Switching Costs, Dynamic Uncertainty, and Buyer-Seller Relationships

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Abstract

We analyze strategic relationships between buyers and sellers in markets with switching costs and dynamic uncertainty by investigating the scenario wherein a representative buyer trades with two foreign sellers located in the same foreign country. We show that, under exchange rate uncertainty, switching costs may lead to switching equilibria where both sellers co-exist in the market with the buyer, or no-switching equilibria where either seller captures the market. Low levels of exchange rate uncertainty facilitate competition by allowing the sellers to co-exist in the market with the buyer. However, if the level of uncertainty is beyond a threshold, the only viable equilibria are those where one of the sellers captures the market. Further, depending on the level of exchange rate uncertainty and the sellers’ variable costs, switching costs may either raise or lower the level of prices in long-term contracts between the buyer and the sellers.

Key Words: Strategic Relationships; Dynamic Uncertainty; Switching Costs; Real Options; International Trade
1. Introduction

One of the important themes in the industrial organization literature is the understanding of how differences in the relative bargaining power of firms arise and affect their strategic relationships. In particular, there is a well-developed body of literature that examines the role of switching costs in influencing the relative bargaining power of buyers and sellers and the resulting effect on their strategic relationships (Klemperer 1995, Farrell and Klemperer 2002). However, the extant literature has not fully explored the impact of switching costs on buyer-seller relationships in the presence of some form of exogenous dynamic uncertainty. We analyze the impact of such uncertainty on the strategic relationships between buyers and sellers. There are several economic scenarios wherein the interplay between both switching costs and dynamic uncertainty significantly affects buyer-seller relationships. For the sake of concreteness, however, we focus on the scenario wherein buyers in one country trade with sellers in a foreign country so that both buyers and sellers are exposed to exchange rate uncertainty.

An investigation of this scenario is particularly relevant due to the dramatic acceleration in globalization that has led to increasing trade between buyers and sellers in different countries. This, in turn, has led to the geographic clustering of firms in particular industries in specific countries that have been able to develop comparative advantages in these industries for various reasons such as government policies, geographic location, natural resources, and skilled low-cost labor (Porter 2000, Monfort and Nicolini 2000). Our model and results have implications for how multi-national firms engaging in business in a foreign country may operationally hedge their

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1 Firms with multiple potential suppliers, banks that screen or monitor customers, consumers in search of the best bargain on a product, have to incur relationship-specific, sunk/set-up/search costs that are offset by lower variable costs. The prices quoted by the suppliers, loan payments made by bank customers, and prices of competing products are all affected by the presence of some form of exogenous, dynamic uncertainty.

2 For example, China in manufacturing, India in business process outsourcing, the Middle East in oil extraction, South Africa in diamond mining, South Korea in shipbuilding, Taiwan in semiconductor manufacturing.
exchange risk exposure by establishing relationships with multiple suppliers in the foreign
country and the resulting effect on the nature of competition between the suppliers.

Our analysis of this problem in a continuous time setting leads to novel predictions
regarding the impact of the interplay between exchange rate uncertainty and switching costs on
buyer-seller relationships. We show that, depending on the exchange rate volatility, the presence
of switching costs may either increase or decrease the level of prices in a mature market. When
exchange rate volatility levels are low, multiple sellers may co-exist in the market. However, the
number of co-existing sellers decreases with exchange rate volatility, implying that seller
concentration increases with exchange rate volatility. Hence, our analysis highlights the
significant impact of the interplay between switching costs and exchange rate uncertainty on
global buyer-seller relationships.

We obtain our results within a parsimonious duopoly model that considers a single,
representative buyer in a country dealing with two potential sellers in a foreign country. The
buyer has a constant, inelastic demand for the product at any instant of time and derives constant
utility from each unit of the product (Dixit 1989). Either seller can completely fulfill the buyer’s
demand so that the buyer will not be in relationships with both sellers simultaneously. We
analyze equilibria of the three-player game between the buyer and the sellers where the buyer
enters into long-term contracts (we model economic scenarios where such contracts are
enforceable) with the sellers and incurs different switching costs vis-à-vis these non-identical
sellers that are observable and verifiable. The sellers quote prices in their currency for each

3 We refer to a mature market as one where switching costs are already present (Klemperer 1995).
4 Joskow (1987) finds that in markets with significant switching costs, buyers and sellers both prefer to enter into
long-term contracts ex ante and Carlton (1986) observes long-term rigidity in the prices of intermediate goods.
unit of a product. The buyer responds to the sellers’ quoted prices by choosing to be either out of the foreign market or in a relationship with one of the sellers whenever it is in the foreign market at any instant of time.

We first show that the buyer’s switching option to switch between the sellers over time has positive value if and only if the ratio of seller prices lies in a non-empty bounded interval that depends on the switching costs and the exchange rate process. The existence of this non-empty interval is therefore a necessary condition for switching equilibria where the sellers co-exist in the market with the buyer, that is, each has a nonzero probability of being in business with the buyer over time. Next, we show that if the interval is non-empty and the sellers have constant variable costs of production in their currency, then a sufficient condition for the sellers to co-exist in any equilibrium with the buyer is that the ratio of their variable costs lies within this interval.

