USING MV CONTROL CONCEPTS IN INVENTORY MANAGEMENT SUBJECT TO UNCERTAIN DEMAND

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Abstract:
This paper is focused on the problem of dynamic inventory management in presence of uncertain future demand. The production system is ATO (given by manufacturing, warehouse and assembly phases) and the model considers: capacity limits; manufacturing lead-times and uncertainties in the future demand and peaks in market demand. Here, a strategy for inventory management is proposed. This strategy has two levels, in the lower one, minimum variance (MV) control theory is used to compute optimal warehouse replacements. In the upper one, a methodology to prevent stockout is proposed in order to deal with lightly unpredictable demand peaks and manufacturing capacity limits.
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1. Introduction

Supply chain of production system is an important issue for many industries in order to be competitive in the market. Some of then have been devoting a great effort to introduce new technologies for supply chain management with the aim of reducing production costs. In this context, inventory management has been cited as one factor to obtain competitiveness in the market. The present paper is focused on the problem of dynamic inventory management in presence of uncertain future demand and an application of minimum variance feedback control methodology is presented.

The production system under control is composed by two stages: one is the manufacturing stage and another is the assembling stage. There is a warehouse for components waiting for assembly between this two phases. The production system model considers: limits in the manufacturing capacity (components); lead-time in the manufacturing stage; uncertainties in the future demand prediction and presence of peaks in market demand. The number of components in the warehouse should be kept in such a level that the market demand (after the assembling stage) is satisfied, so that, the inventory management police is composed by two phases (or levels). The first (or upper) one computes the minimum number of components in the warehouse in order to prevent stockout in presence of demand peaks, that is, the desired warehouse level is computed. The second (or lower) one computes the optimal inventory replacement (components to be manufactured) in such a way that the quadratic error between the desired and actual warehouse level is minimum, considering an uncertain future demand prediction. This strategy uses minimum variance ideas on feedback control to compute the number of components to be manufactured. So, using this optimal control replacement strategy, components are delivered for assembly in such a way that stockout are prevented (even in presence of predicted demand peaks and manufacturing limits) and the actual warehouse level is optimal in relation to a desired level and uncertainties regard to the future market demand.

The present paper is organized as follows. Section 2 reviews some ideas related with to the use of control theory applied to inventory management. Section 3 describes the main characteristics of the inventory system modeling and Section 4 presents the proposed strategy to deal with this problem. Finally, conclusions are addressed in Section 5.

2. The Control System Approach For Stock Management

Inventory can be viewed as dynamic systems and tone approach to understand its behavior (under an inventory control policy) is to use the classical dynamic system theory to derive model and analyze the whole system. In this context, some approaches have been proposed in the literature, as discussed in this section.

The first model of inventory systems and production in continuous time (Inventory and requests based on production control IOBPCS), (Towill, 1982) its based on Laplace transform of the an unique product, one level production to a inventory system, where the decisions in the production level are made based on the current demand and in the deviations of the inventory forecast. This model was extended for one-product, system of production-inventory multi-level through the approach called manufacturing dynamic (Edghill and Towill, 1988).

In the paper (Towill and Del Vecchio, 1994) a case of management inventory using the control theory is analyzed in a supply chain composed of three stages and a product. Each stages of the supply chain are seen as a filter of the behavior demand of the prior stages. In (O'Grady and Bonney, 1985) a model for application of the control theory with discrete time is proposed in several systems productive multi-stages and products. Although the demand pattern can be variable, its forecast needs to be made with accuracy. In (Popplewell and Bonney, 1987) and (Boney, 1994) two systems of control production of the multi-products and multi-levels are represented by means of block diagrams and the output of each system are calculated for different demand patterns. In (Brandolese and Cigolini, 1999) the necessary strategic inventory is dimensioned to deal with a peak market demand. The company assists the demand based on the strategy assembly-to-order (ATO). The production process is constituted of a stage of components production and another of assembly, with the strategic inventory between them. The times to fulfill the demand and of production in each stage is deterministic.

3. Problem Description

The structure of the considered production system is presented in Figure 1.

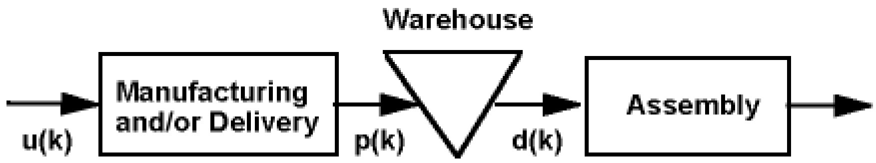

Figure 1: Productive System

It is composed by two stages with a warehouse for components between them. The first stage is related with manufacturing components and/or delivery of components to the warehouse and the second one to finished the products assembly. The availability of products before the first stage is supposed to be limited. The productive system operates in the strategy production Assemble to Order (ATO) or Make to Stock (MTO).

