Research on the equilibrium of competitive supply chain in the presence of customer returns

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Abstract: In this paper, we consider a model with two competing supply chain with customer returns. Each of them includes a manufacturer and a retailer facing the same market. In every level, the retailer supplies the products for customers through its manufacturer, and the competition exists in the retail price and return price of the products. At first, the equilibrium solution is analyzed in three competition models, including II (both choose the integrated management) competition model, DI (one chooses the integrated management, and the other chooses the decentralized management) competition model and DD (both choose the integrated management) competition model. From the competitive supply chain of considering customer returns, we find that the integrated management is superior to the decentralized management in DI model. However, as a whole, the equilibrium retail price and profit in DD model are higher than that in II model.

Key words: competitive supply chain, customer returns, equilibrium, price competition

1 Introduction

In the increasingly competitive global environment, the competition between enterprises is developing into the competition between supply chains. Therefore, it has been far from enough for enterprises if they only focus on their own supply chain performance improvement. They tend to get off the optimal strategy if not considering the decision made by competitors, which will result in failure in competition. In order to survive and develop in the fiercely competitive market, a member enterprise of a supply chain, especially the core enterprise, must study the behavior of its rival supply chain members and pertinently make appropriate decisions to oppose. So, it is very necessary for us to study supply chain in the competitive environment.

In recent years, many scholars have done a lot of research on the equilibrium of the two-layer competitive supply chain. Competitive patterns mainly have the DD competitive mode, DI competitive mode and II competitive mode, as well as RR competitive mode (both choose the pattern retailers manage inventory), RV competitive mode (one of the supply chain chooses the
pattern retailers manage inventory, and the other chooses the pattern vendor manage inventory) and VV competitive mode ((both choose the pattern vendor manage inventory), etc.. Moorthy(1988) and McGuire and Staelin(1983) have traditionally approached the problem from the manufacturer’s perspective. Moreover, Choi(1991) studied three non-cooperative games of different power structures between the two manufacturers and the retailer.

In addition, there are a lot of factors investigated to urge the competition, such as the product quantity, price, quality, service level and so on. Banker(1998) considered a competitive model between retailers where demand was the function of price and quality, and studied the quality equilibrium level in three competitive environment. Boyaci and Gallego(2004) considered a customer competitive supply chain model, and examined three scenario, uncoordinated scenario, coordinated scenario and hybrid scenario respectively. They found that coordination was a dominant strategy for both supply chains, but as in the prisoner’s dilemma. Both supply chains were often worse off under the coordinated scenario relative to the uncoordinated scenario. Santanu and Sarmah(2010) analyzed the coordination issues under three different contexts: price competition without channel coordination, price competition with channel coordination and global coordination.

This paper is closely related to literatures on returns policy. At present, the majority of them mainly involved the problems between the upstream firms and downstream firms. Pasternack(1985) first return policies for a seasonal product with stochastic demand under the newsvendor framework. Marvel and Peck(1995), Emmons and Gilbert(1998) extended Pasternack(1985)’ work. Yue(2007) studied the full returns policy’s impact on supply chains with information asymmetry.

Rare literatures involved on customer returns policy. Nevertheless, customer returns policy exists in competitive market. In general, one of reasons on customer returns is because of dissatisfaction with the product, the other is that product bought back can't meet their taste or reach the expected effect.

Trager, L. ( 2000) and Wood S. L. (2001) reports that customers return policy can enhance customers’ purchasing confidence, stimulate consumption demand, and improve the product's market share. However, customer returns will also make retailers’ or manufacturers’ processing cost increase or delayed sales, which will cause profit loss such as commodity devaluation and inventory redundancy. The double-edged sword characteristic of customer returns policy has caused the academic and business’ attention.
Customer returns will affect the firm’s pricing and order decision. Chen and Bell (2009) examined a retailer offering a full refund customer returns policy facing customer returns that were proportional to sales. But we investigate a retailer giving a partial refund customer returns policy.

