R$ the customers’ total cost in the unregulated and regulated models, respectively. The customers’ total costs in equilibrium in both models depend on the model parameters, e.g., the traveling cost, $t$, and the transaction cost, $c$. The next proposition captures the conditions under which customers are better off in either the regulated or unregulated model.

**Proposition 6 (Consumers’ Preference)** Denote by $\bar{c} = \sqrt{3L^2t^2 - 2L \cdot t \cdot n - n^2 - L \cdot t + n + I} > 0$.

In equilibrium, $TC^R > TC^U$ for $c < \bar{c}$, $TC^R < TC^U$ for $c > \bar{c}$, and $TC^R = TC^U$ for $c = \bar{c}$.

From Corollary 3 we know that under regulation, $TC^R$ decreases in $c$. In the unregulated case, $TC^U$ is independent of $c$, because its effect on $TC^U$ is canceled out by foreign fees and surcharges. Therefore, for example, when $c$ is large and above a certain value $\bar{c}$, the corresponding decrease in the consumers’ total cost under regulation results in consumers being better off compared to the unregulated situation.
6. Concluding Remarks

In this paper we have studied two competing banks in a linear market that decide where to locate their ATMs and then set their prices, including foreign fees and surcharges, if applicable. We have analyzed and compared two models: the unregulated model in which banks are allowed to levy surcharges and the regulated model wherein the surcharges are banned. It has been shown that in both models, banks would always place their ATMs with a certain distance in between to avoid severe price competition and to generate revenues from surcharges and foreign fees. Further, when surcharges are banned, ATMs would be located at the endpoints of the linear market, resulting in the least convenient location arrangement from the viewpoint of the consumers’ total travel cost. In contrast, ATMs in the unregulated model would be closer to each other than those in the regulated model and with possibly asymmetric locations.

One of the main objectives of this paper was to examine the effect of surcharge bans on the banks’ profits and the consumers’ total cost. In this regard, we have shown a surprising result that banks would actually always generate a higher profit when the surcharges are banned, while the consumers could be worse off, relative to the case with the presence of surcharges. It is consistent with the result derived in Massoud and Bernhardt (2002), wherein a circular market is considered and two ATMs are fixed at each end of the diameter of the circle. In their paper, they incorporate the consumers’ deposit decisions, which may lead to different bank sizes. As we stated in the introduction, the banning of surcharges in the model would completely change the pricing dynamics and competition dimensions of both banks, which leads to a beneficial outcome for banks and a possibly detrimental effect on the consumers’ total cost.

There are several limitations of the paper. We have assumed that there is only one ATM for each bank. It would be interesting to consider the situation when banks can choose how many ATMs to install in the market. To facilitate the analysis of the location problem, we have also assumed that both banks are symmetric in terms of the density of their consumers on the market. In view of the existence of heterogeneity of banks in practice, it would be important to extend the model to the case when banks could be asymmetric in the sense that one of them may have a higher density of consumers than that of the other. Finally, we have assumed that banks locate
their ATMs simultaneously. It is also interesting to consider the scenario when banks can locate their ATMs sequentially.

References


Appendix

**Proof of Proposition 1.** Given locations of ATMs, $X_A$ and $X_B$, both banks choose the surcharge and foreign fees simultaneously. Assuming that bank $B$’s prices $S_B$ and $F_B$ are given, we consider
bank A’s problem of setting \( S_A \) and \( F_A \) to maximize its profit:

\[
\Pi_A = (S_A - c + I) q_A^b + (F_A - n + c - I) q_B^0 - c \cdot L,
\]

where \( q_A^b \) and \( q_B^0 = L - q_A^b \) are given as follows:

\[
q_A^b = \begin{cases} 
0 & \text{if } S_A + F_B \geq (X_B - X_A)t + \epsilon, \\
\frac{(X_A + X_B)}{2} - \frac{S_A + F_B}{2t} & \text{if } S_A + F_B \leq (X_B - X_A)t \text{, and }
\end{cases}
\]

\[
q_B^0 = \begin{cases} 
0 & \text{if } S_B + F_A \geq (X_B - X_A)t + \epsilon, \\
\frac{2L - (X_A + X_B) - S_B + F_A}{2t} & \text{if } S_B + F_A \leq (X_B - X_A)t.
\end{cases}
\]