One of our main analytical results is the demonstration of the existence of a critical exchange rate volatility level above which the buyer’s switching option has zero value for all possible prices quoted by the sellers. Therefore, above this level, the only viable equilibria are no-switching equilibria where one of the sellers captures the market, that is, obtains all possible business with the buyer. The level of prices when the seller with the switching-cost advantage (disadvantage) captures the market is typically higher (lower) than the level of prices if there were no switching costs. Therefore, in the presence of exchange rate uncertainty, switching costs may either raise or lower the level of prices. In the absence of uncertainty, one of the two non-identical sellers, in general, captures the market. The presence of uncertainty makes co-existence feasible, but this feasibility disappears when the volatility is beyond a threshold.

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5 This is consistent with considerable empirical evidence that documents the presence of the Grassman bias in bilateral international trade, that is, a majority of international trade contracts are invoiced in the exporters' currency (Grassman 1973, Magee and Rao 1980, Cornell 1980, Viaene and De Vries 1992).
Von Weizsacker (1984) considers a framework with long-term constant price contracts between buyers and sellers in the same country and concludes that switching costs increase competition and lower the level of prices. On the other hand, Klemperer (1987), Beggs and Klemperer (1992), and Padilla (1995) use multi-period deterministic frameworks where sellers compete in each period to conclude that, in general, switching costs raise the level of prices in a mature market and relax competition. Our results that, in the presence of exchange rate uncertainty, switching costs may either raise or lower the level of long-term prices and lead to switching equilibria where both sellers co-exist or no-switching equilibria where one of the sellers captures the market with the buyer, contrast with the above results. The fact that the exchange rate volatility plays a crucial role in determining the actual equilibrium outcome (that is, co-existence or market capture), and the corresponding level of prices, is a novel insight offered by the explicit incorporation of dynamic uncertainty within our framework.6

It is also interesting to compare our result that the buyer’s switching option has zero value beyond a critical value of the exchange rate volatility with a result of Dixit (1989). Dixit (1989) analyzes the entry-exit decisions of a buyer with a single seller in a foreign market, and shows that the buyer delays entry into the foreign market as the exchange rate volatility rises. However, the buyer’s entry option always has nonzero value. In contrast, we show that, with two

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6 In other related work, Farrell and Shapiro (1988) analyze an overlapping generations model of duopolistic competition in the presence of consumer switching costs. They show that, in equilibrium, the market is segmented in that the firm with attached customers specializes in serving them conceding new buyers to its rival. To (1996) examines an infinite-period duopoly market with overlapping generations of consumers. He shows that, if consumers have a finite time horizon, then the two firms may alternate dominance from one period to the next, alternately charging high and low prices. Wang and Wen (1998) examine a three-stage model in which the incumbent firm and the entrant set prices sequentially. They show that the existence of switching costs may be beneficial in that even an entrant with a higher marginal cost may profitably invade part of the market. Lewis and Yildirim (2003) propose a multi-period framework where strategic buyers use switching costs to manage dynamic competition between suppliers. They also incorporate exogenous uncertainty in their framework arising from the process of acquisition of skill by the buyer that in turn affects its switching costs. They show that high switching costs may benefit the buyer by inducing suppliers to price more competitively. Lipman and Wang (2002) show that
competing non-identical foreign sellers, the buyer’s switching option has zero value when the exchange rate volatility exceeds a threshold.

Methodologically, the present study is related to the emerging literature that investigates strategic irreversible investment under uncertainty in multi-period or continuous time frameworks (for example, Dixit and Pindyck 1994, Trigeorgis 1996, Kulatilaka and Perotti 1998, Childs and Triantis 1999, Grenadier 2000). The papers in this stream of the literature have largely focused on the strategic behavior of sellers where the demand is exogenously specified and the sellers are Cournot competitors. Moreover, they have primarily investigated duopolistic timing games where two sellers sequentially enter the market. A distinguishing feature of our framework is that both buyers and sellers behave strategically. Moreover, in our model, the sellers are Bertrand competitors. This entails the analysis of a three-player game that is very different from the games considered so far in this stream of the literature.

Our study is also related to a number of studies in the literature that employ real options techniques to value the long-term operational flexibility of firms that have manufacturing plants or suppliers in foreign countries (Kogut and Kulatilaka 1994, Huchzermeier and Cohen 1996, Dasu and Li 1997, Kouvelis 1999, Kouvelis et al. 2001). These firms have the opportunity to exercise manufacturing or sourcing options over time when faced with stochastic variation in the relative benefits of these options due to exchange rate fluctuations. Since these papers are primarily concerned with a firm (buyer) choosing between various production facilities, or modes of operation, its cost structures are exogenously specified. We complement and extend this stream of the literature by investigating the scenario wherein the buyer’s cost structures are determined by its interaction with active sellers who compete with each other in selling to the

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even small switching costs may make it credible not to change action in finitely repeated games. Klemperer (1995) and Farrell and Klemperer (2002) provide surveys of the extensive "switching costs" literature.
buyer. Hence, the buyer’s sourcing strategies and the sellers’ pricing strategies are together
determined endogenously in a competitive equilibrium between the buyer and the sellers.