Our work is different from the above literatures. We consider customer demand is the linear function of retail price and return price, and further assume the retail price is related to the return price. Then we investigate the equilibrium performance of two-layer supply chain in DD, DI and II competitive model respectively. Finally, we compare the equilibrium profits in three competitive models, and analyzed how return parameters in the demand function influenced the supply chain’s equilibrium profit.

In the real market, customer demand fluctuates with prices. Therefore, it is very meaningful for us to analyze the influence of retail price and return price on customer demand in this paper.

The rest of our paper is organized as follows. In the next section, we introduce the notation about the model. Section 3 studies supply chain competition equilibrium results. Section 4 compares the equilibrium results and gives some examples. Section 5 summarizes the results.

2 Model and Notation

We consider two competitive supply chains in the market, and each one includes a manufacturer and a retailer facing a group of customers. We assume that manufacturers set their wholesale price \( w_i \) \((i = 1, 2)\). Customers choose the product in which supply chain depends on the retail price \( p_i \) \((i = 1, 2)\) and return price \( r_i \) \((i = 1, 2)\). We provide \( r_i < w_i < p_i \) \((i = 1, 2)\).

We assume that the retailer’s demand \( D_i(p, r) \) \((i = 1, 2)\) is relative to retail price and return price, and is the linear function of the two.

\[
D_i(p, r) = a - bp_i + cp_j + \xi r_i - \eta r_j.
\]

Further, we assume the return price is proportional to the retail price. That is \( r_i = \beta_i p_i, r_2 = \beta_2 p_2 \).

So, we have

\[
D_1(p, r) = D_1(p) = a - (b - \xi \beta_1)p_1 + (c - \eta \beta_2)p_2, \text{ and}
\]

\[
D_2(p, r) = D_2(p) = a - (b - \xi \beta_2)p_2 + (c - \eta \beta_1)p_1.
\]

Let the parameters \( a, b > 0, c \geq 0, \xi \geq 0, \eta \geq 0 \) are constants. \( a \) is the initial market demand,
and it reflects the products’ whole developing level. \( b \) denotes the demand responsiveness to the supply chain's own retail price, while \( c \) denotes the demand responsiveness to the competitor's retail price. \( \xi \) denotes the demand responsiveness to the supply chain's own return price, while \( \eta \) denotes the demand responsiveness to the competitor's return price. The parameters \( \beta_1, \beta_2 \) are called return factors, and reflect that return prices are effected by retail prices. We require \( \xi < b, \eta < c \), \( 0 < \beta_1, \beta_2 < 1 \), and \( b - \xi \beta_i > c - \eta \beta_i \), \( b - \xi \beta_i > c - \eta \beta_i \).

In this paper, the production cost is not considered. We assume the inventory quantity is adequate, and the return rate is \( H_i, i = 1, 2 \) respectively.

3 Supply Chain Competition Equilibrium Results

In this section, we will orderly analyze equilibrium results in II competition model, DI competition model and DD competition model.

3.1 Equilibrium Results in II competition model

Two supply chains select the centralized management, and manufacturers in each supply chain set their own retail prices, which in fact form a duopoly market.

In this case, two supply chains’ channel profit is

\[
\Pi_{\text{SC1}}^I = (p_1 - \beta_1 p_1 H_1) D_1(p) = (1 - \beta_1 H_1) p_1 [a - (b - \xi \beta_1) p_1 + (c - \eta \beta_2) p_2], \quad \text{and}
\]
\[
\Pi_{\text{SC2}}^I = (p_2 - \beta_2 p_2 H_2) D_2(p) = (1 - \beta_2 H_2) p_2 [a - (b - \xi \beta_2) p_2 + (c - \eta \beta_1) p_1],
\]
respectively.

The first order condition on their retail price is

\[
\frac{\partial \Pi_{\text{SC1}}^I}{\partial p_1} = (1 - \beta_1 H_1) [a - 2(b - \xi \beta_1) p_1 + (c - \eta \beta_2) p_2] = 0 \quad \text{and}
\]
\[
\frac{\partial \Pi_{\text{SC2}}^I}{\partial p_2} = (1 - \beta_2 H_2) [a - 2(b - \xi \beta_2) p_2 + (c - \eta \beta_1) p_1] = 0,
\]
respectively.