Clearly, the optimal value of \( S_A \) depends on \( F_B \), but not on \( S_B \), and it can be derived from the first term of \( \Pi_A \). Similarly, the optimal value of \( F_A \) depends on \( S_B \), but not on \( F_B \), and it can be derived from the second term of \( \Pi_A \). Let us first solve for the optimal \( S_A \) to maximize \((S_A - c + I) q_A^b \). Two cases are considered: (i) \( S_A + F_B > (X_B - X_A)t \) and (ii) \( S_A + F_B \leq (X_B - X_A)t \).

(i) For \( S_A + F_B \geq (X_B - X_A)t + \epsilon, q_A^b = 0 \), which leads to \((S_A - c + I) q_A^b = 0 \), for any \( S_A \). For simplicity, let us assume that \( S_A + F_B = (X_B - X_A)t + \epsilon \).

(ii) For \( S_A + F_B \leq (X_B - X_A)t \), \( S_A \) is set to maximize \((S_A - c + I) \left( \frac{(X_A + X_B) - S_A + F_B}{2} \right) \). It is a constrained optimization problem with a concave objective function and a linear constraint. With simple algebra, let \( A_{1s} = (X_B - 3X_A)t - c + I \), we can derive:

\[
S_A = \begin{cases} 
\frac{(X_A + X_B)t - F_B + c - I}{2} & \text{if } F_B \leq A_{1s}, \\
(X_B - X_A)t - F_B & \text{if } F_B \geq A_{1s}.
\end{cases}
\]

Comparing cases (i) and (ii), we characterize the optimal value of \( S_A \) for any given \( F_B \):

\[
S_A(F_B) = \begin{cases} 
\frac{(X_A + X_B)t - F_B + c - I}{2} & \text{if } F_B \leq A_{1s}, \\
(X_B - X_A)t - F_B & \text{if } A_{1s} \leq F_B \leq A_{2s}, \\
(X_B - X_A)t - F_B + \epsilon & \text{if } F_B \geq A_{2s},
\end{cases}
\]

where \( A_{2s} = (X_B - X_A)t - c + I \). Note that \( A_{1s} \leq A_{2s} \).

Using a similar approach, we can derive the reaction function \( F_A(S_B) \) accordingly, which is presented below:

\[
F_A(S_B) = \begin{cases} 
\frac{2Lt - (X_A + X_B)t - S_B + n - c + I}{2} & \text{if } S_B \leq A_1, \\
(X_B - X_A)t - S_B & \text{if } A_1 \leq S_B \leq A_2, \\
(X_B - X_A)t - S_B + \epsilon & \text{if } S_B \geq A_2.
\end{cases}
\]
where \( A_1 = (3X_B - X_A - 2L) t - n + c - I \), \( A_2 = (X_B - X_A) t - n + c - I \), and \( A_1 \leq A_2 \).

Due to symmetry of bank \( B \)'s problem of choosing optimal values of \( S_B \) and \( F_B \) to bank \( A \)'s \( S_A \) and \( F_A \), respectively, we can easily write the optimal value of \( S_B \) as a function of \( F_A \). To obtain \( S_B(F_A) \), we only need to replace \( X_A \) and \( X_B \) with \( L - X_B \) and \( L - X_A \), respectively, in \( S_A(F_B) \) given by (14). Accordingly, we have:

\[
S_B(F_A) = \begin{cases} 
\frac{2Lt - (X_A + X_B)t - F_A + c - I}{2} & \text{if } F_A \leq B_{1s}, \\
(X_B - X_A)t - F_A & \text{if } B_{1s} \leq F_A \leq B_{2s}, \\
(X_B - X_A)t - F_A + \epsilon & \text{if } F_A \geq B_{2s},
\end{cases}
\]

(16)

where \( B_{1s} = (3X_B - X_A - 2L) t - c + I \), \( B_{2s} = (X_B - X_A) t - c + I \), and \( B_{1s} \leq B_{2s} \).