The plan for the paper is the following. In Section 2, we present the model. In Section 3,
we derive the optimal switching policy of the buyer for given seller prices. In Section 4, we use
the results of Section 3 to characterize equilibria of the game between the buyer and the sellers.
Section 5 concludes the paper. All detailed proofs are provided in the Appendix.

2. The Model

We consider two competing sellers located in the same country selling a product with
several non-identical potential buyers possibly located in different countries. For simplicity and
concreteness, we focus on a representative buyer in a country dealing with two competing
foreign sellers (hereafter referred to as seller 1 and seller 2) located in the same foreign country.
The buyer and the sellers are all value maximizers in their respective currencies and the foreign
currency is tradeable. In order to develop a tractable model for our analysis, we make some
simplifying assumptions that we now describe and motivate.

As in the canonical “real options” model of Dixit (1989), the buyer has a constant,
inelastic demand of one unit of the product per unit time and derives a constant utility of one
from each unit of the product. We assume that either seller can completely fulfil the buyer's
demand; an assumption that is realistic in several industries such as semi-conductors, electronics,
and general manufacturing, where an increase in contract manufacturing in global trade has led
to the growth of major suppliers (assemblers and fabricators) that cater to the needs of a large
pool of buyers. If the buyer is not in a relationship with either of the sellers, we assume that the
buyer fulfils its demand for the product from a seller in its own country. Without loss of
generality, we assume that the cost (per unit) of the product in the buyer’s own country is one

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unit of the buyer’s currency. For expositional convenience, we hereafter refer to this state as the *idle* state.

We examine the scenario where the buyer and sellers enter into long-term contracts where the sellers quote prices $Q_1, Q_2$ in their currency (we model economic scenarios where long-term contracts are enforceable). This is consistent with substantial empirical evidence that, in markets with significant switching costs, buyers and sellers prefer to enter into long-term contracts *ex ante* (for example, Joskow 1987), a majority of bilateral international trade is invoiced in the *exporters’* currency (Grassman 1973, Magee and Rao 1980, Cornell 1980, Viaene and De Vries 1992), and prices, especially those of intermediate goods, exhibit long-term rigidity (for example, Carlton 1986). 7 From a theoretical standpoint, our assumptions are similar to those made by the real options literature on valuing global operational flexibility wherein procurement and manufacturing costs (that include prices paid to suppliers) are assumed to be constant in the local currency (Kogut and Kulatilaka 1994, Dasu and Li 1997, Kouvelis et al. 2001). As we discuss later, we can extend our model to allow for renegotiation of prices over time. Under alternative simplifying assumptions, we can show that our main results are not qualitatively affected (see Conclusions).

Since the buyer is a value-maximizer in its own currency, by the fundamental theorem of asset pricing (Duffie 2001), the value it derives from entering into relationships with the sellers is the discounted expectation of the corresponding cash flows where the discounting is at the buyer's risk-free rate, and the expectation is under the buyer’s *risk-neutral or equivalent martingale* probability measure. As in Dixit (1989) and Dixit and Pindyck (1994) we can,

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7 We recognize that a more general theoretical model (that may, unfortunately, be intractable in a multi-period setting) would be one where the nature of contracts between the buyer and sellers were justified endogenously. However, we believe that it is nevertheless instructive to examine the economic consequences of typically observed contractual structures. Such an analysis could provide insights into the reasons for their existence and prevalence.
therefore, derive the buyer's value-maximizing policy by modeling the foreign exchange rate, that is, the value \( q(.) \) of the sellers' currency in the buyer’s currency, under the buyer's risk-neutral probability measure. Since the buyer's value from following such a policy is, by construction, the maximum possible value it can derive from relationships with the sellers, the value-maximizing policy incorporates, in particular, the possibility of financial hedging where the buyer may enhance its value by hedging its cash flows using other traded securities, but incurs the costs of using these additional securities (see Dixit and Pindyck 1994, Duffie 2001).

We assume that the foreign exchange rate evolves as a lognormal process under the buyer's risk-neutral probability measure:

\[
\frac{dq(t)}{q(t)} = \left(\beta - \beta'\right)dt + \sigma dB(t)
\]

In the above, \( B(.) \) is a Brownian motion. Note that since the foreign currency is traded, its drift under the buyer's risk-neutral measure must be equal to the difference between the domestic (buyer's) and foreign (sellers’) risk-free rates (see Duffie 2001). The price (per unit) of the product \( p(.) \) demanded by seller 2 in the buyer’s currency is given by \( p(.) = Q_2 q(.) \) and also evolves as in (2.1) with drift \( \mu = \beta - \beta' \) and volatility \( \sigma \). Therefore, the price per unit of the product demanded by seller 1 is proportional to the price demanded by seller 2 and is given by \( \lambda p \) where \( \lambda = Q_1 / Q_2 \).

