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Financial Scheduling: Feasibility and Optimality

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Abstract

Personal financial planning is the preparation of target-oriented decisions concerning personal assets, incomes, and expenses. Important part of personal financial planning is personal financial scheduling that is about choosing timepoints for realization of extraordinary expenses and incomes. As people have different preferences for different financial goals, and the goals are flexible, personal financial planning may be formulated as a multicriteria decision making problem that is usually addressed by trial calculations under different scenarios. We describe a mathematical programming approach for optimal decision making provided that the weights are prescribed to different objectives. To simplify the multiriteria decision making we examine a sum of the weighted goals as an objective function to be maximized via a mathematical program. Solving the mathematical program problem gives an optimal financial schedule concerning personal financial goals and preferences.

Key Words: Financial Scheduling, Multicriteria Optimization

1 Introduction

We are forced to recognize that States in Europe are withdrawing more and more from a comprehensive social responsibility for their citizens. These social and economic changes along with complex financial marketplace make it difficult for people to both keep up with all these changes and understand how this or that change may affect them. As a result, the complexity of but also the need for a comprehensive personal financial planning (PFP) has increased. PFP is not only for wealthy people, everyone needs PFP, and a careful and target-oriented planning of financial decisions is one of the most urgent economic questions at the beginning of the 21st century. It should be noted however that currently the importance of managing and controlling personal financial affairs using optimization technique is not yet generally accepted in Europe.

Often PFP is reduced to the single aspect of answering the question of how to get rich. PFP should instead investigate the questions of how expenses should be used (view on the expenses), how incomes should be achieved (view on the incomes), and how assets should be invested (view on the assets). In each of the three cases, the preferences of a person play a crucial role, and PFP is defined as the preparation of target-oriented decisions concerning personal financial affairs. PFP has to be examined as a multi-stage dynamic process of meeting financial goals through the management of a family's financial affairs. Financial goals can include buying a home, buying a car, saving for children's education, planning for future retirement. The latter is the most important and rather complex part of the PFP as follows from Certified Financial Planner Board of Standards [4].

In this paper, we adopt the view on realization timepoints of additional incomes and expenses. The corresponding problem is the financial scheduling problem of personal financial planning, i.e. personal financial scheduling (PFS). The basic objective is to reach as many financial goals as possible by obeying liquidity restrictions. Hereby, the person might have various preferences for their different financial goals and these goals might be contrary in the sense that earlier realization of one goal means later realization of another goal. As a result, PFS can be considered as a multicriteria decision making problem, and mathematical programming appears as an appropriate method for formulating and solving this problem. Obviously, it is not realistic to assume all the input numerical parameters of the mathematical program are exactly known before mid-term and long-term PFS. Even in the case of short-term PFS, some input data can be changed considerably. Therefore stability analysis seems to be useful and even necessary to estimate the credibility of the concrete personal financial schedule. High degree of stability of personal finance fitness can make life more saved and comfortable due to protecting family's future from unexpected financial accidents or illnesses circumstances. It should be noted that stability aspects of PFS obtained a limited attention in the literature so far and the main aim of this paper is to propose the operational research model and algorithms for stability analysis of personal financial scheduling as a part of PFP.

The paper is organized as follows. Section 2 gives a brief review of related works to the field of PFP. Problem setting and notations are given in Section 3. Feasibility test of PFS is considered in Section 4. Section 5 describes an integer programming approach and algorithm for constructing optimal personal financial schedules. Concluding remarks are given in Section 8.

2 Related work

The PFP related literature is vast and originates a number of disciplines that investigate specific topics of importance for PFP (e.g. (socio-) economics [7], psychology [15], and medicine [26]). In addition, the field is well supplied with a lot of Textbooks [9, 17]. Most books can be used as guides to handle personal financial problems, e.g. maximizing wealth, achieving various financial goals, determining emergency savings, maximizing retirement plan contributions, etc. There are several PFP models in the literature covering mainly the asset's view on PFP [2, 6, 11, 21]. Warschauer [29] underlines the need for academic research in the development of the personal financial planning profession.

Ashford et al. [1] give a review of the use of OR in financial management. The authors state that recent years have seen the development of numerous applications for OR models and algorithms in the financial world, as computer capacity and power were growing exponentially. In addition, financial institutions, large corporations and research centers are increasingly devoting important resources to research and development in financial modeling and optimization. Within the field of OR, Multicriteria Decision Making (MCDM) has evolved as one major discipline. The development of MCDM is based on the simple finding that a single objective, goal, criterion or point of view is rarely used to make real-world decisions. State-of-the-art reviews [24, 27, 30] of the research made on the application of the techniques of MCDM to problems and issues in finance show that the multidimensional nature of financial decisions has already motivated researchers to explore the potential of MCDM in addressing financial decision-making problems because the necessity to address financial problems in a broader and more realistic context has been noted. Especially the assets view on PFP is well supplied with a lot of research contributions. Ehrgott et al. [8] describe an MCDM approach to portfolio optimization. For further references on the use of optimization models for portfolio selection, the reader is referred to Pardalos et al. [20]. All those optimization models are only for the view on the assets. To the best of our knowledge, there is no portfolio optimization model for the views on the incomes (as we are doing in this contribution) and on the expenses [3]. Multicriteria decision making models for the asset's view on personal financial planning are presented by Puelz and Puelz [19], and Gao et al. [12].

ChiangLin and Lin [5] describe PFP based on fuzzy multiple objective programming. They argue that solving the financial planning problem by trial-and-error gives a satisfying suggestion, but not necessarily the (concerning the preferences of the decision maker) best solution that can be found. Their approach with fuzzy goal programming is motivated by the fact that financial goals set by the decision maker or by the financial planner might be flexible. So it seems to be preferable to provide an acceptable range for a goal instead of an exact value. They close their motivation for their study by the conclusion that in view of the above difficulties, mathematical programming appears a promising approach for PFP. In more detail, ChiangLin and Lin [5] formulate a decision model for PFP that considers the incomes from salary and investment and the expenses for living, purchasing a house and raising children. Four objectives are considered, including the level of living expense, the time to buy a house, the value of the house, and the pension available at retirement. All the objectives that contribute to one´s life quality before and after retirement are to be maximized except the time to buy a house. Numerical examples are provided to show the effectiveness of their approach to PFP.

3 Problem setting and notations

A financial planner has to analyze client's (who acts as the final decision maker) information to assess the current financial situation and determine what the client must do to meet her (his) financial goals. Depending on what services a client has asked for, this could include analyzing assets, liabilities and cash flows, current insurance coverage, investments or tax strategies. The main planning and control instrument of personal financial planning is the *finance plan*, i.e. a listing of

- all essential assets (estate, wealth) and liabilities (debts),
- all essential expected regular (fixed) and additional (variable) incomes,
- all essential expected regular (fixed) and additional (variable) expenses,

along with the most suitable time periods for realizations of the incomes and expenses.