The second order condition is \( \frac{\partial^2 \Pi_{\text{SCI}}^I}{\partial p_i^2} = 2(1 - \beta_i H_i)(\xi \beta_i - b) < 0 \) \( (i = 1, 2) \). Therefore, they are strictly concave functions on their own retail price.

The equilibrium retail price is

\[
p_1^{I\ast} = [(c - \eta \beta_2) + 2(b - \xi \beta_1)] a \quad \text{and} \quad p_2^{I\ast} = [(c - \eta \beta_1) + 2(b - \xi \beta_2)] a,
\]
respectively.

For the convenience of expression, let
\[ U = \frac{1}{4(b - \xi\beta_1)(b - \xi\beta_2) - (c - \eta\beta_1)(c - \eta\beta_2)}. \]

Therefore, the equilibrium demand is

\[ D_{1e}^* (p) = (b - \xi\beta_1)[2(b - \xi\beta_2) + (c - \eta\beta_2)]Ua \quad \text{and} \quad D_{2e}^* (p) = (b - \xi\beta_2)[2(b - \xi\beta_1) + (c - \eta\beta_1)]Ua, \]

respectively.

The equilibrium profit of the whole supply chain 1 and supply chain 2 is

\[ \Pi_{\text{SC1}}^* = (1 - \beta_1H_1)a^2U^2(b - \xi\beta_1)[2(b - \xi\beta_2) + (c - \eta\beta_2)]^2 \quad \text{and} \quad \Pi_{\text{SC2}}^* = (1 - \beta_2H_2)a^2U^2(b - \xi\beta_2)[2(b - \xi\beta_1) + (c - \eta\beta_1)]^2, \]

respectively.

### 3.2 Equilibrium Results in DI competition model

In DI competition model, we assume one of the supply chains makes decentralized decision, the manufacturer first sets the wholesale price, then the retailer decide the retail price to make his own profit maximization. Then, the manufacturer knows the market demand and the retail price, and renews his wholesale price. However, the other supply chain chooses the centralized decision.

In this first supply chain, retailer 1’s profit is

\[ \Pi_{\text{DI}R_1} (p) = (p_1 - \beta_1p_1H_1 - w_1)D_1 (p). \]

Manufacturer 1’s profit is

\[ \Pi_{\text{DI}M_1} (p) = w_1D_1 (p). \]

In the supply chain 2, the supply chain’s channel profit is

\[ \Pi_{\text{DI}SC_2} (p) = (1 - \beta_2H_2)p_2D_2 (p). \]

Retailer 1’ first order on his retail price is

\[ \frac{\partial \Pi_{\text{DI}R_1}}{\partial p_1} = (1 - \beta_1H_1)[a - 2(b - \xi\beta_1)p_1 + (c - \eta\beta_2)p_2] + w_1(b - \xi\beta_1) = 0. \]

The supply chain 2’ first order on his retail price is

\[ \frac{\partial \Pi_{\text{DI}SC_2}}{\partial p_2} = (1 - \beta_2H_2)[a - 2(b - \xi\beta_2)p_2 + (c - \eta\beta_1)p_1] = 0. \]

Retailer 1’ second order on his retail price \( \frac{\partial^2 \Pi_{\text{DI}R_1}}{\partial p_1^2} = 2(1 - \beta_1H_1)(\xi\beta_1 - b) < 0, \) and the supply chain 2’ second order on his retail price\( \frac{\partial^2 \Pi_{\text{DI}SC_2}}{\partial p_2^2} = 2(1 - \beta_2H_2)(\xi\beta_2 - b) < 0, \) strictly concave.
functions on their own retail price.