By solving equations (15) and (16) simultaneously, we are able to derive the equilibrium surcharge and foreign fees for both banks under a given set of locations of ATMs. These equilibrium prices can be characterized in five different regions structured by different values of \( X_A \) and \( X_B \), and these regions are presented in Figure 2. By simple algebra, the expressions of the equilibrium prices can be computed and they are summarized in Table 3.

**Proof of Corollary 1.** Let us take Region 4 as an example to show this result. The proof in Regions 2 and 5 are analogous to that in Region 4.

In Region 4, the multiple equilibria of prices satisfy \( S_B^* + F_A^* = (X_B - X_A)t \). The total profit of both banks \( A \) and \( B \) which is affected by the multiple equilibria is

\[
(F_A^* - n + c - I)q_B^* + (S_B^* - c + I)q_B^* = ((X_B - X_A)t - n)(L - X_B),
\]

(17)

where the first term on the left-hand-side of the equation is bank \( A \)'s profit from its own customers who switch to bank \( B \) for ATM services, and the second term is bank \( B \)'s profit from serving bank \( A \)'s customers. To satisfy Assumption 1, i.e., to equally split the sum of the profit, given in (17), between the two banks, \( F_A^* \) has to satisfy \( F_A^* - n + c - I = \frac{1}{2}((X_B - X_A)t - n) \), which leads to \( F_A^* = \frac{1}{2}[(X_B - X_A)t - 2(c - I) + n] \). Similarly, we can derive \( S_B^* = \frac{1}{2}[(X_B - X_A)t + 2(c - I) - n] \). \( \Box \)

**Proof of Proposition 2.** Note that the equilibrium prices under a given set of ATM locations \((X_A, X_B)\) are classified into five different regions (see Figure 2). Thus, we consider the location problem in each of these five regions.
Region 1: In this region, we have \((X_B - X_A)t \leq n\). For any given value of \(X_A\), let us consider bank \(B\)'s decision on \(X_B\). If it is set such that \((X_B - X_A)t \leq n\), then \(\Pi_B = -c \cdot L\), which is the least possible profit that bank \(B\) can ever realize in equilibrium. Apparently, bank \(B\) has an incentive to set a value for \(X_B\) such that \((X_B - X_A)t > n\), under which its profit will be strictly greater than \(-c \cdot L\). Thus, we can conclude that there will no equilibrium for the locations.

Region 2: In this region, we have \((5X_B - X_A)t \leq 4Lt + n\), \((X_B - 5X_A)t \leq n\), and \((X_B - X_A)t \geq n\). Substituting the equilibrium prices in Corollary 1 and demand functions in Table 3 into banks \(A\) and \(B\)'s profit functions, given in (5) and (6), respectively, we have

\[
\Pi_A = \Pi_B = \frac{1}{2}(L - (X_B - X_A))((X_B - X_A)t - n) - c \cdot L. \tag{18}
\]

The equal profit for both \(A\) and \(B\) stems from the equal profit splitting rule made in Assumption 1. Clearly, each bank’s profit function is quadratic and concave in its corresponding location decision and one can easily derive the best reaction functions, \(X_A^*(X_B)\) and \(X_B^*(X_A)\), which are listed below.

\[
X_A(X_B) = \begin{cases} 
\frac{X_Bt-n}{5t} & \text{if } X_B \leq \frac{5Lt+3n}{8t}, \\
\frac{2X_Bt-Lt-n}{2t} & \text{if } X_B \leq \frac{7Lt+n}{8t}, \\
\frac{5X_Bt-4Lt-n}{t} & \text{otherwise, and,}
\end{cases} \tag{19}
\]

\[
X_B(X_A) = \begin{cases} 
\frac{5X_At+n}{t} & \text{if } X_A \leq \frac{Lt-n}{8t}, \\
\frac{Lt+2X_At+n}{2t} & \text{if } X_A \leq \frac{3Lt-3n}{8t}, \\
\frac{4Lt+X_At+n}{5t} & \text{otherwise.}
\end{cases} \tag{20}
\]