In the *personal financial scheduling* problem of *personal financial planning*, the client has to decide about the realization timepoints of additional (variable) incomes and expenses reflecting her (his) personal goals and wishes (e.g. buying or selling a car or a house). As a result, *financial scheduling* consists in determining the best timepoints (according to given preferences of the client) for realization of additional incomes and expenses. It is sufficient to consider only the most important (costly) incomes and expenses, and there is no need to consider each income an expense.

As usual, we consider n time periods with equal length for financial planning (scheduling) which are defined by $n+1$ equidistant timepoints $j = 0, 1, \ldots, n$. The difference between sequential timepoints j and $j+1$ (with $0 \leq j < n$) is equal to the length of planning period $(j, j+1]$, e.g. one to five years for long-time or mid-time financial planning, and one month (or one week) for short-time financial planning. Period $(0, n]$ means total period of financial planning (called also *planning horizon*). The starting wealth (starting surplus income) for financial planning is equal to $S_0 = I_0 - E_0$, where I_0 (E_0) are total incomes (total expenses) received (spent) until timepoint 0.

We will distinguish two types of incomes and expenses included in total incomes I_j and total expenses E_j in period $(j-1, j]$, namely, *regular* (also called *fixed*) incomes and expenses (like wages, average bonuses, alimony payments, investment income, unemployment benefits, mortgage or rent, car payments, average food bills, medical expenses, clothing payments, tax payments), and *additional* (also called *variable*) incomes and expenses (like buying or selling a house or an own flat or a vacation house, buying or selling a car, payment for children's education, buying furniture, specific saving's amount at the beginning of retirement, fulfilling an expensive wish, expensive entertainment).

While regular expenses E_j^{fix} (incomes I_j^{fix}) have to be definitely included in the sum of total expenses E_j (incomes I_j) in period $(j-1, j]$, there is a choose for selecting the most suitable timepoints for the additional expenses $e_1, e_2, \ldots, e_{emax}$ and for the additional incomes $i_{imin}, i_{imin+1}, \ldots, i_{imax}$. In this way, a financial scheduling problem arises which consists of determining an optimal (according to the preferences of a decision maker) sequence of realization timepoints of additional incomes and expenses. In more detail, we define the sum of expenses E_j and the sum of incomes I_j in period $(j-1, j]$ as follows:

$$E_j = E_j^{fix} + \sum_{i=1, x_i=j}^{emax} e_i \quad \text{and} \quad I_j = I_j^{fix} + \sum_{k=imin, y_k=j}^{imax} i_k,$$

where x_i (y_k) means the realization timepoint of expense e_i (income i_k). For simplicity, we assume that realization timepoint j occurs always just at the end j of a period $(j-1, j]$. In the financial scheduling problem, the optimal timepoints x_i and y_k of the realization of the additional expense e_i and income i_k have to be determined.

The main notations summarized in Table 1 will be used in the problem formulation and in the algorithms for financial planning and scheduling based on the mathematical programming models.

4 Feasibility of a personal financial plan

A financial plan is *feasible* if the total expenses E_j within each planning period $(j-1, j]$, $j \in \{1, 2, \ldots, n\}$, can be financed by the sum of total current incomes I_j and the previous surplus incomes S_{j-1}.

Table 1: Notations

$n+1$	number of timepoints $j = 0, 1, \ldots, n$ in personal financial scheduling, the difference between sequential timepoints j and $j+1$ being equal to the length of one period (e.g. one week, one month or one year)
I_j	total sum of all incomes in period $(j-1, j]$, i.e. the sum of all fixed and variable incomes within period $(j-1, j]$, we have always $I_j \geq 0$
I_0	liquidated wealth at timepoint 0
I_j^{fix}	amount of regular (fixed) incomes in period $(j-1, j]$
$imin$	minimal index of additional (variable) income
$i_{imin}, , \ldots, i_{imax}$	additional (variable) incomes
$imax - imin + 1$	number of additional (variable) incomes in planning horizon $(0, n]$
y_k	realization timepoint of additional (variable) income i_k
E_j	total sum of all expenses in period $(j-1, j]$, i.e. the sum of all fixed and variable expenses within period $(j-1, j]$, we have always $E_j \geq 0$
E_0	liquidated debts at timepoint 0
E_j^{fix}	amount of regular (fixed) expenses in period $(j-1, j]$
$emax$	number of additional (variable) expenses in planning horizon $(0, n]$
$e_1, \ldots, e_{emax}$	additional (variable) expenses
x_i	realization timepoint of additional (variable) expense e_i
S_j	sum of the surplus incomes at timepoint j, $S_j := S_{j-1}(1 + r_j^+) - C_{j-1} r_j^- + I_j - E_j + C_j$
C_j	overdraft credit taken from a bank at timepoint j
r_j^+	average inflation adjusted interest savings rate in period $(j-1, j]$
r_j^-	average lending rate for the overdraft credit in period $(j-1, j]$
D_j	upper bound of total debt allowed in period $(j-1, j]$
L_j	liquidity reserve required in period $(j-1, j]$
α_k	weight of additional income i_k (decision maker's time preference)
β_i	weight of additional expense e_i (decision maker's time preference)
γ	weight of the final surplus income (final wealth) at timepoint n

For simplicity, we assume that all payments occur at the end j of the corresponding period $(j-1, j]$. Let r_j^+ denote the inflation adjusted interest savings rate in period $(j-1, j]$. Therewith, we set $S_0 = I_0 - E_0$ and compute $S_j, j = 1, 2, \ldots, n$, as follows:

$$S_j = S_{j-1} \cdot (1 + r_j^+) + I_j - E_j. \tag{1}$$

Here, the positive surplus income S_{j-1} of the former period $(j-2, j-1]$ will be interested in period $(j-1, j]$, and the surplus income will be increased at the end j of period $(j-1, j]$ by $S_{j-1} r_j^+$ (e.g. from call money of a banking account).

Regarding to commonly used practice, in what follows, we assume the possibility of using overdraft credits. In particular, if client's own money combined with her (his) savings currently available and current incomes are not sufficient to cover all the expenses in the period $(j-1,j]$, where $j \in \{1,2,\dots,n\}$, i.e. if $I_j+S_{j-1}\cdot(1+r_j^+) < E_j$, then overdraft credits C_j can be taken (usually of a bank) to cover the difference $E_j - \big(I_j + S_{j-1}\cdot(1+r_j^+)\big)$. For ease of use we regard only one credit (total credit) per period $(j-1,j]$.

Let r_j^+ (r_j^-) be the interest (lending) rate in period $(j-1,j]$. Then we compute $S_j, j = 1,2,\dots,n$, as follows:

$$S_j = S_{j-1}(1+r_j^+) - C_{j-1}(1+r_j^-) + C_j + I_j - E_j. \tag{2}$$

In (2), C_j is a kind of slack variable and is the credit taken at timepoint j that has to be repaid at timepoint $j+1$ or later. If credit C_j has been taken within period $(j-1,j]$, then it will be necessary to return to the bank not only value C_j but also value $C_j r_{j+1}$ as the bank borrows money in the amount of C_j during period $(j,j+1]$.