We assume that the buyer incurs different relationship-specific sunk costs or switching costs \( k_1, k_2 \) respectively vis-à-vis the non-identical sellers 1 and 2 each time it enters into a relationship with them. Note that the switching cost \( k_i; i = 1, 2 \) is incurred when the buyer enters into a relationship with seller \( i \). The switching costs the buyer incurs vis-à-vis the non-identical sellers could be different, for example, due to the sellers having differing geographical locations,
product differentiation, or one of the sellers being less “developed” than the other, thereby requiring the buyer to incur higher costs to set up a relationship with it. Without loss of generality, we assume that

\[ 0 \leq k_1 < k_2. \]

We consider a mature market where the switching costs incurred by the buyer are already present (see Klemperer 1995 for a definition of mature markets), and assume that these costs are *publicly observable* and *verifiable*. In particular, this eliminates the possibility of hold-up where either seller exploits the buyer by reneging on its contract and raising prices after the buyer has incurred switching costs. To simplify the analysis and the exposition, we also assume that the buyer incurs no switching costs when it exits the foreign market from a relationship with either seller. We can incorporate such exit costs in the model, but this complicates the analysis without qualitatively affecting our results. Given the difference (2.2) in switching costs, seller 2 can compete with seller 1 only by quoting a lower price. Therefore, it suffices to consider situations where the ratio of the seller prices \( \lambda \) satisfies

\[ \lambda > 1. \]

We first assume that the prices \( Q_1, Q_2 \) quoted by the sellers are *exogenously specified*. The *variable costs* of the buyer with either seller are therefore constant in the sellers’ currency.\(^8\) In Section 4, these prices are endogenously determined in equilibrium. As mentioned earlier, our framework may also accommodate several non-identical buyers possibly located in different countries with different pairs of switching costs vis-à-vis the two sellers. Moreover, the sellers may quote the same (publicly observed) prices to all buyers or they may enter into separate

\(^8\) In fact, the results of the first part of the paper are more generally applicable to scenarios wherein the "sellers" are not active economic agents who compete with each other. For example, the "sellers" could represent different
private contracts with each buyer. To simplify the exposition, we assume the latter scenario. Hence, we focus on the equilibrium between the sellers and the representative buyer described by the pair of switching costs in (2.2).

In this paper, we consider the scenario wherein the buyer may re-enter the market after exiting it. We can modify our framework to analyze the situation where the buyer, after exiting the market from a relationship with either seller, does not re-enter it. We can also analyze the scenario wherein the buyer, after exiting the foreign market from a relationship with either seller, renegotiates its contracts with the sellers before re-entering it. Under alternative simplifications, we can show that our main results and insights are not qualitatively affected (see Conclusions).

For given prices $Q_1, Q_2$, since either foreign seller can completely fulfil the buyer’s demand for the product at any instant, it is sub-optimal for the value-maximizing buyer to source from both sellers simultaneously at any instant of time, that is, partially meet its demand for the product with either seller. Therefore, at any time $t$, the buyer may either be idle, or in a relationship with one of the foreign sellers. We use the variable $s$ to denote these three possibilities so that $s$ takes on values in the set $\{0,1,2\}$. The feasible policies of the buyer are given by

\[ (2.4) \quad \Gamma \equiv \{\tau_1, \tau_2, \ldots\} \]

where $\{\tau_n\}$ is an increasing sequence of stopping times representing the instants at which the buyer switches between the various states. The value the buyer derives from policy $\Gamma$ is

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production facilities, or different modes of operation for the buyer whose cost structures are exogenously specified (for example, Kogut and Kulatilaka 1994, Huchzermeier and Cohen 1996, Dasu and Li 1997).
\[
U_T(p, s_0) = E \sum_{i=0}^{\infty} \{ 1_{s=1}[-\exp(-\beta \tau_i)k_1 + \int_{\tau_i}^{r_{i+1}} \exp(-\beta s)(1 - \lambda p(s))ds] + 1_{s=2}[-\exp(-\beta \tau_i)k_2 + \int_{\tau_i}^{r_{i+1}} \exp(-\beta s)(1 - p(s))ds] \}
\]

(2.5)

In the above, \(p_0\) is the initial price offered by seller 2, and \(s_0\) is the initial state of the buyer.

Note that, consistent with our assumption that the buyer does not incur switching costs when it exits the foreign market from a relationship with either seller (that is, it enters state 0), switching costs are only incurred when the buyer enters either state 1 or state 2. Each term in the summation above represents the total discounted cash flows of the buyer from using either of the sellers over the time interval \((\tau_i, r_{i+1})\).

The goal of the buyer is to choose its switching policy \(\Gamma\) to maximize its value. Since the variable costs incurred by the buyer with the two sellers are proportional to each other, the “state of the buyer” is described by the price \(p\) demanded by seller 2 and the value of the variable \(s\). For expositional convenience, hereafter we shall refer to the buyer being in “state 0, state 1, or state 2” as the buyer being idle, with seller 1, or with seller 2 respectively. From (2.5), it is clear that at any time \(t\), the optimal decision of the buyer does not depend on time, but only on the current value of the variable \(s\) of the buyer and the price \(p\) demanded by seller 2.