We can obtain the equilibrium retail price

\[ p_{2i}^{eq} = U \left\{ [(c-\eta \beta_i) + 2(b-\xi \beta_i)]a + \frac{w_i(b-\xi \beta_i)(c-\eta \beta_i)}{1-\beta_i H_i} \right\} \]

and

\[ p_{1i}^{eq} = U \left\{ [2(b-\xi \beta_i) + (c-\eta \beta_i)]a + \frac{2w_i(b-\xi \beta_i)(b-\xi \beta_i)}{1-\beta_i H_i} \right\}. \]

Then the equilibrium demand becomes

\[ D_{1i}^{eq}(p) = U(b-\xi \beta_i)\left\{ [(c-\eta \beta_i) + 2(b-\xi \beta_i)]a + \frac{w_i}{1-\beta_i H_i}\left[(c-\eta \beta_i)(c-\eta \beta_i) - 2(b-\xi \beta_i)(b-\xi \beta_i)\right]\right\} \quad \text{an} \]

\[ D_{2i}^{eq}(p) = U(b-\xi \beta_i)\left\{ [2(b-\xi \beta_i) + (c-\eta \beta_i)]a + \frac{w_i}{1-\beta_i H_i}(b-\xi \beta_i)(c-\eta \beta_i)\right\}, \quad \text{respectively.} \]

Manufacturer 1’ profit is

\[ \Pi_{M1}^{eq} = w_i D_{i1}^{eq}(p) \]

\[ = w_i \left\{ U(b-\xi \beta_i)\left\{ [(c-\eta \beta_i) + 2(b-\xi \beta_i)]a + \frac{w_i}{1-\beta_i H_i}\left[(c-\eta \beta_i)(c-\eta \beta_i) - 2(b-\xi \beta_i)(b-\xi \beta_i)\right]\right\}\right\}. \]

Manufacturer 1’ first order on his wholesale price \( w_i \) is

\[ \frac{\partial \Pi_{M1}^{eq}}{\partial w_i} = U(b-\xi \beta_i)\left\{ [(c-\eta \beta_i) + 2(b-\xi \beta_i)]a + \frac{2w_i}{1-\beta_i H_i}\left[(c-\eta \beta_i)(c-\eta \beta_i) - 2(b-\xi \beta_i)(b-\xi \beta_i)\right]\right\} \]

\[ = 0. \]

Then, we have the equilibrium wholesale price

\[ w_i = \frac{(1-\beta_i H_i)a[2(b-\xi \beta_i) + (c-\eta \beta_i)]}{4(b-\xi \beta_i)(b-\xi \beta_i) - 2(c-\eta \beta_i)(c-\eta \beta_i)}. \]

Therefore, supply chain 1’ and supply chain 2’ equilibrium profit is

\[ \Pi_{SC1}^{eq} = (1-\beta_i H_i) p_{1i}^{eq} D_{i1}^{eq}(p) \]

\[ = \frac{1}{2} \left\{ (1-\beta_i H_i)a^2 U^2 (b-\xi \beta_i)[(c-\eta \beta_i) + 2(b-\xi \beta_i)]^2 \left[1 + \frac{(b-\xi \beta_i)(b-\xi \beta_i)}{2(b-\xi \beta_i)(b-\xi \beta_i) - (c-\eta \beta_i)(c-\eta \beta_i)} \right]\right\} \quad \text{and} \]

\[ \Pi_{SC2}^{eq} = (1-\beta_i H_i) p_{2i}^{eq} D_{i2}^{eq}(p) \]

\[ = (1-\beta_i H_i)a^2 U^2 (b-\xi \beta_i)\left\{ [2(b-\xi \beta_i) + (c-\eta \beta_i)] + \frac{(c-\eta \beta_i)(b-\xi \beta_i)(2(b-\xi \beta_i) + (c-\eta \beta_i))}{2(b-\xi \beta_i)(b-\xi \beta_i) - (c-\eta \beta_i)(c-\eta \beta_i)} \right\}^2, \]
3.3 Equilibrium Results in DI competition model

The two supply chain choose the decentralize decision. The decision sequence is as follows.