Solving these two reaction functions jointly we can obtain the equilibrium values of locations in Region 2. The location equilibrium can be characterized into three different cases based on the ranges of \(X_A\) and \(X_B\). For example, if \(X_A \leq \frac{Lt-n}{8t}\) and \(X_B \leq \frac{5Lt+3n}{8t}\) (say, case (a)), then we have multiple location equilibria as long as the equation \(5X_At = X_Bt - n\) is satisfied. Among all of these multiple equilibria, one can easily verify that \((X_A^* = \frac{Lt-n}{8t}, X_B^* = \frac{5Lt+3n}{8t})\) maximizes both banks’ profits, given in (18), and the maximum profit is \(\frac{(Lt-n)^2}{8t} - c \cdot L\). Following the same analysis procedure, we can similarly derive location equilibria for case (b) and case (c) in Proposition 2. Note that in case (b), both banks realize the same profit across all possible location equilibria, and
in case (c), both banks’ profits are maximized at \((X_A^* = \frac{3Lt-3n}{8t}, X_B^* = \frac{7Lt+n}{8t})\), and the maximum profit is \(\frac{(L-t-n)^2}{8t} - c \cdot L\).

Region 3: In this region, we have \((5X_B - X_A) t \geq 4Lt + n \) and \((X_B - 5X_A) t \geq n\). Substituting the equilibrium prices and demand functions, given in Table 3, into banks A and B’s profit functions, given in (5) and (6), respectively, we obtain

\[
\Pi_A = \Pi_B = \frac{1}{18t}((X_A + X_B)t - n)^2 + \frac{1}{18t}((2Lt - (X_A + X_B)t - n)^2 - c \cdot L. \tag{21}
\]

Note that in this region, both banks always realize an equal profit, regardless of the locations of the ATMs. Note further that the profit function is quadratic and convex in location decisions. Since Region 3 is compact and convex, with some algebra, one can derive the optimal location reaction functions, \(X_A^*(X_B)\) and \(X_B^*(X_A)\). Solving these two reaction functions simultaneously, we can obtain the two possible equilibria. They are \((X_A^* = 0, X_B^* = \frac{4Lt+n}{8t})\) and \((X_A^* = \frac{Lt-n}{8t}, X_B^* = L)\).

Region 4: In this region, we have \((5X_B - X_A) t \leq 4Lt + n \) and \((X_B - 5X_A) t \geq n\). Given the equilibrium prices in Table 3 and Corollary 1, banks’ profit functions can be written as

\[
\Pi_A = \Pi_B = \frac{((X_A + X_B)t - n)^2}{18t} + \frac{1}{2}((X_B - X_A)t - n)(L - X_B) - c \cdot L. \tag{22}
\]

Note that bank A’s profit function is quadratic and convex in \(X_A\). Thus, for any given \(X_B\), the profit maximizing \(X_A\) for bank A is at either of the two extreme points of the region. That is that the optimal reaction function, \(X_A^*(X_B)\), is set such that either \((5X_B - X_A) t = 4Lt + n\) or \((X_B - 5X_A) t = n\). If the former is satisfied, the problem reduces to that in Region 3. If the latter condition happens, the problem reduces to that in Region 2.