The warehouse level at time *k+1* can be represented by the following difference equation.

$$y(k+1) = y(k) + p(k) - d(k) \tag{1}$$

where, $y(k)$ it is the warehouse level at time k (k is an integer); $p(k)$ it is the product replacement of the warehouse; $d(k)$ it is the products withdraw of the warehouse for assembly. The first stage can be modelate as pure delay of d, instants, so Equation (1) can be written as follows:

$$\begin{aligned} y(k+1) &= y(k) + p(k) - \hat{d}(k) + \zeta(k+1) \\ p(k) &= u(k-d) \end{aligned} \tag{2}$$

In this equation, $u(k)$ is the order production; d is delay between request production and replacement stock; and $\hat{d}(k)$ it is the predicted demand at time k. By defining $e(k)$ as random number equivalent to the prediction demand error at time k, that is,

$$e(k) = d(k) - \hat{d}(k)$$

it follows that $\zeta(k)$ is equal to $-e(k\text{-}1)$. Figure 2 illustrates this structure.

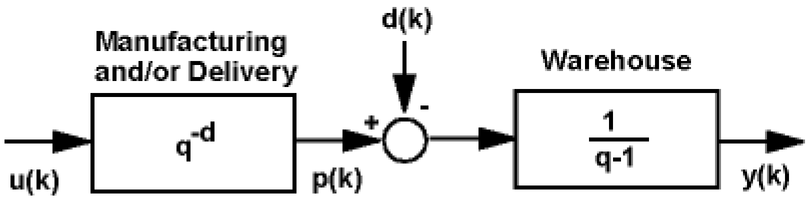

Figure 2: Productive System with delay

4. A Control Theory Based Approach For Inventory Management

4.1 The Basic Idea

The number of components in the warehouse should be kept in such a level that the market demand (after the assembling stage) is satisfied, so that, the inventory management police is characterized by the presence of two phases (or levels). The lower one computes, the optimal inventory replacement (components to be manufactured) product in order to satisfy the market demand. This is done in such a way that the quadratic error between the desired and actual warehouse level is minimum, considering an uncertain future demand prediction. This strategy uses minimum variance ideas on feedback control to compute the number of components to be manufactured. The computation if performed by means of a reference value for the inventory and with some information about level the future demand provided by a prediction (or forecast) phase. The upper level computes the minimum number of components in the warehouse in order to prevent stockout in presence of demand peaks, that is, the desired warehouse level is computed. So, this level deals with the problem of delivery and/or manufacturing limitations and changes the reference value in order to prevent stockout in presence of predicted demand peaks. Figure 3 summarizes these ideas and the two levels are described in following.

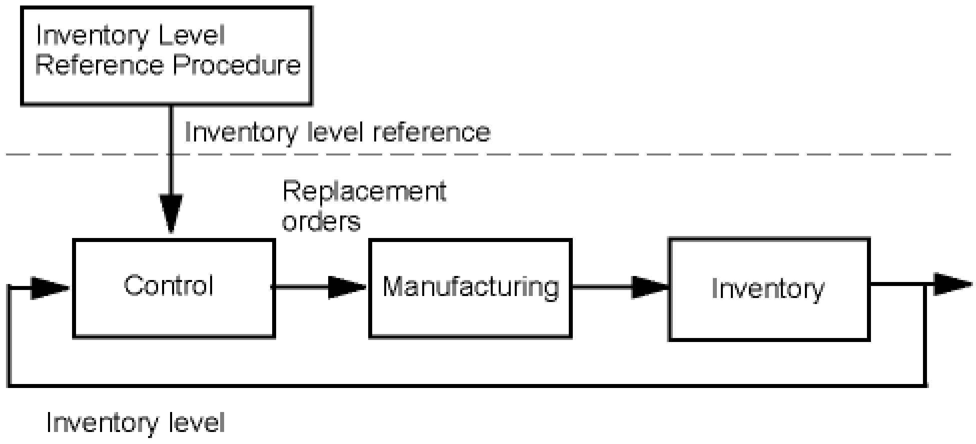

Figure 3 – Inventory control system

4.2 Upper level: dealing with stockout problem

Usually, the inventory level control system can't deal with demand peaks if these peaks are bigger than the manufacturing or delivery capacity. In these cases, a common strategy is to increase the inventory components before the peak arrives. This is done by changing the pre-defined reference level (initially set as a constant value) and them, a strategic inventory is create to satisfy the needs of market demand.