(i) The Stackelberg manufacturers set the wholesale price.

(ii) The retailers give the retail price according to the wholesale price to make their own profit maximized.

(iii) The manufacturers know the two retail prices in the market, and refresh their each wholesale price.

Retailer 1’ and retailer 2’ profit is

\[ \Pi_{DD}^{R1} = (p_1 - \beta_1 p_1, H_1 - w_1)D_1 = (p_1 - \beta_1 p_1, H_1 - w_1)[a + (b - \xi \beta_1) p_1 + (c - \eta \beta_1) p_2] \quad \text{and} \]

\[ \Pi_{DD}^{R2} = (p_2 - \beta_2 p_2, H_2 - w_2)D_2 = (p_2 - \beta_2 p_2, H_2 - w_2)[a + (b - \xi \beta_2) p_2 + (c - \eta \beta_1) p_1], \]

respectively.

Manufacturer 1’ and manufacturer 2’ profit is

\[ \Pi_{DD}^{M1} (p) = w_1 D_1^{DD} (p) \quad \text{and} \]

\[ \Pi_{DD}^{M2} (p) = w_2 D_2^{DD} (p), \quad \text{respectively.} \]

Supply chain 1’ and supply chain 2’ equilibrium profit is

\[ \Pi_{DD}^{SC1} = (1 - \beta_1 H_1) p_1 D_1^{DD} (p) \quad \text{and} \]

\[ \Pi_{DD}^{SC2} = (1 - \beta_2 H_2) p_2 D_2^{DD} (p), \quad \text{respectively.} \]

Retailer 1’ and Retailer 2’ first order on their own retail price

\[ \frac{\partial \Pi_{DD}^{R1}}{\partial p_1} = (1 - \beta_1 H_1)[a - 2(b - \xi \beta_1) p_1 + (c - \eta \beta_1) p_2] + w_1(b - \xi \beta_1) = 0, \]

\[ \frac{\partial \Pi_{DD}^{R2}}{\partial p_2} = (1 - \beta_2 H_2)[a - 2(b - \xi \beta_2) p_2 + (c - \eta \beta_1) p_1] + w_2(b - \xi \beta_2) = 0, \]

, respectively.

The second order \( \frac{\partial \Pi_{DD}^{Ri}}{\partial p_i^2} = 2(1 - \beta_i H_i)(\xi \beta_i - b) < 0 (i = 1, 2) \), and is strictly concave functions on their own retail price.

Therefore, we have the equilibrium retail price

\[ p^{DD}_i = U\{[c - \eta \beta_2] + 2(b - \xi \beta_2)]a + (b - \xi \beta_i)\left(\frac{2w_i(b - \xi \beta_i)}{1 - \beta_i H_i} + \frac{w_i(c - \eta \beta_2)}{1 - \beta_2 H_2}\right) \quad \text{and} \]
\[ p^*_2 = U \left\{ \left( c - \eta \beta_1 \right) + 2 \left( b - \xi \beta_1 \right) \right\} a + \left( b - \xi \beta_1 \right) \left[ \frac{w_1(c - \eta \beta_1)}{1 - \beta_1 H_1} + \frac{2w_2(b - \xi \beta_1)}{1 - \beta_2 H_2} \right], \]

respectively.

Then the equilibrium demand becomes
\[ D^{DD*}_1(p) = a_1 - \left( b - \xi \beta_1 \right) p_1^{DD*} + \left( c - \eta \beta_1 \right) p_2^{DD*} \]
\[ = U \left( b - \xi \beta_1 \right) \left\{ \left( c - \eta \beta_1 \right) + 2 \left( b - \xi \beta_1 \right) \right\} a + \frac{w_1}{1 - \beta_1 H_1} \left[ \left( c - \eta \beta_1 \right) \left( c - \eta \beta_2 \right) - 2 \left( b - \xi \beta_1 \right) \left( b - \xi \beta_2 \right) \right] \]
\[ + \frac{w_2 \left( c - \eta \beta_1 \right) \left( b - \xi \beta_2 \right)}{1 - \beta_2 H_2}, \]