Region 5: What happens in this region is similar to that in Region 4. In this region, we have \((5X_B - X_A) t \geq 4Lt + n \) and \((X_B - 5X_A) t \leq n\). Taking into account the equilibrium prices in Table 3 and Corollary 1, banks’ profit functions can be written as

\[
\Pi_A = \Pi_B = \frac{(2Lt - (X_A + X_B)t - n)^2}{18t} + \frac{1}{2}((X_B - X_A)t - n)X_A - c \cdot L. \tag{23}
\]

Note that bank B’s profit function is quadratic and convex in \(X_B\). Thus, for any given value of \(X_A\), bank B’s optimal reaction function of \(X_B^*(X_A)\) is chosen so that either \((5X_B - X_A) t = 4Lt + n\) or \((X_B - 5X_A) t = n\). If the former occurs, then the problem coincides with that in Region 2. If the latter happens, then the problem reduces to that in Region 3. □
Proof of Proposition 3. By adding the cost of banks A and B’s customers calculated right before Proposition 3, we immediately have the consumers’ total cost function,

\[ TC = \frac{L \cdot t + n}{2} (L - X_B^* + X_A^*) + (X_B^*)^2 t + (X_B^* - X_A^*)^2 t + (L - X_B^*)^2 t. \]  

Since \( 2X_B^* t = L \cdot t + 2X_B^* t + n \), substituting \( X_B^* \) into the cost function above, we have \( TC = \frac{4X_A^* n t + 8 t^2 (X_A^*)^2 + 3 L^2 t^2 + n^2 - 4 X_A^* L t^2}{4t} \), which is convex in \( X_A^* \) and minimized at \( X_A^* = \frac{L t - n}{4 t} \). Accordingly, \( X_A^* = \frac{3L - n + n}{4t} \) and the minimum total cost is \( \frac{5(L t)^2 + 2L t n + n^2}{8t} \). □

Proof of Lemma 1. We follow the analysis of the pricing game in the unregulated model to prove this result. Note that the difference between the regulated and unregulated models is that in the regulated model, the surcharges, \( S_A \) and \( S_B \), are both zero, while they are decisions in the unregulated model. By substituting \( S_B = 0 \) into equation (15), we immediately have bank A’s optimal foreign fee, for any given locations of ATMs, which is:

\[
F_A^* = \begin{cases} \frac{2L - (X_A + X_B) t + n - c + I}{2} & \text{if } 0 \leq A, \\ (X_B - X_A) t & \text{if } A_1 \leq 0 \leq A_2, \\ (X_B - X_A) t + \epsilon & \text{if } A_2 \leq 0, \end{cases}
\]  

where \( A_1 = (3X_B - X_A - 2L) t - n + c - I \) and \( A_2 = (X_B - X_A) t - n + c - I \).

By substituting \( X_A \) with \( L - X_B \) and \( X_B = L - X_A \) into equation (25), we derive bank B’s optimal foreign fee, for any given locations of ATMs, which is:

\[
F_B^* = \begin{cases} \frac{(X_A + X_B) t + n - c + I}{2} & \text{if } 0 \leq B_1, \\ (X_B - X_A) t & \text{if } B_1 \leq 0 \leq B_2, \\ (X_B - X_A) t + \epsilon & \text{if } B_2 \leq 0, \end{cases}
\]  

where \( B_1 = (X_B - 3X_A) t - n + c - I \) and \( B_2 = (X_B - X_A) t - n + c - I \). This completes the proof of Lemma 1. □

Proof of Proposition 4. Let us first consider bank A’s reaction. For a given location \( X_B \), Bank A chooses \( X_A \) that maximizes its profit, given in (7). We consider three possible cases for \( X_A \).

(i) If \( X_A \) is set such that \( A_1 = (3X_B - X_A - 2L) t - n + c - I \geq 0 \), then from equation (8) or (25), \( F_A^* = \frac{2L - (X_A + X_B) t + n - c + I}{2} \), which is less than or equal to \( (X_B - X_A) t \), since \( A_1 \leq 0 \). Substituting \( F_A^* \) and \( S_B = 0 \) into \( q_B^a \), given in (4), we have

\[
q_B^a = \frac{2L - ((X_A + X_B))}{2} - \frac{F_A^*}{2t} = \frac{2L t - ((X_A + X_B)) t - n + c - I}{4t}.
\]
Bank A’s corresponding profit becomes

\[
\Pi_A = \frac{(2Lt - n + c - I - (X_A + X_B)t)^2}{8t} - cL,
\]

which is convex and decreasing in \( X_A \), when \( X_A \geq 0 \) and \( X_A \) satisfies the condition that \( A_1 \geq 0 \). Therefore \( X_A = 0 \) is the optimal location for bank A.

