Determining the Optimal Amount and Location of Inventory in a Two-Stage Supply Chain

Track Title: Models and Methods for Operations Systems

Jishnu Hazra
Indian Institute of Management
Bangalore 560076, India
Email: hazra@iimb.ernet.in

Peruvemba S. Ravi
School of Business and Economics
Wilfrid Laurier University
Waterloo, Canada N2L 3C5
Email: pravi@wlu.ca

Abstract

We consider a supply chain consisting of a manufacturer and a distributor. The manufacturer has finite capacity and the distributor faces random but known demand distribution. Demand that cannot be fulfilled is backordered. The supply chain is centrally controlled and two decisions are made every period: how much to release to the factory and how much to ship to the distributor. The objective is to minimize the total costs which include both fixed and variable costs. The model provides insight into the impact of various parameters on the optimal amount and location of inventory in a two-stage supply chain.

Introduction

With increased availability of information because of ERP and point-of-sales data, companies must develop decision support systems to exploit this information for better decision-making. For example, many companies that have implemented ERP either do not use the available information to its full potential or it is simply "business as before ERP". This leads to disappointment as it does not result in significant inventory, lead time or cost reduction as advertised by ERP vendors.

In this context, we formulate a simple model that determines optimal operating policies in a two-stage supply chain with real-time demand information. Specifically, demand is stochastic with known probability distribution and is available to the decision-maker after it has been realized. Both production and distribution systems are capacitated with certain cost structure. The objective of this modeling exercise is to get insights into the effect of real time demand information and its usefulness under different scenarios such as capacity, demand variability and manufacturing and transportation flexibility.

Recently, there has been great deal of interest in coordination of supply chain. Readers may refer to Tayur (1998) for detailed reviews on supply chain coordination.

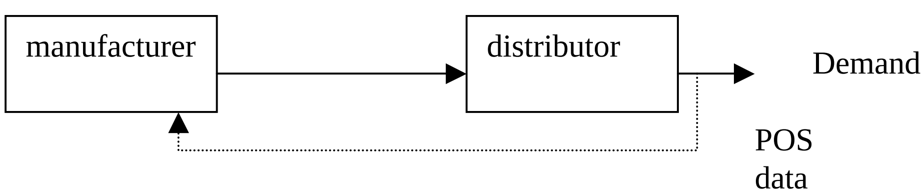
The Problem Description

We consider a two stage system consisting of a manufacturer and a distributor dealing with a single product. The distributor faces a random demand whose probability mass function is known. Demand that cannot be fulfilled immediately is backordered. If the backorder reaches a given threshold value and if a demand occurs then that demand is lost. The manufacturer collects demand information every period (through point of sales data). In other words, on-hand inventory level is known to the manufacturer. Finished goods inventory is also stocked at the manufacturer's warehouse.

The manufacturer makes the product based on current inventory at its end, inventory at the distributor (this information is known) and cost. We assume infinite raw material is available for the manufacturer. The manufacturer has finite capacity given by *C* units per period. It also has to make another decision: that is whether to ship to the distributor or not. In other words, the manufacturer has decide every period as whether to release a production order, the size of the production order (how much to make) and whether to ship from its finished foods (FG) to distributor and how much to ship. Note that products can be shipped only if there is FG inventory at the manufacturer's end.

Sequence of events every period:

i) Shipment reaches the distribution center at the start of the period.
ii) A demand of random amount (possibly zero) takes place.
iii) Unsatisfied demand results in backlogging or lost sales.
iv) Manufactured amount (if any) at the factory becomes FG inventory at the factory.
v) At the end of the period, review the state of the system, decisions are made and costs are incurred.



- Decision Set:
  - To release an order to the shop floor and its size
  - To ship an order to the distribution centre and its size.
- Cost Structure: Costs are measured as "cost per period". It consists of the following components and is a combination of both variable and fixed costs.
  - Inventory cost at factory and distribution centre

- Work-in-process inventory carrying cost
- Lost Sales
- Backlog cost at the distribution centre
- Fixed cost of releasing an order
- Fixed cost of transportation

- Response Time
  - Manufacturing lead time is incurred because there is a finite capacity of *C* units per period.
  - Shipment lead time from the factory to the distributor centre is effectively zero because products are shipped at the end of the period and received at the beginning of the next period.

- Service Level
  - Service level is defined as the fraction of demand that is met directly “off the shelf” at the distributor centre.

Stochastic Dynamic Program Formulation

Infinite horizon average cost per period minimization model.

Notation:

Decision Vector: $(\delta_1, \delta_2) =$ (Amount Shipped, Order release amount)

State space description: (x_1, x_2, y)

x_1 : Factory FG inventory at the end of the current period.

x_2 : Distribution Centre FG inventory at the end of the current period.

y : WIP inventory in the factory at the end of the current period.

h_1 : Inventory holding cost ($/unit/period) at the factory warehouse

h_2 : Inventory holding cost ($/unit/period) at the distribution centre

h_3 : *WIP* Inventory holding cost ($/unit/period) at the shop floor

b_1 : Backorder cost ($/unit/period)

b_2 : Lost sales cost ($/unit)
FC_1 : Fixed cost of shipping order from factory to warehouse
FC_2 : Fixed cost of releasing an order to shop floor.
C : Capacity of the Factory (units/period)
$\Pr(D = d_k) = p_k$ is the probability distribution of demand.
$I(k)$: Indicator function, takes a value 1, if condition k is true, else 0.