\[ D^{DD*}_2(p) = U \left( b - \xi \beta_2 \right) \left\{ \left( c - \eta \beta_1 \right) + 2 \left( b - \xi \beta_1 \right) \right\} a + \frac{w_1 \left( c - \eta \beta_1 \right) \left( b - \xi \beta_2 \right)}{1 - \beta_1 H_1} + \]
\[ \frac{w_2}{1 - \beta_2 H_2} \left[ \left( c - \eta \beta_1 \right) \left( c - \eta \beta_2 \right) - 2 \left( b - \xi \beta_1 \right) \left( b - \xi \beta_2 \right) \right]. \]

Then, manufacturer 1’ and manufacturer 2’ profit is
\[ \Pi^{DD*}_{M_1}(p) = w_1 D^{DD*}_1(p) \quad \text{and} \quad \Pi^{DD*}_{M_2}(p) = w_2 D^{DD*}_2(p) , \]

respectively.

Therefore, manufacturer 1’ and manufacturer 2’ first order on their own wholesale price is
\[ \frac{\partial \Pi^{DD*}_{M_1}(p)}{w_1} = U \left( b - \xi \beta_1 \right) \left\{ \left( c - \eta \beta_2 \right) + 2 \left( b - \xi \beta_2 \right) \right\} a + \frac{w_2 \left( c - \eta \beta_1 \right) \left( b - \xi \beta_2 \right)}{1 - \beta_2 H_2} + \]
\[ \frac{2w_1}{1 - \beta_1 H_1} \left[ \left( c - \eta \beta_1 \right) \left( c - \eta \beta_2 \right) - 2 \left( b - \xi \beta_1 \right) \left( b - \xi \beta_2 \right) \right] = 0 \]

and
\[ \frac{\partial \Pi^{DD*}_{M_2}(p)}{w_2} = U \left( b - \xi \beta_2 \right) \left\{ \left( c - \eta \beta_1 \right) + 2 \left( b - \xi \beta_1 \right) \right\} a + \frac{w_1 \left( c - \eta \beta_1 \right) \left( b - \xi \beta_2 \right)}{1 - \beta_1 H_1} + \]
\[ \frac{2w_2}{1 - \beta_2 H_2} \left[ \left( c - \eta \beta_1 \right) \left( c - \eta \beta_2 \right) - 2 \left( b - \xi \beta_1 \right) \left( b - \xi \beta_2 \right) \right] = 0 \]

Further, we get the equilibrium wholesale price
\[ w_1^* = (1 - \beta_1 H_1) a * \]
\[ \frac{\left( c - \eta \beta_1 \right) \left( b - \xi \beta_2 \right) \left[ 2 \left( b - \xi \beta_2 \right) + \left( c - \eta \beta_1 \right) \right]}{2 \left[ \left( c - \eta \beta_1 \right) \left( c - \eta \beta_2 \right) - 2 \left( b - \xi \beta_1 \right) \left( b - \xi \beta_2 \right) \right]} \]
\[ - \left( c - \eta \beta_1 \right) \left( c - \eta \beta_2 \right) \left[ \left( c - \eta \beta_1 \right) \left( c - \eta \beta_2 \right) - 2 \left( b - \xi \beta_1 \right) \left( b - \xi \beta_2 \right) \right] \]
\[= \frac{w_2^*}{1 - \beta_2 H_2} a * \]
\[ \frac{\left( c - \eta \beta_1 \right) \left( b - \xi \beta_2 \right) \left[ 2 \left( b - \xi \beta_2 \right) + \left( c - \eta \beta_1 \right) \right]}{2 \left[ \left( c - \eta \beta_1 \right) \left( c - \eta \beta_2 \right) - 2 \left( b - \xi \beta_1 \right) \left( b - \xi \beta_2 \right) \right]} \]
\[ - \left( c - \eta \beta_1 \right) \left( c - \eta \beta_2 \right) \left[ \left( c - \eta \beta_1 \right) \left( c - \eta \beta_2 \right) - 2 \left( b - \xi \beta_1 \right) \left( b - \xi \beta_2 \right) \right]. \]
Therefore, supply chain 1’ and supply chain 2’ equilibrium profit is
\[
\Pi_{SC1}^{II} = (1 - \beta_1 H_1)a^2 U^2 [2(b - \xi \beta_2) + (c - \eta \beta_2)]\{[2(b - \xi \beta_2) + (c - \eta \beta_2)] + (b - \xi \beta_2) \\
\times \{4(b - \xi \beta_2)(b - \xi \beta_2) - (c - \eta \beta_2)(c - \eta \beta_2)\}[2(b - \xi \beta_2)(b - \xi \beta_2) + (c - \eta \beta_2)(c - \eta \beta_2) + 3(c - \eta \beta_2)(b - \xi \beta_2)]\} \\
\times \{2[(c - \eta \beta_2)(c - \eta \beta_2) - 2(b - \xi \beta_1)(b - \xi \beta_2)]^2 - (c - \eta \beta_2)(c - \eta \beta_2)(b - \xi \beta_1)(b - \xi \beta_2)\} \\
\times \{2[(c - \eta \beta_2)(c - \eta \beta_2) - (c - \eta \beta_2)(c - \eta \beta_2)]^2\}
\]