Therefore, it suffices to consider policies of the buyer that are described as follows:

\[
\Lambda = \{p_{01}, p_{10}, p_{12}, p_{21}, p_{02}, p_{20}\}
\]

where \(p_{ij}\) is the switching point for switching from state \(i\) to state \(j\), i.e. \(p_{ij}\) is the price of seller 2 at which the buyer will switch from state \(i\) to state \(j\). It is not difficult to see that it is never optimal for the buyer to switch from state 2 to state 1. It therefore suffices to consider policies of the buyer that are described as follows:
\{p_{01}, p_{10}\}$, i.e. the buyer only uses seller 1  (Case 1)  
\{p_{02}, p_{20}\}$, i.e. the buyer only uses seller 2  (Case 2)  
\{p_{01}, p_{10}, p_{12}, p_{20}\}$, i.e. the buyer may use both sellers over time  (Case 3).

Since we have assumed that the buyer is initially in the idle state and the initial price $p_0 > 1$, it follows that in the Case 3 above, it suffices to consider policies where

\begin{align}
(2.8) \quad p_{12} & \leq p_{01} \leq 1 \quad \text{and} \\
(2.9) \quad p_{20} & \geq 1, \lambda p_{10} \geq 1.
\end{align}

We denote the \textit{optimal value functions} of the buyer, (i.e. the buyer’s optimal expected utilities when it uses policies described by (2.7)) by $v_1, v_2, \text{ and } v_{12}$ respectively. The optimal value function $v$ of the buyer over all feasible policies is therefore given by

\begin{align}
(2.10) \quad v = \max(v_1, v_2, v_{12}).
\end{align}

For the sellers to co-exist, the buyer’s value function $v$ must be equal to $v_{12}$ and be \textit{strictly greater} than $\max(v_1, v_2)$ so that its corresponding optimal policy must involve switching between both sellers over time as described by Case 3 in (2.7). If $u$ is the value function of a policy (not necessarily optimal) of the buyer, then it is well known that $u$ satisfies the following system of ordinary differential equations:

\begin{align}
(2.11) \quad -\beta u + \mu pu_p + \frac{1}{2} \sigma^2 p^2 u_{pp} + 1_{s=1}(1 - \lambda p) + 1_{s=2}(1 - p) = 0,
\end{align}

with appropriate boundary conditions for the transitions between different states. Any solution to the system of equations above is of the form:

\begin{align}
(2.12) \quad u(p) = A_i p^{\eta_i} + B_i p^{\eta_i} + \frac{1}{\beta} - 1_{i=1} \frac{\lambda p}{\beta - \mu} - 1_{i=2} \frac{p}{\beta - \mu}
\end{align}
where $A_i, B_i, i = 0, 1, 2$ are constants determined by the boundary conditions and $\eta^+, \eta^-$ are the positive and negative root respectively of the quadratic equation:

$$(2.13) \quad \frac{1}{2} \sigma^2 x^2 + (\mu - \frac{1}{2} \sigma^2)x - \beta = 0.$$ 

We can now write down the functional forms for the value functions corresponding to the various types of policies the buyer may choose. For the sake of brevity, we only illustrate the case where the buyer uses policies where it switches between both sellers over time, i.e., Case 3 in (2.7). The other situations follow as special cases. Using (2.12) we see that the value function of a policy $\{p_{01}, p_{12}, p_{10}, p_{20}\}$ is given by

$$u_{12}(p) = \begin{cases} 
A_{12} p^{\eta^+}; & p > p_{01} \text{ and the buyer is in state 0} \\
B_{12} p^{\eta^+} + C_{12} p^{\eta^-} + \frac{1}{\beta} \frac{\lambda p}{\beta - \mu}; & p_{12} < p < p_{10} \text{ and the buyer is in state 1} \\
D_{12} p^{\eta^-} + \frac{1}{\beta} \frac{p}{\beta - \mu}; & p < p_{20} \text{ and the buyer is in state 2}
\end{cases}$$

(2.14)

with the coefficients $A_{12}, B_{12}, C_{12}, D_{12}$ determined by value matching (continuity) conditions at the switching points. If $v_{12}(p_0)$ is the optimal value function of the buyer over the class of policies where it may switch between both sellers over time, we have

$$v_{12}(p_0) = \sup_{\{p_{01}, p_{12}, p_{10}, p_{20}\}} u_{12}(p_0).$$

(2.15)

If the policy defined by $\{p_{01}, p_{12}, p_{10}, p_{20}\}$ is optimal within the class of policies where both sellers are used, then $\{p_{01}, p_{12}, p_{10}, p_{20}\}$ are determined by additional smooth pasting or differentiability conditions at the switching points.