The algorithm is presented in following. Let *wc* be a constant level for reference inventory level, so *w(k)=wc*. Assuming that a demand peak is predicted for the future, *p* time instants before the peak and until it ends, the reference level is changed as follows:

$$w(k+1) = w(k) + pd(k) \tag{3}$$

where *pd(k),* is computed as follows:

$$pd(k) = \frac{demand\ peak\ height}{p} \tag{4}$$

4.3 Lower level: the optimal control law

The inventory model under analysis is given by equation (2), which can be written as:

$$y(k+1) = y(k) + u(k-d) - \hat{d}(k \mid k) + e(k+1) \tag{4}$$

The aim of the control level (lower level) is to compute a replacement order at time *k*, that is '*u(k)*', that minimizes the quadratic error between the inventory actual level '*y(k)*' and the inventory reference level '*w(k)*'. Since there is a time delay in the system due to manufacturing (or delivery) procedure, the following cost function can be defined:

$$J_k = (\hat{y}(k+j \mid k) - w(k+j))^2 \tag{5}$$

where *j=d+1*. This cost function represents the difference between the actual and desired inventory level at time *k+j* and the optimal replacement order is the one that makes it minimum. So, the minimization, in relation to *u(k)*, of this cost function at time *k* gives the optimal replacement order.

To solve this problem, it is necessary to compute *j*-step ahead the prediction equation for the inventory level, considering the information available at time *k*. The *j*-step ahead prediction is:

$$s(k+j)-s(k)=u(k-d+j)-\hat{d}(k+j)+e(k+1+j) \tag{6}$$

By using the forward operator *q*, that is, $qx(k)=x(k+1)$. This is the same as:

$$s(k+j)=\frac{q^{-d}}{q-1}u(k+j)-\frac{1}{q-1}\hat{d}(k+j)+\frac{q}{q-1}e(k+j) \tag{7}$$

By defining

$$A(q)=q-1;\quad B(q)=q^{-d};\quad C(q)=q;\quad D(q)=-1 \tag{8}$$

the following equation is obtained:

$$s(k+j)=\frac{B(q)}{A(q)}u(k+j)+\frac{D(q)}{A(q)}\hat{d}(k+j)+\frac{C(q)}{A(q)}e(k+j) \tag{9}$$

Some terms of the above equation are unknown at time *k*, for instance *e(k+j)*. As it is classical in *j*-step ahead prediction computations [Astrom95], to deal with this problem the following Diofantine equation is defined:

$$\frac{q^{d-1}C(q)}{A(q)}=F(q)+\frac{G(q)}{A(q)} \tag{10}$$

and its solution for *A*(*q*) and *C*(*q*) given by equation (8) is:

$$\begin{aligned}F(q)&=q^{d}+q^{d-1}+q^{d-2}+\cdots+1\\G(q)&=q^{d}\end{aligned} \tag{11}$$

By using equations (9), (10) and (11) and after some calculations, one obtains the best prediction of the level inventory at time *k+j*, made using the information available at time *k*. This prediction is given by:

$$\hat{s}(k+j\,|\,k)=\left(\frac{\left(q^{j}-G(q)\right)B(q)}{A(q)}\right)u(k)-\left(\frac{q^{j}-G(q)}{A(q)}\right)\hat{d}(k\,|\,k)+Gs(k) \tag{12}$$

So equation (10) is the *j*-step ahead inventory level prediction that is used in the minimization of the cost function (4). To solving this minimize in relation *u(k),* that is, $\frac{\partial J_k}{\partial u(k)}=0$, the optimal value for the inventory replacement order is:

$$u(k) = \frac{D}{B}\hat{d}(k) - \frac{G}{BF}y(k) + \frac{C}{BF}w(k+d) \tag{13}$$

By using equations (8) and (11), *u(k)* can be written as follows:

$$u(k) = q^d \hat{d}(k) - \frac{q^d}{q^d + q^{d-1} + q^{d-2} + \cdots + 1}s(k) + \frac{q^{d+1}}{q^d + q^{d-1} + q^{d-2} + \cdots + 1}w(k+d) \tag{14}$$

which is the same as:

$$\begin{aligned} u(k) \ = \ & -u(k-1) - u(k-2) - \cdots - u(k-d) + \hat{d}(k+d) + \hat{d}(k+d-1) \\ & + \cdots \hat{d}(k) - s(k) + w(k+d+1) \end{aligned} \tag{15}$$

5. Conclusions

This paper proposed a strategy for dynamic inventory management in presence of uncertain future demand. The dynamic inventory system is modelated by two stages with a warehouse between them. The first one is the delivery and/or manufacturing products stage and this stage was modelated by a pure delay. The warehouse was modelated by an accumulator. The inventory level control strategy is characterized by the presence of two levels. The lower one computes, by feedback, an optimal replacement product order to satisfy the market demand. The computation if performed by means of a reference value for the inventory and with some information about level the future demand provided by a prediction (or forecast) phase. The upper level deals with the problem of delivery and/or manufacturing limitations and changes the reference value in order to prevent stockout in presence of predicted demand peaks.

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