C_j will be equal to zero, if $(I_j - E_j) + S_{j-1}(1+r_j^+) - C_{j-1}(1+r_j^-) \geq 0$. In this case, no overdraft credit is necessary to reach a non-negative S_j. For ease of use we assume that at the end of each period a client can either repay parts of overdraft credit (or the whole credit) or take additional overdraft credits to updated expenses and interest rates.

We denote

$$h_j = S_{j-1}(1+r_j^+) - C_{j-1}(1+r_j^-) + I_j - E_j. \tag{3}$$

The height of the overdraft credit C_j borrowed at timepoint j is defined as follows:

$$C_j = \begin{cases} |h_j|, & \text{if } h_j < 0, \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

Figure 1 gives an example of a simple financial schedule.

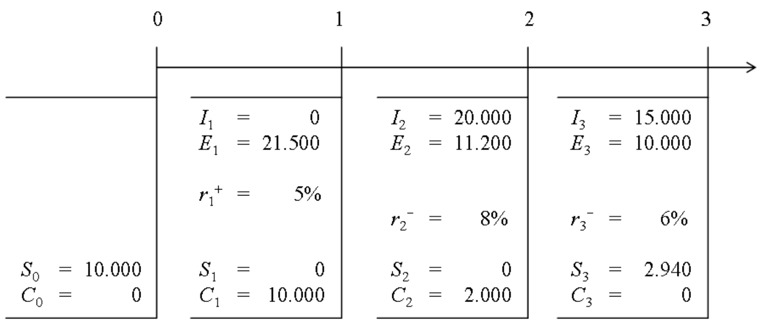

Figure 1: Savings S_j and credits C_j

Let a client have $S_0 = I_0 - E_0 = 10.000$ *MU (hereafter, MU means monetary units, e.g. Euro, US\$) at timepoint* 0, *and* $C_0 = 0$. *This positive surplus income will be interested by* $r_1^+ = 5\%$ *in period* $(0, 1]$, *and a client has* $S_0(1.05) + (I_1 - E_1) = -10.000$ *MU. Therefore, a client has to take an overdraft credit at timepoint* 1 *in height of* $C_1 = 10.000$ *MU. The lending rate* $r_2^- = 8\%$ *leads to a negative surplus income after period* $(1, 2]$, *and a client has to take again an overdraft credit at timepoint* 2 *in height of* $C_2 = 2.000$ *MU. After period* $(2, 3]$, *the final surplus income is as follows:* $S_3 = I_3 - E_3 + C_2(1 + r_3^-) = 2.940$ *MU.*

In order to regulate possible liquidity reserve (which are admissible by a client), the model of personal financial scheduling will include a lower limit of possible total liquidity reserve

L_j allowed to be hold in period $(j-1, j]$, $j \in \{1, 2, \ldots, n\}$, and inequality $S_j \geq L_j$ must hold for each $j = 0, 1, \ldots, n$.

The other constraint is in case of allowing to take credits: Here, we restrict the maximum credit amount by a total debt restriction, $C_j \leq D_j$. Note that in case of required positive liquidity reserve, i.e. $L_j \geq 0$, it is not allowed to take a credit in period $(j-1, j]$, i.e. $D_j = 0$. The other way around, if we allow to take credits, i.e. $D_j \geq 0$, then we always have $L_j = 0$.

The following remark given in [4] highlights a possible rule for choosing appropriate values D_j, $j \in \{1, 2, \ldots, n\}$, (see also the papers of Greninger et al. [10], Mason and Griffith [18], and DeVaney [28] defining personal financial ratios and benchmarks).

> *"How much debt is too much debt? Debt isn't necessarily bad, but too much debt is. Add up what you pay monthly in car loans, student loans, credit card and charge card loans, personal loans everything but your mortgage. Divide that total by the money you bring home each month. The result is your "debt ratio". Try to keep that ratio to 10 percent or less. Total mortgage and non-mortgage debt should be no more than 36 percent of your take-home pay. What's the difference between "good debt" and "bad debt"? Yes, there is such a thing as good debt. That's debt that can provide a financial pay off. Borrowing to buy or remodel a home, pay for a child's education, advance your own career skills, or buy a car for getting to work can provide long-term financial benefits. Bad debt is when you borrow for things that don't provide financial benefits or that*

don't last as long as the loan. This includes borrowing for vacations, clothing, furniture, or dining out."

As a result, we can formulate the feasibility conditions of a financial plan as follows:

$$C_j \leq D_j \text{ and } S_j \geq L_j, j = 0, 1, \ldots, n, \tag{5}$$

where values of D_j and L_j have to be defined by the client.

Because of the slack variables C_j, every $S_j, j = 0, 1, \ldots, n$, is non-negative. Maybe the client has to take a credit in the last period $(n-1, n]$ at timepoint n, i.e. $C_n > 0$. In this case, the credit cannot be repaid in the planning horizon.

A person may agree that the financial schedule is feasible if the credits C_j do not exceed the given limits D_j for all $j \in \{0, 1, \ldots, n\}$ including n.

For simplicity of the problem, we make the agreement that at least one of two equalities $S_j = 0$ or $C_j = 0$ must hold in each period $(j-1, j]$. We can argue such an agreement as follows. On the one hand, if $r_j^- > r_j^+$ (and this is always the case in call markets), then there exists an optimal financial plan such that inequality $C_j > 0$ implies equality $S_j = 0$. In such a case using savings is definitely preferable to a loan. On the other hand, the inequality $S_j > 0$ implies always equality $C_j = 0$, reducing the loan to zero (i.e. re-paying all the credit) is definitely preferable to holding a part of the credit and investing the rest in a call market. For some special kinds of credits the opposite inequality $r_j^- < r_j^+$ may hold, but we consider only call markets, so we can assume $r_j^- > r_j^+$. Of course, this is a simplification of the real-word situation since conditions for some kinds of credits do not allow to return the whole debt or even its main part in any timepoint currently desired by a client. However for the rough financial planning, the above agreement may be practically reasonable and definitely useful from computational reasons. Every credit C_{j-1} taken at timepoint $j-1$ has to be repaid at timepoint j or later. Then client can borrow a new credit C_j (or add a necessary part to previous credit) at timepoint j or later and so on. We prove the following auxiliary claim.

Lemma 1: If it is assumed that for each $j \in \{0, 1, \ldots, n\}$, at least one of equalities $S_j = 0$ or $C_j = 0$ holds, then

$$S_j = h_j + C_j \tag{6}$$

where C_j is defined in (3) and (4).

Proof.

If either equality $C_{j-1} = 0$ or $S_{j-1} = 0$ holds for each $j \in \{1, 2, \ldots, n\}$, we have to treat four possible cases (i), (ii), (iii) and (iv) as follows.

(i) Let $C_{j-1} = 0$, $S_{j-1} \geq 0$, $I_j - E_j \geq S_{j-1}(1 + r_j^+)$.