**The Value of the Switching Option**

As stated earlier, the buyer holds the option of switching between the two sellers over time. We can use the notation introduced above to define the value of this option as follows:
(2.16) \[ \text{Value of Switching Option} = (v(p_0) - \max(v_1(p_0), v_2(p_0))), \]

where \( p_0 > 1 \) is the initial price demanded by seller 2. In the above equation,

\[ v(p_0) = \max(v_1(p_0), v_2(p_0), v_{12}(p_0)) \]

is the maximum value to the buyer from using both sellers and \( \max(v_1(p_0), v_2(p_0)) \) is the maximum value from using only one of the two sellers. The switching option of the buyer has strictly positive value if and only if there exists a solution \((p_{01}, p_{12}, p_{10}, p_{20})\) of the value matching and smooth pasting conditions.

### 3. Optimal Policy of the Buyer

We analyze equilibria between the buyer and the sellers by following the standard approach of first deriving the optimal switching policy of the buyer for given seller prices. We derive explicit conditions on the switching costs and the sellers’ prices for the buyer’s switching option (2.16) to have strictly positive value. If the buyer’s switching option does not have strictly positive value, it is optimal for the buyer to use only one of the two sellers. To simplify the analysis and exposition, we assume that the switching cost \( k_1 \) vis-à-vis seller 1 is zero in the following. We can show that our results, and the intuition underlying them, are valid in the more general scenario wherein the switching costs vis-à-vis both sellers are nonzero (details available). We relax the assumption that \( k_1 = 0 \) in our subsequent numerical simulations.

The optimal policy of the buyer is completely characterized in the following theorem whose proof follows from that of Proposition 3.1 later in the section.

**Theorem 3.1**

a) For each \( k_2 > 0 \), there exists an interval of seller price ratios \((\lambda_{\min}, \lambda_{\max})\) such that the buyer’s optimal policy has the following form: If \( \lambda \leq \lambda_{\min} \), the buyer will use seller 1 alone; if \( \lambda_{\min} < \lambda < \lambda_{\max} \), the buyer will switch between both sellers over time; if \( \lambda \geq \lambda_{\max} \), the buyer will
use seller 2 alone. We may have $\lambda_{\text{min}} = \lambda_{\text{max}}$ in which case the buyer’s switching option has zero value for all $\lambda$.

b) For each $\lambda > 1$, there exists an interval of switching costs $(k_{\text{min}}, k_{\text{max}})$ vis-à-vis seller 2 such that the buyer’s optimal policy has the following form: if $k_2 \leq k_{\text{min}}$, the buyer will use seller 2 alone; if $k_{\text{min}} < k_2 < k_{\text{max}}$, the buyer will switch between both sellers over time; if $k_2 \geq k_{\text{max}}$, the buyer will use seller 1 alone. We may have $k_{\text{min}} = k_{\text{max}}$ in which case the buyer’s switching option has zero value for all $k_2$.

The intuition for Theorem 3.1 is that, for given switching costs, if the seller price ratio $\lambda$ is very high, seller 1’s price is much higher than that of seller 2 so that the buyer is willing to pay the higher switching costs and use seller 2 alone. On the other hand, for $\lambda = 1$, i.e. equal variable costs, the buyer will only use seller 1 due to the lower switching costs. Therefore, we would intuitively expect the existence of thresholds $\lambda_{\text{min}}$ and $\lambda_{\text{max}}$ such that for $\lambda < \lambda_{\text{min}}$, the buyer will only use seller 1, and for $\lambda > \lambda_{\text{max}}$, the buyer will only use seller 2. In the intermediate region, i.e. if $\lambda_{\text{min}} < \lambda < \lambda_{\text{max}}$, the buyer will enter the market with seller 1 and switch to seller 2 if the price becomes more favorable so that both sellers will be used over time. The intuition for part b) of the theorem is analogous.

The conditions under which the interval $(\lambda_{\text{min}}, \lambda_{\text{max}})$ is non-empty are *not* obvious. The non-emptiness of the interval guarantees the existence of seller prices for which the optimal policies of the buyer involve switching between the sellers over time. When the prices quoted by the sellers are endogenously determined in equilibrium between the buyer and the sellers, the non-emptiness of the interval is therefore, a necessary (but not sufficient) condition for the
sellers to co-exist in equilibrium with the buyer. We now introduce some analytics essential to the proof of Theorem 3.1.

For each $\lambda$, let $z_1^\cdot(p_0)$ denote the optimal value the buyer obtains from the class of policies where it always enters the market with seller 1, but may optimally switch to seller 2 if the exchange rate becomes more favorable. Since $k_1 = 0$, it is easy to show that the optimal policy within this class must involve the buyer entering and exiting the market from a relationship with seller 1 when $p(\cdot) = 1/\lambda$. However, the buyer may optimally switch to seller 2 if the process $p(\cdot)$ falls to a level $p_{12}$ lower than $1/\lambda$ and exit the market from a relationship with seller 2 when $p(\cdot)$ rises to some level $p_{20} > 1$. The value function $z_1^\cdot(p_0)$ has the functional form given by (2.14) with $p_{01} = p_{10} = 1/\lambda$, and the entry and exit triggers $p_{12}, p_{20}$ may be determined using value matching and smooth pasting conditions. We denote the optimal value functions of the buyer from policies where it only uses either seller 1 or seller 2 by $v_1^\cdot(p_0), v_2(p_0)$ respectively where the superscript in $v_1^\cdot(p_0)$ indicates the explicit dependence of the value function on the seller price ratio $\lambda$. The following proposition characterizes the switching interval thresholds $\lambda_{\min}, \lambda_{\max}$ and thereby proves Theorem 3.1.