(ii) If \( X_A \) is such that \( A_1 = (3X_B - X_A - 2L)t - n + c - I \leq 0 \leq (X_B - X_A)t - n + c - I = A_2 \), then from equation (8) or (25), \( F^*_A = (X_B - X_A)t + \epsilon \). Similarly, substituting \( F^*_A \) and \( S_B = 0 \) into \( q^b_B \), given in (4), we have \( q^b_B = L - X_B \), and Bank A’s profit becomes

\[
\Pi_A = (L - X_B)((X_B - X_A)t - n + c - I) - cL,
\]

which is decreasing in \( X_A \). Therefore, the optimal location for A is \( X_A = 3X_Bt - 2Lt - n + c - I \).

(iii) If \( X_A \) is such that \( A_2 = (X_B - X_A)t - n + c - I \leq 0 \), then from equation (8) or (25), \( F^*_A = (X_B - X_A)t + \epsilon \). Thus, from (4), \( q^b_B = 0 \), and therefore, \( \Pi_A = -cL \).

Comparing the three cases for \( X_A \) above, apparently, the optimal location for bank A is \( X_A = 0 \).

For bank B’s optimal \( X_B \), we can do a similar analysis as that for bank A. We also consider three cases for \( X_B \) in the following.

(i) When \( X_B \) is chosen such that \( B_2 = (X_B - X_A)t - n + c - I \leq 0 \), then from equation (9) or (26), \( F^*_B = (X_B - X_A)t + \epsilon \). Thus, from (3), \( q^b_A = 0 \), and therefore, \( \Pi_B = 0 \).

(ii) When \( X_B \) is such that \( B_1 = (X_B - 3X_A)t - n + c - I \leq 0 \leq (X_B - X_A)t - n + c - I = B_2 \), then from equation (9) or (26), \( F^*_B = (X_B - X_A)t \). Substituting \( F^*_B \) and \( S_A = 0 \) into \( q^b_A \), given in (3), we have \( q^b_A = X_A \). Bank B’s profit becomes

\[
\Pi_B = (X_A)((X_B - X_A)t - n + c - I) - cL,
\]

which is increasing in \( X_B \). Therefore, bank B chooses \( X_B = 3X_At + n - c + I \).

(iii) If \( X_B \) is such that \( B_1 = (X_B - 3X_A)t - n + c - I \geq 0 \), then from equation (9) or (26), \( F^*_B = \frac{(X_A + X_B)t + n - c + I}{2} \). Similarly, substituting \( F^*_B \) and \( S_A = 0 \) into \( q^b_A \), given in (3), we have \( q^b_A = \frac{(X_A + X_B)t - n + c - I}{4t} \). Bank B’s profit is then

\[
\Pi_B = \frac{((X_A + X_B)t - n + c - I)^2}{8t} - cL,
\]
which is convex and increasing in $X_B$, when $X_B$ satisfies the condition that $B_1 \geq 0$. Hence, $X_B = L$.

By comparing the three choices for the value of $X_B$ above, one can conclude that the optimal location for bank $B$ is $X_B^* = L$, for any given location of bank $A$’s ATM.

Therefore, there is a unique location equilibrium for the regulated model which precludes the surcharges. The equilibrium is $(X_A^* = 0, X_B^* = L)$. Given the equilibrium locations, we can derive the prices, demand and profits for both banks, and the consumers’ total cost, $TC^R$, in equilibrium accordingly. It is clear that $A_1 > 0$ and $B_1 > 0$ at $(X_A^* = 0, X_B^* = L)$. Thus, from equations (8) and (9), we have $F_A^* = F_B^* = \frac{Lt+n-c+I}{2}$. Substituting $(F_A^*, F_B^*)$ and $(S_A = 0, S_B = 0)$ into the demand functions, $q_A^b$ and $q_B^b$, given in (3) and (4), we have $q_B^b = q_A^b = \frac{Lt-n+c-I}{4t}$. From the profit functions of both banks, given in (7), we have $\Pi_A^* = \Pi_B^* = \frac{(Lt-n+c-I)^2}{8t} - cL$.