$$v_i = \min_{a\in A}(c_i(a) - g + \sum_j P_{ij}(a)v_j)$$

v_i : relative value function of state i
g : cost per time of the optimal policy
$c_i(a)$: cost incurred at state i by selecting decision a
$P_{ij}(a)$: Transition Probability vector for decision a

When decisions δ_1, δ_2 are made then the next state is given by $(x_1^{next}, x_2^{next}, y^{next})$.

$$(x_1^{next}, x_2^{next}, y^{next}) = (x_1 + \min(C, S_1 - x_1 + \delta_1, y + \delta_2) - \delta_1, \max(-S_{22}, x_2 + \delta_1 - d_k), y + \delta_2 - \min(C, S_1 - x_1 + \delta_1, y + \delta_2)$$

$$\delta_1 = 0,1,\cdots,\min(x_1, S_{21} - x_2)$$

$$\delta_2 = 0,1,\cdots,(W - y)$$

$$0 \le x_1 \le S_1$$

$$-S_{22} \le x_2 \le S_{21}$$

$$0 \le y \le W$$

In the above formulation, we have assumed that the x_1, the finished goods at the factory cannot exceed S_1. Similarly, the on-hand inventory at the distribution centre cannot exceed S_{21} and the backorder cannot exceed S_{22}. Any demand that results in backorder exceeding S_{22} is lost. Work-in-process (WIP) inventory at the factory cannot exceed W; in other words if the WIP inventory is W then no more order is released.

The reason for having such threshold is two-fold:
a) Make the problem computationally tractable,
b) It is not uncommon for companies to have such limits on various types of inventory (and backorders) because of lean manufacturing and distribution practices.

Cost function: At the end of the current period if the state of the system is given by (x_1, x_2, y) then the cost incurred is given by the following expression:

$$h_1 x_1 + h_2 x_2 I(x_2 \geq 0) + h_3 y - b_1 x_2 I(x_2 < 0) + b_2 \sum_{\forall d_k} \max(0, -S_{22} - x_2 - \delta_1 + d_k) \Pr(D = d_k)$$

$$+ FC_1 I(\delta_1 > 0) + FC_2 I(\delta_2 > 0)$$

The first three terms are inventory holding costs at the factory warehouse, distribution centre and work-in-process stage. The next two terms are the backorder costs and the expected lost sales cost (lost sales will occur only if backorder exceeds S_{22}). Finally the last two terms are fixed cost of shipping an order from the factory to the distribution centre and releasing an order to the shop-floor respectively.

Computational Algorithm

As the state space is typically large we utilize the value iteration (Puterman,1994 and Tijms, 1994) described below.

1. Initialize $V^0(i) = \min_a c_i(a), \ \forall i.$

2. For each i, compute $V^n(i) = \min_a \left\{ c_i(a) + \sum_{\forall j} p_{ij} V^{n-1}(j) \right\}$.

3. Compute bounds $m_n = \min_i (V^n(i) - V^{n-1}(i))$ and $M_n = \max_i (V^n(i) - V^{n-1}(i))$.
If
$0 \leq M_n - m_n \leq \varepsilon m_n$, where ε is a pre-specified tolerance value, then stop else go to step 2.

The optimal cost per period is given by $g = Lim_{n \to \infty} (V^n(i) - V^{n-1}(i))$.
Size of the state space is given by $(W+1)(S_1+1)(S_{21}+S_{22}+1)$.

Numerical Results

We have carried to some numerical analysis to get some insights in the behavior of the optimal solution.

$S_1=12$; $S_{22}=10$;*S21=14*; $W=14$; $h_1=h_2=1$; $h_3=0.5$; $b_1=1.5$; $b_2=2$; $FC_1=FC_2=3$;
$C=5,6,7$ units

We assume a 3-point distribution: $\Pr(D = d_k) = p_k$

d_k	p_k	p_k	p_k	p_k
1	0.05	0.1	0.3	0.4
4	0.9	0.8	0.4	0.1
7	0.05	0.1	0.3	0.5

CoV	Cost/ Utilization	Cost (67 % Utilization)	Cost (57 % Utilization)
0.237	11.842/80%	11.715/67%	11.671/57%
0.335	13.533/80%	13.356/67%	13.280/57%
0.581	17.202/80%	17.131/67%	16.757/57%
0.658	18.940/86 %	18.532/72%	17.626/61.4

Table 1: Effect of Variability and Utilization on Cost

CoV	Factory Inventory	DC Inventory	Avg. Backorder	Factory /Total	Service level
0.237	4.451	0.4769	O.457	0.903	0.886
0.335	4.944	0.8126	0.899	0.859	0.775
0.581	6.189	2.435	1.583	0.718	0.603
0.658	6.960	2.594	1.969	0.728	0.542

Table 2: Variability and Optimal Inventory Placement

We find that utilization does not affect significantly the cost per period, unless utilization is extremely high in which lost sales becomes the dominant component of the total cost. However, we find that variability (as measured by coefficient of variation) does indeed affect total cost primarily through higher inventories. This is a well known trade-off between inventory and uncertainty. We also found that as uncertainty increases inventory is shifted from the factory warehouse to the distribution center and a decrease in service level.

References

Puterman, M.L. *Markov Decision Processes*, John Wiley and Sons, 1994
Tayur S., Ganeshan R., Magazine M. (Editors). *Quantitative Models for Supply Chain Management*, Kluwer Academic Press, 1998.
Tijms, H.C. *Stochastic Models*, John Wiley and Sons, 1994