In this case it is not necessary to borrow a credit at timepoint j, i.e., $C_j = 0$. The surplus income S_j increases to $S_{j-1}(1 + r_j^+) + I_j - E_j$. Equation (6) holds.

(ii) Let $C_{j-1} = 0$, $S_{j-1} \geq 0$, $I_j - E_j < S_{j-1}(1 + r_j^+)$.

It is necessary to borrow a credit at timepoint j, i.e. $C_j = S_{j-1}(1 + r_j^+) + I_j - E_j$. The surplus income S_j becomes 0. Equation (6) holds.

(iii) Let $C_{j-1} \geq 0$, $S_{j-1} = 0$, $I_j - E_j \geq C_{j-1}(1 + r_j^-)$.

In this case we can repay the credit C_{j-1} completely at timepoint j, and we have $C_j = 0$. The surplus income S_j increases to $I_j - E_j - C_{j-1}(1 + r_j^-)$. Equation (6) holds.

(iv) Let $C_{j-1} \geq 0$, $S_{j-1} = 0$, $I_j - E_j < C_{j-1}(1 + r_j^-)$.

In this case we have to borrow a credit $C_j = I_j - E_j - C_{j-1}(1 + r_j^-)$ at timepoint j. The surplus income does not change: $S_j = 0$. Equation (6) holds.

Thus, it is proven that in all possible cases equality (6) holds. □

If all the input data are exactly known before timepoint 0, then (due to 4) the sum of the surplus incomes S_j can be computed in constant time $O(1)$ using equation (6) for each $j \in \{1, 2, \ldots, n\}$. As a result, one can test feasibility of the concrete financial plan in linear time $O(n)$ using Algorithm 1 given in Fig. 2, where $[A]$ means the value of variable A.

To illustrate the usage of the Algorithm 1, we consider the following example.

A couple (both 34 years old) has a regular net income of 60.000 MU (MU) per year. They plan to retire when they will be 65, and the net retirement payment will be 38.000 MU per year. The rent for the flat (100 m^2) is equal to 1.000 MU per month (12.000 MU per year). Other living expenses (including insurance and provision for retirement) are currently 45.000 MU per year. Living expenses at retirement are assumed to be about 32.000 MU per year. Incomes and expenses are expected to increase from year to year with the inflation rate. Therefore, the computations are all done in current prices. The couple cur-

Algorithm 1: *Feasibility check*

Input: $E_0, \ldots, E_n,\ I_0, \ldots, I_n,\ D_1, \ldots, D_n,\ r_1^+, \ldots, r_n^+,\ r_1^-, \ldots, r_n^-$
begin
$S_0 = I_0 - E_0$; $C_0 = 0$; we assume $I_0 \geq E_0$
for $(j = 1, \ldots, n)$
{
  $S_j := S_{j-1}(1 + r_j^+) - C_{j-1}(1 + r_j^-) + I_j - E_j$;
  $C_j = 0$;
  if $(S_j < 0)$
  {
    $C_j = |S_j|$;
    if $(C_j > D_j$ OR $S_j < L_j)$
    {
      print ("Financial plan is not feasible:
        Debt in period [j] is grater than that allowed.");
      stop;
    }
  }
}
print ("Financial plan is feasible.");
end;
Output:
"Financial plan is feasible." or
"Financial plan is not feasible. Debt in period [j] is grater than that allowed." — $O(n)$

Figure 2: Feasibility check of a financial plan.

rently has 250.000 MU in investment with an annual inflation-adjusted rate of return of 3.5%. All surplus incomes will be reinvested in the same investment. There are no debt and liquidity reserve restrictions.

The planning horizon $(0, n]$ consists of $n = 46$ years, starting from age 34 of the couple and ending at their age 80. The financial plan derived from this basic scenario is feasible as all the expected expenses can be financed by the incomes. The couple has never borrowed a credit from a bank to finance their planned expenses (see Table 2). The value of the starting investment increases to 1.359.448 MU at age 80. We have to note that these computations are very sensitive against small changes in the savings interest rate (because of the long planning horizon). E.g., if the interest rate will be set to $r_j^+ = 2.0\%$ instead of $r_j^+ = 3.5\%$, the couple would end up at age 80 with an investment value of 676.891 MU. Whereas, if the interest rate will be set to $r_j^+ = 5.0\%$ instead of $r_j^+ = 3.5\%$, the couple would end up at age 80 with an investment value of 2.651.703 MU.

Table 2: Cash flow table for basic scenario.

Time-point j	Age	Regular net income	Regular expenses	Rent for flat	Investment	Credit
	34				250.000	0
1	35	60.000	45.000	12.000	261.750	0
2	36	60.000	45.000	12.000	273.911	0
3	37	60.000	45.000	12.000	286.498	0
4	38	60.000	45.000	12.000	299.526	0
5	39	60.000	45.000	12.000	313.009	0
6	40	60.000	45.000	12.000	326.964	0
7	41	60.000	45.000	12.000	341.408	0
8	42	60.000	45.000	12.000	356.357	0
9	43	60.000	45.000	12.000	371.830	0
10	44	60.000	45.000	12.000	387.844	0
11	45	60.000	45.000	12.000	404.418	0
12	46	60.000	45.000	12.000	421.573	0
13	47	60.000	45.000	12.000	439.328	0
14	48	60.000	45.000	12.000	457.705	0
15	49	60.000	45.000	12.000	476.724	0
16	50	60.000	45.000	12.000	496.410	0
17	51	60.000	45.000	12.000	516.784	0
18	52	60.000	45.000	12.000	537.871	0
19	53	60.000	45.000	12.000	559.697	0
20	54	60.000	45.000	12.000	582.286	0
21	55	60.000	45.000	12.000	605.666	0
22	56	60.000	45.000	12.000	629.865	0
23	57	60.000	45.000	12.000	654.910	0
24	58	60.000	45.000	12.000	680.832	0
25	59	60.000	45.000	12.000	707.661	0
26	60	60.000	45.000	12.000	735.429	0
27	61	60.000	45.000	12.000	764.169	0
28	62	60.000	45.000	12.000	793.915	0
29	63	60.000	45.000	12.000	824.702	0
30	64	60.000	45.000	12.000	856.566	0
31	65	38.000	32.000	12.000	880.546	0
32	66	38.000	32.000	12.000	905.365	0
33	67	38.000	32.000	12.000	931.053	0
34	68	38.000	32.000	12.000	957.640	0
35	69	38.000	32.000	12.000	985.157	0
36	70	38.000	32.000	12.000	1.013.638	0
37	71	38.000	32.000	12.000	1.043.115	0
38	72	38.000	32.000	12.000	1.073.624	0
39	73	38.000	32.000	12.000	1.105.201	0
40	74	38.000	32.000	12.000	1.137.883	0
41	75	38.000	32.000	12.000	1.171.709	0
42	76	38.000	32.000	12.000	1.206.719	0
43	77	38.000	32.000	12.000	1.242.954	0
44	78	38.000	32.000	12.000	1.280.458	0
45	79	38.000	32.000	12.000	1.319.274	0
46	80	38.000	32.000	12.000	1.359.448	0

The situation changes if the couple considers financial goals exceeding regular expenses as follows.