\[
\Pi_{SC2}^{II} = (1 - \beta_2 H_2)a^2 U^2 [2(b - \xi \beta_2) + (c - \eta \beta_2)]\{[2(b - \xi \beta_2) + (c - \eta \beta_2)] + (b - \xi \beta_2) \\
\times \{4(b - \xi \beta_2)(b - \xi \beta_2) - (c - \eta \beta_2)(c - \eta \beta_2)\}[2(b - \xi \beta_2)(b - \xi \beta_2) + (c - \eta \beta_2)(c - \eta \beta_2) + 3(c - \eta \beta_2)(b - \xi \beta_2)]\} \\
\times \{2[(c - \eta \beta_2)(c - \eta \beta_2) - 2(b - \xi \beta_1)(b - \xi \beta_2)]^2 - (c - \eta \beta_2)(c - \eta \beta_2)(b - \xi \beta_1)(b - \xi \beta_2)\} \\
\times \{2[(c - \eta \beta_2)(c - \eta \beta_2) - (c - \eta \beta_2)(c - \eta \beta_2)]^2\}
\]

4 Comparison of the equilibrium results

Based on the Section 3, we compare the supply chains’ equilibrium results in II competition model and DI competition model. Therefore, we have the proposition 1.

**Proposition 1** \( \Pi_{SC1}^{II} > \Pi_{SC1}^{DI} , \ \Pi_{SC1}^{II} < \Pi_{SC2}^{DI} \).

**Proof.** From the results of Section 3, we know
\[
\Pi_{SC1}^{II} = (1 - \beta_1 H_1)a^2 U^2 (b - \xi \beta_1)[(c - \eta \beta_2) + 2(b - \xi \beta_2)]^2 \\
\times \frac{1}{2} \left[1 + \frac{(b - \xi \beta_1)(b - \xi \beta_2)}{2(b - \xi \beta_1)(b - \xi \beta_2) - (c - \eta \beta_1)(c - \eta \beta_2)} \right]
\]

From the assumption condition, \( (c - \eta \beta_1)(c - \eta \beta_2) < (b - \xi \beta_1)(b - \xi \beta_2) \), we have
\[
\frac{(b - \xi \beta_1)(b - \xi \beta_2)}{2(b - \xi \beta_1)(b - \xi \beta_2) - (c - \eta \beta_1)(c - \eta \beta_2)} < 1,
\]

Further,
\[
\frac{1}{2} \left[1 + \frac{(b - \xi \beta_1)(b - \xi \beta_2)}{2(b - \xi \beta_1)(b - \xi \beta_2) - (c - \eta \beta_1)(c - \eta \beta_2)} \right] < 1,
\]

Therefore, we have \( \Pi_{SC1}^{II} > \Pi_{SC1}^{DI} \).