**Proposition 3.1**

\( a) \quad \lambda_{\max} = \sup(\lambda : z_1^\cdot(p_0) > v_2(p_0)) \)

\( b) \quad \lambda_{\min} = \min(\lambda_0, \lambda_{\max}) \) where $\lambda_0 = \inf(\lambda : z_1^\cdot(p_0) > v_1^\cdot(p_0))$

**Proof.** In the Appendix.

We can use analogous arguments to characterize the interval $(k_{\min}, k_{\max})$ analytically and prove part b) of Theorem 3.1. We shall omit these for the sake of brevity. **Figures 1a and 1b** graphically illustrate the result of part a) of Theorem 3.1 where the switching costs $k_1, k_2$ are
both nonzero. Figure 1a illustrates a scenario wherein the switching option of the buyer given by (2.16) has strictly positive value. Figure 1b illustrates a scenario where the interval is empty so that the value of the buyer’s switching option is zero for all values of λ. Figures 2a and 2b graphically illustrate the intuition underlying part b) of Theorem 3.1.

Dependence of Switching Interval \((\lambda_{\min}, \lambda_{\max})\) and Buyer’s Switching Option on Exchange Rate Volatility

We now determine economic conditions under which the switching interval \((\lambda_{\min}, \lambda_{\max})\) is degenerate and, therefore, the buyer’s switching option has zero value. In this case, the only viable equilibria are those where one of the sellers captures the market.

**Proposition 3.2** There exists an exchange rate volatility level \(\sigma_T\) such that, for all values of the sellers’ price ratio \(\lambda\), the buyer’s switching option has zero value if \(\sigma > \sigma_T\). Hence, for \(\sigma > \sigma_T\), the only viable equilibria are those where one of the sellers captures the market.

**Proof.** In the Appendix.

The intuition for this result is the following. Since it is sub-optimal for the buyer to switch from seller 2 to seller 1, the buyer’s switching option has positive value only when it enters the market with seller 1. In this scenario, as the exchange rate volatility increases, the buyer delays switching to seller 2 so that the variable cost advantage of seller 2 declines. Beyond a critical volatility level, the variable cost savings from switching to seller 2 after entering the market with seller 1 do not justify the increased switching costs incurred. Therefore, the buyer will directly enter the market with either seller 2 or seller 1 and never switch between them. Hence, either one of the two sellers may capture the market. In the absence of uncertainty, one of the two sellers captures the market in general. The presence of uncertainty makes their co-existence feasible, but this feasibility disappears if the level of uncertainty is “too
high”. Figure 3 graphically illustrates these results (where $k_1$ is again chosen to be nonzero). It shows the variation of $\lambda_{\text{min}}$ and $\lambda_{\text{max}}$ with the exchange rate volatility $\sigma$ for a given value of $k_2$. The results clearly show that there exists a threshold value $\sigma^T$ below which the switching interval $(\lambda_{\text{min}}, \lambda_{\text{max}})$ is non-empty and above which it becomes empty.

It is interesting to compare the result of Proposition 3.2 with that of Dixit (1989) who analyzes the entry-exit decisions of a buyer with a single seller in a foreign market. He shows that the buyer delays entry into the foreign market as the exchange rate volatility increases, but there is always a nonzero probability of the buyer entering the foreign market regardless of the exchange rate volatility level. In other words, the buyer’s entry option always has nonzero value. Proposition 3.2 shows that, for a buyer faced with two competing sellers in a foreign market, the switching option, that is, the option of switching between both sellers over time has zero value if the exchange rate volatility exceeds a threshold.

4. Equilibria between the Buyer and Sellers

We now investigate equilibria of the game between the buyer and the sellers incorporating the variable costs of the sellers. We assume that the switching costs $k_1, k_2$ the buyer incurs with the sellers are exogenously specified. However, the prices quoted by the sellers are determined competitively. In the game between the buyer and its sellers, the sellers’ strategies are to quote prices per unit of the product in the sellers’ currency and the buyer’s response is to choose its optimal switching policy. Each seller has constant variable costs of production (in the sellers’ currency) and, therefore, adopts a “markup pricing” policy by quoting a price at a premium to its cost. We denote the variable costs of the sellers by

$$C_1, C_2 > 0,$$
where seller 2’s costs may well exceed seller 1’s costs. The equilibrium prices set by the two sellers are given by \( Q_1^*, Q_2^* \) with \( Q_1^* > C_1, Q_2^* > C_2 \).