The couple plans to save a certain amount of their income each year for general costs and for buying a house with 150 m^2 in 5 years (when they are 39). The price of the house should be about 250.000 MU. They plan to spend additional 10.000 MU per year for the children education for the next 15 years (i.e. last saving when they are 49). For unspecified wishes, the couple would like to spend 3.000 MU per year up to the date of their retirement (i.e. when they are 64). Finally, the couple calculates for cars (including maintenance, incidental and running costs) 25.000 MU every 6 years (first car in one year and the last car when they will be no more than 75 years old).

It turns out that it will be necessary to use an overdraft credit from a bank to finance the planned additional expenses.

Let the upper bound of total debt allowed (by the decision maker) in each period $(j-1, j], j = 1, 2, \dots, n,$ *be the same for all periods and be equal to* $D_j =$ 100.000 *MU. Let the average interest rate for the overdraft credit in each period* $(j-1, j], j = 1, 2, \dots, n,$ *be assumed to be equal to* $r_j^- = 6\%$.

The financial plan derived from this extended scenario is not feasible as the credit taken at age 53 is a little bit too high (105.008 > 100.000). The financial planner could speak to the couple and increase the total debt allowed from $D_j = -100.000$ MU to $L_j = 106.000$ MU. Then the financial plan would be feasible as shown in Table 3. Nevertheless one can see that the couple has always debts from their age 39 on. Only the last few years the debts will again decrease, but at age 80, they will still have 83.089 MU debts.

Table 3: Cash flow table for extended scenario.

Time-point j	Age	Regular net income	Regular expenses	Rent for flat	House	Children	Wish	Cars	Investment	Credit
	34								210.000	0
1	35	60.000	45.000	12.000	0	10.000	3.000	25.000	223.750	0
2	36	60.000	45.000	12.000	0	10.000	3.000	0	221.581	0
3	37	60.000	45.000	12.000	0	10.000	3.000	0	219.337	0
4	38	60.000	45.000	12.000	0	10.000	3.000	0	217.013	0
5	39	60.000	45.000	12.000	250.000	10.000	3.000	0	0	35.391
6	40	60.000	45.000	0	0	10.000	3.000	0	0	35.515
7	41	60.000	45.000	0	0	10.000	3.000	25.000	0	60.646
8	42	60.000	45.000	0	0	10.000	3.000	0	0	62.284
9	43	60.000	45.000	0	0	10.000	3.000	0	0	64.021
10	44	60.000	45.000	0	0	10.000	3.000	0	0	65.863
11	45	60.000	45.000	0	0	10.000	3.000	0	0	67.814
12	46	60.000	45.000	0	0	10.000	3.000	0	0	69.883
13	47	60.000	45.000	0	0	10.000	3.000	25.000	0	97.076
14	48	60.000	45.000	0	0	10.000	3.000	0	0	100.901
15	49	60.000	45.000	0	0	10.000	3.000	0	0	104.955
16	50	60.000	45.000	0	0	0	3.000	0	0	99.252
17	51	60.000	45.000	0	0	0	3.000	0	0	93.207
18	52	60.000	45.000	0	0	0	3.000	0	0	86.800
19	53	60.000	45.000	0	0	0	3.000	25.000	0	105.008
20	54	60.000	45.000	0	0	0	3.000	0	0	99.308
21	55	60.000	45.000	0	0	0	3.000	0	0	93.267
22	56	60.000	45.000	0	0	0	3.000	0	0	86.863
23	57	60.000	45.000	0	0	0	3.000	0	0	80.074
24	58	60.000	45.000	0	0	0	3.000	0	0	72.879
25	59	60.000	45.000	0	0	0	3.000	25.000	0	90.251
26	60	60.000	45.000	0	0	0	3.000	0	0	83.667
27	61	60.000	45.000	0	0	0	3.000	0	0	76.687
28	62	60.000	45.000	0	0	0	3.000	0	0	69.288
29	63	60.000	45.000	0	0	0	3.000	0	0	61.445
30	64	60.000	45.000	0	0	0	3.000	0	0	53.132
31	65	38.000	32.000	0	0	0	0	25.000	0	75.320
32	66	38.000	32.000	0	0	0	0	0	0	73.839
33	67	38.000	32.000	0	0	0	0	0	0	72.269
34	68	38.000	32.000	0	0	0	0	0	0	70.605
35	69	38.000	32.000	0	0	0	0	0	0	68.842
36	70	38.000	32.000	0	0	0	0	0	0	66.972
37	71	38.000	32.000	0	0	0	0	25.000	0	89.990
38	72	38.000	32.000	0	0	0	0	0	0	89.390
39	73	38.000	32.000	0	0	0	0	0	0	88.753
40	74	38.000	32.000	0	0	0	0	0	0	88.078
41	75	38.000	32.000	0	0	0	0	0	0	87.363
42	76	38.000	32.000	0	0	0	0	0	0	86.605
43	77	38.000	32.000	0	0	0	0	0	0	85.801
44	78	38.000	32.000	0	0	0	0	0	0	84.949
45	79	38.000	32.000	0	0	0	0	0	0	84.046
46	80	38.000	32.000	0	0	0	0	0	0	83.089

The couple might feel more comfortable (in financial sense) when they buy a smaller house for about 200.000 MU. In this case, their wealth would increase to 285.937 MU at age 80. On the other hand, if the couple plans to buy a house of about 300.000 MU, the situation changes to worse again. In that case, the debt increases from year to year up to 628.232 MU at age 80. The last financial plan is clearly not feasible. It turns out that either some of the planned expenses have to be reduced or timepoints for the realization of some of the expenses have to be redefined. In addition, the couple might come to additional income by selling some of their assets. In this case again, the couple has to choose the appropriate assets for selling and they have to determine optimal timepoints of selling. As a result, the financial planner has to develop a feasible financial plan such that the couple can maximize the utility of the incomes and expenses by using money the best way for themselves. In the next section, we develop a mathematical program to solve such a problem.

5 Optimality of a personal financial schedule

We define a financial plan as *optimal* if it is feasible and if the sum of utility of the incomes and expenses and final surplus income in the last point n of the planning horizon takes maximal value. The optimization process involves identifying incomes and expenses, weighting each of the additional essential incomes and expenses, and defining optimal timepoints for the realization of these incomes and expenses. The optimization problem is usually harder than the problem of testing feasibility of the concrete financial plan as described in Section 4.

We will consider the following objective function

$$f(x,y) = v_i \cdot \sum_{k=imin}^{imax} \alpha_k \cdot t(y_k) + v_e \cdot \sum_{i=1}^{emax} \beta_i \cdot t(x_i) + v_s \cdot hv(F_n) \tag{7}$$

which value has to be maximized. The objective function $f(x,y)$ is equal to the sum of three summands. The first two summands show how good the level of life is during the planning horizon, the last summand predicts how good the level of future life (after the planning horizon $(0,n]$) may be, due to financial conditions defined by difference of total savings and total debt at timepoint n.