We further compare \( \Pi_{SC1}^{II} = (1 - \beta_1 H_2)a^2 U^2 (b - \xi \beta_2)[(c - \eta \beta_2) + 2(b - \xi \beta_2)]^2 \), and
\[
\Pi_{SC2}^{II} = (1 - \beta_2 H_2)a^2 U^2 (b - \xi \beta_2)[(c - \eta \beta_2) + 2(b - \xi \beta_2)]^2 \\
\times \frac{(c - \eta \beta_1)(b - \xi \beta_2)[2(b - \xi \beta_2) + (c - \eta \beta_2)] + (c - \eta \beta_1)(b - \xi \beta_2)[2(b - \xi \beta_2) + (c - \eta \beta_2)]}{2(b - \xi \beta_1)(b - \xi \beta_2) - (c - \eta \beta_1)(c - \eta \beta_2)}
\]
We know \( \frac{(c-\eta \beta_1)(b-\xi \beta_1)[2(b-\xi \beta_2)+(c-\eta \beta_2)]}{2(b-\xi \beta_1)(b-\xi \beta_2)-(c-\eta \beta_1)(c-\eta \beta_2)} > 0. \)

Meanwhile, \( [2(b-\xi \beta_1)+(c-\eta \beta_1)] < \)
\[ \left\{ \frac{(c-\eta \beta_1)(b-\xi \beta_1)[2(b-\xi \beta_2)+(c-\eta \beta_2)]}{2(b-\xi \beta_1)(b-\xi \beta_2)-(c-\eta \beta_1)(c-\eta \beta_2)} \right\}^2. \]

Therefore, we obtain \( \Pi_{sc2}^U < \Pi_{sc2}^{D} \).

From the result of proposition 1, we can find that the integrated management is superior to the decentralized management in DI model. Further, because the expression of the equilibrium results is very complicated, we will give some examples to compare the results and analyze how the change of return factors affects the two supply chain’s equilibrium profit.

**Example 1** Let market demand parameters \( a_1 = a_2 = 200, \ b = 15, \ c = 12, \ \beta_1 = 0.5 \), the wholesale price \( w_1 = 15 \) and \( w_2 = 20 \), the return rate \( H_1 = \frac{1}{4} \), \( H_2 = \frac{1}{3} \).

We can draw the curve where profit functions in II competition model and DD competition model change with the parameter \( \beta_1 \) in [0,1].

![Figure 1](image)

**Figure 1** The relation of supply chain profit and the return factor \( \beta_1 \).

**Example 2** Let market demand parameters \( a_1 = a_2 = 200, \ b = 15, \ c = 12, \ \beta_1 = 0.6 \), the
wholesale price \( w_1 = 15 \), \( w_2 = 20 \), the return rate \( H_1 = \frac{1}{4} \), \( H_2 = \frac{1}{3} \). We can draw the curve where profit functions in II competition model and DD competition model change with the parameter \( \beta_2 \) in \([0,1]\).

From figure 1 and figure 2, we can attain the following conclusion.

1. When the return factor is defined on \([0,1]\), that is the return price defined on \([0,\rho]\), the profit of supply chain channel profit in DD competition model is higher than in II competition model, respectively. It means DD competition model is superior to II competition model.

2. In the competition environment, the profit of the own supply chain is reduced with the return price increased, but is lightly influenced. The profit of the competitive supply chain is reduced with the return price increased,

5 Conclusion

The customer returns policy is an important decision which made by the enterprise in the fierce competition. In this paper, we study that the effect of the customer returns on two-layer competitive supply chain. From the competitive supply chain of considering customer returns, we find that the
integrated management is superior to the decentralized management in DI model. However, as a whole, the equilibrium retail price and profit in DD model are higher than that in II model. In finally, we conclude that when considering the customer returns, it is optimal for both of the competitive supply chain to employ the policy of the decentralized management.

References