We next calculate the consumers’ total cost. This is analogous to the procedure presented in §3.3 in the unregulated model. Consider first bank $A$’s consumers’ total cost. Since $A$’s ATM is located in the left extreme of the line, and $q_A^b = \frac{Lt-n+c-I}{4t}$, which implies that any $A$’s consumer residing in the region of $[0, L - q_A^b]$ will visit its own deposit bank, i.e., bank $A$, for ATM services. Their cost is the traveling cost: $\int_0^{L-q_A^b} xt \cdot 1 dx$. For consumers residing in the region of $[L - q_A^b, L]$, they would visit the other bank, i.e., bank $B$, for the service. Thus, they need to pay for the transportation and the foreign fee, and their total cost is $\int_{L-q_A^b}^L (F_A^* + (L - x)t) \cdot 1 dx$. Therefore, the total cost of bank $A$’s consumers is the sum of these two costs, which is:

$$\int_0^{L-q_A^b} xt \cdot 1 dx + \int_{L-q_A^b}^L (F_A^* + (L - x)t) \cdot 1 dx = \frac{(3Lt + n - c + I)(5Lt - n + c - I)}{16t} - \frac{L^2t}{2}.$$  

Similarly, we can calculate bank $B$’s consumers’ total cost, which is identical to bank $A$’s consumers’ total cost. Thus, the consumers’ total cost is

$$TC = \frac{(3Lt + n - c + I)(5Lt - n + c - I)}{8t} - L^2t. \quad (27)$$

This completes the proof of Proposition 4. □

**Proof of Corollary 3.** Taking the first-order derivative of $\Pi_A^*$ and $\Pi_B^*$ with respect to $c$, respectively, we have $\frac{\partial \Pi_A^*}{\partial c} = \frac{\partial \Pi_B^*}{\partial c} = -(3Lt + n - c + I)/4t < 0$. Thus, both banks’ equilibrium profits decrease in $c$. Similarly, taking the first-order derivative of $TC^R$ with respect to $c$, we have $\frac{\partial TC^R}{\partial c} = -(Lt - n + c - I)/4t < 0$, which implies that consumers’ total cost decreases in $c$. □
Proof of Proposition 6. Recall that the consumers’ total cost in the unregulated model (Region 2) is given in Proposition 3, and the consumers’ total cost in the regulated model is given in Proposition 4. Recall further that $c > I$ and $Lt > n$. Thus, the difference in the consumers’ total cost between the regulated and the unregulated case can be simplified to:

$$TC^R - TC^U = \frac{2n^2 - 2L^2t^2 + 2Ltc - 2LtI - 2nc + 2nI + c^2 - 2cI - 1}{8t}.$$  

It is easy to verify that the difference in the consumers’ total cost is decreasing in $c$ for $c \geq 0$ since $Lt > n$. It is clear that $(TC^R - TC^U)|_{c=I} = \frac{(Lt+n)(Lt-n)}{4t} > 0$, since $Lt > n$. Thus, there must exist a value, $\bar{c}$, such that $TC^R - TC^U > 0$ when $c < \bar{c}$, $TC^R - TC^U < 0$ when $c > \bar{c}$, and $TC^R - TC^U = 0$ when $c = \bar{c}$. The value of $\bar{c}$ is the unique solution to $TC^R - TC^U = 0$, which satisfies the condition that $c \geq I$, and $\bar{c} = \sqrt{3L^2t^2 - 2L \cdot t \cdot n - n^2} - L \cdot t + n + I > 0$. □