Let v_i, v_e and v_s be values that indicate the importance of the classes of incomes, expenses, and final surplus income, respectively, for the decision maker. These preference values have to be determined by multicriteria decision making tools such as AHP, the Analytic Hierarchy Process [14, 23]. AHP consists of electing pairwise comparisons from the decision maker and then applying the Eigenvector theory in order to obtain the set of weights most consistent with the pairwise comparisons. The greater the weight determined with AHP, the greater the relative importance of the class of incomes, expenses or final surplus income.

In *financial scheduling*, we have to decide about realization timepoints x_i and y_k of additional expenses and incomes, respectively. We assume that the heights of planned incomes and expenses are fixed and do not consider *height preferences*.

In equation (7), the unknown variables are integer vectors x and y. As there are finite number of possible vectors x and y, we can enumerate them as follows. Let $x^{(t)} = \left(x_1^{(t)}, x_2^{(t)}, \ldots, x_{emax}^{(t)}\right)$ and $y^{(t)} = \left(y_{imin}^{(t)}, y_{imin+1}^{(t)}, \ldots, y_{imax}^{(t)}\right)$, be vectors of realization timepoints of additional expenses and incomes, respectively. Integer $x_i^{(t)}$ (integer $y_k^{(t)}$, respectively) means the end of period $(k-1, k]$ chosen for the realization of a specific expense e_i (income i_k), $k \in \{0, 1, \ldots, n\}$. An ordered pair $(x^{(t)}, y^{(t)})$ of these two vectors is called a *personal financial schedule* (PFS). The personal financial schedule with maximum objective value $f(x^{(t)}, y^{(t)})$ is denoted by σ^*.

If we assume that an expense e_i (or a not realized income i_k) gives equal value in each period of using, we can define $t(x_i)$ and $t(y_k)$ as

$$t(x_i) = 1 - \frac{x_i}{n} \tag{8}$$

and for the incomes as

$$t(y_k) = \frac{y_k}{n}. \tag{9}$$

As an example, let e_a mean the expense of buying a car. The larger x_a will be, the more years will be given to the client to use this car in the planning horizon. Almost the same we can say about buying a house or other assets. If e_a is realized at timepoint $x_a = j$, then the client has usage of this car in the rest of the planning horizon, i.e., for $n - j$ periods. This reflects the "earlier is better" principle.

Similarly, let i_a mean the income of selling jewellery or the discounted amount of a second job. If the timepoint y_k for selling the jewellery is equal to j, then the client can delight in the jewellery for j periods before saling it. The larger y_k will be, the more years will be given to the client to use this asset. This reflects the "later is better" principle. Almost the same can be said about selling a summer house, a second flat, or jewellery.

It is clear that if $x_i = n$ ($y_k = 0$, respectively), buying (selling) a corresponding asset would have no influence on the first (second) summand in the objective function (7). In fact, equality $x_i = n$ ($y_k = 0$) will mean that the client would not have any use of the corresponding asset during the planning horizon. Equality $x_i = n$ can be interpreted as the fact that the corresponding asset x_i will be not bough in the planning horizon. Similarly, for each asset i_k for sale action, equality $y_k = 0$ will mean that i_k will be not sold in the planning horizon.

The coefficients $\alpha_k,\ k \in \{imin, imin+1, \ldots, imax\}$ and $\beta_i,\ i \in \{1, 2, \ldots, emax\}$ are the *time preferences* concerning additional incomes i_k and expenses e_i. They have to be defined with the help of multicriteria decision analysis methods. A small β means that a specific expense for buying an asset can be realized late, and a small α means that a specific income from selling an asset can realized early in the planning horizon (it is not so important for the decision maker to own that specific asset for a long time).

Final surplus income F_n is equal to S_n (in case of $S_n \geq 0$) or $-C_n$ (in case of $C_n \geq 0$). As greater the final surplus income, as better the level of future life (after the planning horizon $(0, n]$) is assumed. In the following, we describe the derivation of a linear height preference function $hv(F_n)$. Because of interest rate effects, final surplus income F_n is minimal if all of the expenses are realized at timepoint 0 and if all of the incomes are realized at timepoint n. As a result, minimum final surplus income F_n^- can be computed in time $O(n+m)$ as shown in Fig. 3:

Calculating minimum final surplus income

$F_n^- = S_0;$
for $(k = 1, \ldots, emax)$ $F_n^- = F_n^- - e_k;$
for $(k = 2, \ldots, n)$
$\{$
    if $(F_n^- > 0)$
        $F_n^- = F_n^- \cdot (1 + r_k^+) + I_k^{fix} - E_k^{fix};$
    else
        $F_n^- = F_n^- \cdot (1 + r_k^-) + I_k^{fix} - E_k^{fix};$
$\}$
for $(k = imin, \ldots, imax)$ $F_n^- = F_n^- + i_k;$

Figure 3: Calculating minimum final surplus income

Similarly, final surplus income is maximal if the expenses are realized at timepoint n and if the incomes are realized at timepoint 0. The procedure is similar to the procedure above. Note that F_n^- and F_n^+ may both be positive or both negative. Assuming a linear height preference for the final surplus income, we obtain

$$hv(F_n) = \frac{F_n - F_n^-}{F_n^+ - F_n^-}. \tag{10}$$

Moreover, we allow to buy and sell the same asset during the planning horizon $(0, n]$. Therefore, one has to introduce both, an expense e_a (with realization timepoint x_a) and an income i_a (with realization timepoint y_a) with the same index a. Such an asset (e.g., a car) may be bought and sold during the same planning horizon. For such type of assets one has to introduce additional linear equality or inequality like $x_a \leq y_a + c_a$ where c_a is constant input parameter. As a result, we come to the integer programming problem for financial scheduling depicted in Fig. 4.

Let m denote the number of variables in the mathematical programming problem shown at Fig. 3: $m = emax + imax - imin + 1$. Next we prove the following claim.

Lemma 2: There are totally $O(n^m)$ possible schedules of buying and selling maximum m assets at timepoints $0, 1, \ldots, n$.

Proof. The largest number of possible values for each of m variable x_i and y_k is $n+1$ (in this case there are no equation restricted possible values for each variable, i.e., $imin = emax + 1$). Thus, there exist at most $(n+1)^m$ possible schedules. And so the number of schedules is asymptotically bounded by $O(n^m)$. $\square$

Integer programming problem for calculating optimal PFS

Maximize $f(x, y) = v_i \cdot \sum_{k=imin}^{imax} \alpha_k \cdot t(i_k) + v_e \cdot \sum_{i=1}^{emax} \beta_i \cdot t(e_i) + v_s \cdot hv(F_n)$
subject to

(1)–(n)	$I_j = I_j^{fix} + \sum_{y_k=j} i_k$	$j = 1, 2, \ldots, n;$
(n+1)–($2n$)	$E_j = E_j^{fix} + \sum_{x_i=j} e_i$	$j = 1, 2, \ldots, n;$
($2n$+1)–($3n$)	$h_j = S_{j-1} \cdot (1 + r_j^+) - C_{j-1} \cdot (1 + r_j^-) + I_j - E_j$	$j = 1, 2, \ldots, n;$
($3n$+1)–($4n$)	$C_j = \max\{0,\ -h_j\}$	$j = 1, 2, \ldots, n;$
($4n$+1)–($5n$)	$S_j = \max\{0,\ h_j\}$	$j = 1, 2, \ldots, n;$
($5n$+1)–($6n$)	$C_j \leq D_j$	$j = 1, 2, \ldots, n;$
($6n$+1)–($7n$)	$S_j \geq L_j$	$j = 1, 2, \ldots, n;$
($7n$+1)–($7n$+$emax$–$imin$+1)	$x_a\ \{\leq, =, \geq\}\ y_a + c_a$	$a = imin, imin + 1, \ldots, emax;$
	$x_i \in \{0, 1, \ldots, n\}$	$i = 1, 2, \ldots, emax;$
	$y_k \in \{0, 1, \ldots, n\}$	$k = imin, imin + 1, \ldots, imax.$

Figure 4: Integer programming problem for calculating optimal personal financial schedule

Calculation of the objective function for each fixed schedule is restricted by $O(n)$. And due to Lemma 5, to solve mathematical programming problem shown at Fig. 3 via direct enumerative method takes $O(n^{m+1})$ time.

It should be noted that in real financial scheduling one can assume that numbers $imax$ and $emax$ of essential incomes and expenses which have to betreated for a family are not large. Therefore, assuming that m is restricted by a constant we can conclude that direct enumerative method has polynomial complexity $O(n^m)$ where m is constant. One can also assume that number n is restricted because for a large planning horizon the most numerical data are uncertain and so there is no sense (even not possible) to construct reliable financial plan with the help of a financial schedule for a large planning horizon. We consider the following example.

Let only $m = emax = 2$ expenses (kitchen 12.000 MU and car 10.000 MU) and $n = 2$ timepoints be considered. At timepoint 0, we have $S_0 = 100.000$ MU in investment. Regular incomes − regular expenses are 5.000 MU each year. The savings and credits interest rates are $r_1^+ = r_2^+ = 5\%$ and $r_1^- = r_2^- = 8\%$. Debt limits are $D_1 = D_2 = 0$ money units.

As depicted in Fig. 5, there are $(n+1)^m = 9$ possible schedules.

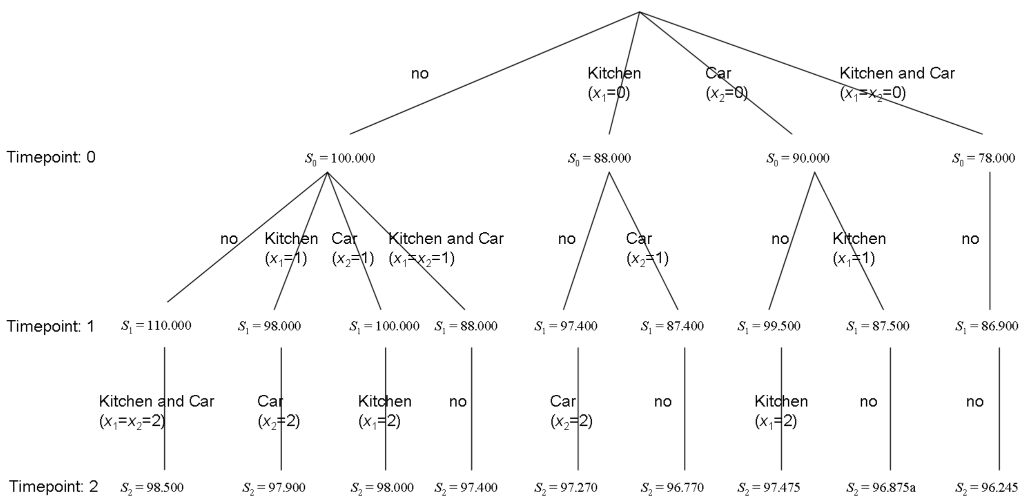

Figure 5: $m = 2$ assets and $n = 2$ timepoints.

Provided that numbers $imax$, $emax$, and n are sufficiently small, one can use direct enumeration scheme for solving the integer programming problem given in Fig. 4 with small problem size. The algorithm for such complete enumeration of all feasible financial schedules is given in Fig. 6.

Algorithm 2: *Optimal financial schedule*

Input: $n, emax, imax, E_0, I_0,\ E_1^{fix}, \ldots, E_{emax}^{fix},\ I_1^{fix}, \ldots, I_{imax}^{fix},$
$i_{imin}, \ldots, i_{imax},\ e_1, \ldots, e_{emax},\ r_1^+, \ldots, r_n^+,\ r_1^-, \ldots, r_n^-,$
$\alpha_1, \ldots, \alpha_{imax},\ \beta_1, \ldots, \beta_{emax},\ \gamma,\ D_1, \ldots, D_n,\ L_1, \ldots, L_n,\ v_i, v_e, v_s$

begin

1. $start = I_0 - E_0;\quad f^* = 0;\quad m = emax + imax - imin + 1;$
2. **for** $(j = 1, \ldots, n)\ \ E[j] = E_j^{fix};$
3. **for** $(j = 1, \ldots, n)\ \ I[j] = I_j^{fix};$
4. **for** $(t = 1, \ldots, (n+1)^m)$
5. {
6. $S_0 = start;$
7. $E_0 = 0;$ **for** $(j = 1, \ldots, n)\ E_j = E_j^{fix};$
8. $I_0 = 0;$ **for** $(j = 1, \ldots, n)\ I_j = I_j^{fix};$
9.   **for** $(j = 1, \ldots, emax)\ \ E[A[j][t]] = E[A[j][t]] + e_j;$
10.   **for** $(j = imin, \ldots, imax)\ \ I[A[m - imax + j][t]] = I[A[m - imax + j][t]] + i_j;$
11.   **for** $(j = 0, \ldots, n)$
12.   {
13.   **if** $(j = 0)$
14. $h_0 = I_0 - E_0 + S_0;$
15.   **else**
16. $h_j = (I_j - E_j + S_{j-1} \cdot (1 + r_j^+) - C_{j-1} \cdot (1 + r_j^-);$
17.   **if** $(h_j < 0)$
18. $C_j = -h_j;$
19.   **else**
20. $C_j = 0;$
21. $S_j = h_j + C_j;$
22.   **if** $(C_j > D_j$ OR $S_j < L_j)$
23.   {
24. $f = 0;$ (financial schedule is not feasible)
25.   **goto** line 4;
26.   }
27.   }
28.   // Compute objective value
29. $f = v_i \cdot \sum_{k=imin}^{imax} \alpha_k \cdot t(y_k) + v_e \cdot \sum_{i=1}^{emax} \beta_i \cdot t(x_i) + v_s \cdot hv(F_n);$
30.   **if** $(f > f^*)$
31.   {
32. $f^* = f;$
33.   **for** $(j = 1, \ldots, emax)\ \ x^{(j)} = A[j][i];$
34.   **for** $(j = emax + 1, \ldots, m)\ \ y^{(j)} = A[j][i];$
35.   }
36. }

. . . to be continued on next page

```
37. if (f* > 0)
38.       print ("Optimal financial schedule: [(x*, y*)].";
39. else
40.       print ("There is no feasible financial schedule.");
end;

Output:
"Optimal financial schedule: [(x*, y*)]." or
"There is no feasible financial schedule." — O(n^m + m)
```

Figure 6: Construction of the optimal financial schedule.

Lines 6-8 initialize starting surplus income S_0 and sets the values of incomes and expenses to fixed incomes and expenses. In lines 9-10, the variable incomes and expenses are added to the fixed incomes and expenses. We consider all of the $(n+1)^m$ permutations of $m = imax + emax$ assets that can be bought or sold over the planning horizon. Therefore, we introduce for each asset a an array $A[a][t]$ that gives the realization timepoints of each asset a where $t = 1, 2, \ldots, n^m$ is the number of the financial schedule. Furthermore we assume that the assets are arranged in an order such that at first the expenses occur in A and afterwards the incomes, i.e. we have the order $a = 1, \ldots, emax, emax + 1, \ldots, m$, where $m = emax + imax - imin + 1$. The assignment of the expenses in line 10 is evident. We assign for each $j = 1, \ldots, emax$: $E[A[j][t]] = E[A[j][t]] + e_j$. A little bit more complicated is the assignment of the incomes. Here, we have to obeye that $imin$ may be less or equal to $emax$, and we assign for each $j = imin, \ldots, imax$: $I[A[m - imax + j][t]] = I[A[m - imax + j][t]] + i_j$. In lines 22-26, the liquidity constraints are checked.

A financial schedule is not feasible if there is no solution that follows the liquidity restrictions during the whole planning horizon. Lines 11-27 compute the surplus income S_j

or surplus credit C_j, respectively. The objective value f is computed in line 29. In lines 30-35, the best found schedule is stored together with its objective value f. Timepoints of realization of the incomes and expenses are stored in lines 33 and 34.

We continue the above example. Let us assume that with the help of AHP we have determined the following values: The realization of the assets (kitchen and car) is for the decision maker of equal importance as her (his) final surplus income, i.e. $v_e = v_s = 0.5$*). Concerning the time preferences for kitchen and car, we come to the conclusion that buying a kitchen is slightly more important for the decision maker than buying a car (*$\beta_1 = 0.55$ *for the kitchen,* $\beta_2 = 0.45$ *for the car). The optimal solution is to buy the kitchen and the car at timepoint 1 of the planning horizon. Table 4 shows the results. In the first row, the realization timepoints of kitchen and car are displayed. In the second row the objective values are displayed. The optimal financial schedule is* $(1, 1)$ *with value* 0.5061*, marked with a star. If we change the values* β_1 *to* $\beta_1 = 0.70$ *and* β_2 *to* $\beta_2 = 0.30$*, we come to an optimal schedule* $(0, 2)$*, i.e. kitchen should be realized immediately (before starting financial scheduling), and car should be realized after the planning period. If we further change the importance of expenses in comparison to final surplus income to values* $v_e = 0.30$ *and* $v_s = 0.70$*, the optimal schedule would be* $(2, 2)$*, i.e. the expenses should not be realized during the planning horizon. And, finally, if we change these values again to* $v_e = 0.30$ *and* $v_s = 0.70$*, the result is a schedule* $(0, 0)$*, i.e. both assets should be bought* *immediately.*

Table 4: Objective values for different realization timepoints

(0,0)	(1,0)	(2,0)	(0,1)	(1,1)	(2,1)	(0,2)	(1,2)	(2,2)
0.5000	0.5022	0.4977	0.5039	0.5061*	0.5016	0.5023	0.5045	0.5000

Let us have a look a second example.

We have $n = 5$ *periods where each period is* 5 *years. Starting capital is* 10.000 *MU, fixed incomes - fixed expenses are* 20.000 *MU in each period, debt restriction is* -25.000 *MU. The savings and credits interest rates are* $r_j^+ = 5\%$ *and* $r_j^- = 8\%, j = 1, \ldots, n$. *There are four variable expenses in heights of* $e_1 = 20.000$ *MU,* $e_2 = 30.000$ *MU,* $e_3 = 30.000$ *MU,* $e_4 = 40.000$ *MU and one variable income in height of* $h_5 = 20.000$ *MU to decide of. The relative importance of the expenses for the decision maker is* $0.10, 0.20, 0.25, 0.45$*, i.e.,* e_1 *gets the value* 0.10*,* e_2 *gets* 0.20*,* e_3 *gets* 0.25*, and* e_4 *gets* 0.45*. Concerning the relative importance of the classes of incomes, expenses, and final surplus income, the decision maker's preferences can be quantified as* 0.05 *for the incomes,* 0.40 *for the class of the expenses, and* 0.55 *for the class of the final surplus income. Totally, there are* $6^5 = 7776$ *financial schedules to be investigated. It turns out that the following schedule is optimal:*

$$\sigma^* = (5, 4, 2, 0, 0) \text{ with objective value } f^* = 0.6231.$$

We have $x^* = (5, 4, 2, 0)$ *and* $y^* = (0)$. e_1, e_2, e_3, e_4 *have to be realized at time periods* $5, 4, 2, 0$*, i.e.,* e_1 *has to be realized after 25 years,* e_2 *in the third period*

(10-15 years), e_3 in the second period (5-10 years), and e_4 directly. i_5 has also be realized directly. The optimal financial schedule is always a feasible schedule, i.e., the credit limit of 25.000 *MU is never exceeded. Final surplus income lays between* −24.294 *and* 28.801 *MU, and the optimal schedule leads to a final surplus income of* 11.145 *MU.*

6 Summary

Personal financial planning investigates the question of how solid planning of financial affairs can contribute to our fundamental goals. Part of comprehensive personal financial planning consists of planning all future incomes and expenses during a planner's lifetime. In this paper, we adopt the view on realization timepoints of extraordinary and variable incomes and expenses. The corresponding problem is the financial scheduling problem of personal financial planning, i.e. personal financial scheduling (PFS). Conflicting objectives with different goals of varying levels of importance for the decision maker might be involved in this decision problem. As a result, PFS can be considered as a multicriteria decision making problem, and mathematical programming appears as an appropriate method for formulating and solving it. We develop a model of personal financial scheduling, derive a mathematical program, and show the effectiveness of this approach by providing a numerical example. It is clear that some input data for personal financial scheduling are random in nature. Moreover, it is not realistic to obtain a reliable probability distribution for some random numerical parameters. In usual practice, one can only assume that reliable lower and upper bounds for real values of the uncertain numerical parameters [22] are known before personal financial scheduling. As a result, further research is necessary to handle the personal financial scheduling with uncertain numerical parameters.

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