**The Structure of the Game between the Buyer and the Sellers**

The variable costs of both sellers are common knowledge between all the players. *At its discretion*, the buyer first elicits a price quote from either one of the two sellers, and then obtains a price quote from the other seller after revealing the first quote. Thus, we have a coupled leader-follower game structure where one of the sellers is chosen as the leader and the other the follower *at the behest* of the buyer. The sellers rationally anticipate the buyer’s choice of leader/follower and its optimal policy (as determined in previous sections) in response to their quoted prices.

We now introduce some analytics essential to a detailed analysis of this game. The sellers’ and buyer’s value functions given the seller prices and the initial value of the exchange rate are denoted by \( V_1(Q_1, Q_2, q(0)), V_2(Q_1, Q_2, q(0)), V(Q_1, Q_2, q(0)) \) respectively. The buyer’s value function \( V \) has been derived in earlier sections. We now derive the sellers' value functions that depend on cash flows in the sellers’ currency.

**Analysis of the Game**

Since the sellers are assumed to be value-maximizers in their currency, we can, as in the case of the buyer, derive their value-maximizing price-quoting strategies by modeling their respective cash flow processes under the sellers’ risk-neutral or equivalent martingale probability measure. We can show that, *under the sellers’ risk-neutral measure*, the process \( q^*(.) \) representing the value of one unit of the sellers' currency in the buyer's currency evolves as a lognormal process:

\[
dq^*(t) = q^*(t)[(\beta - \beta^* + \sigma^2)dt + \sigma dB^*(t)] = q^*(t)[(\mu + \sigma^2)dt + \sigma dB^*(t)]
\]

(4.2)
The exchange rate is denoted by $q^*(.)$ to emphasize the point that its evolution is described in the sellers’ risk-neutral world. Note that $q^*(.)$ is not the price process of a traded asset in the sellers' currency since it is the value of one unit of the sellers' currency in the buyer's currency.

Given the switching costs $k_1,k_2$ and prices $Q_1,Q_2$ quoted by the sellers, we have seen that both sellers co-exist in the buyer’s market if and only if $\lambda_{\min} < Q_1/Q_2 < \lambda_{\max}$ where $(\lambda_{\min},\lambda_{\max})$ is the interval of seller price ratios where the buyer’s switching option has strictly positive value. The optimal policy of the buyer is described by entry, exit, and switching points (expressed in terms of seller 2’s quoted price $Q_2 q^*(.)$), namely,

$$\{p_{01}(Q_1/Q_2), p_{12}(Q_1/Q_2), p_{10}(Q_1/Q_2), p_{20}(Q_1/Q_2)\}.$$  Thus, the entry, exit, and switching points are functions of the ratio of seller prices. When the buyer uses only one of the two sellers, only the corresponding entry and exit price triggers appear. We can now use standard arguments (that we omit for the sake of brevity) to show that the value functions $V_1(p), V_2(p)$ of the sellers as a function of the price $p$ quoted by seller 2 in the buyer’s currency must have the following functional forms:

$$V_1(p) = A_1 p^{\rho_1^+}; p \geq p_{01}(\lambda) \text{ and the buyer is in state 0}$$

$$= B_1 p^{\rho_1^+} + C_1 p^{\rho_1^-} + \frac{Q_1-C_1}{\beta}; p_{10}(\lambda) \geq p \geq p_{12}(\lambda) \text{ and the buyer is in state 1}$$

$$= D_1 p^{\rho_1^-}; p \leq p_{20}(\lambda) \text{ and the buyer is in state 2}$$

$$V_2(p) = A_2 p^{\rho_2^+}; p \geq p_{12}(\lambda) \text{ and the buyer is in state 0 or state 1}$$

$$= D_2 p^{\rho_2^-} + \frac{Q_2-C_2}{\beta}; p \leq p_{20}(\lambda) \text{ and the buyer is in state 2}$$

with $\rho_1^+, \rho_1^-$ being the positive and negative roots of (2.13) with $\beta, \mu$ replaced by $\beta^+, \mu + \sigma^2$, respectively and $\lambda = Q_1/Q_2$. The value functions of the sellers
$V_1(Q_1, Q_2, q^*(0)), V_2(Q_1, Q_2, q^*(0))$ may be discontinuous functions of the arguments $Q_1, Q_2$. The only possible discontinuities of the value functions are at the indifference points where $Q_1 / Q_2 = \lambda_{\min}$ or $Q_1 / Q_2 = \lambda_{\max}$. Due to the nature of the optimal policies of the buyer, if the sellers' value functions are discontinuous at $\lambda_{\min}$ and $\lambda_{\max}$, then seller 1's value function falls at these points and seller 2's value function rises.

In order to ensure that the value functions of the sellers are well defined we introduce an additional technical rule of the game. At $\lambda = \lambda_{\min}$ and $\lambda = \lambda_{\max}$, the buyer always chooses the policy that maximizes the value of the follower. This rule ensures that the value functions $V_1(q^*(0), Q_2)$ and $V_2(q^*(0), Q_1, ..)$ are left continuous that, in turn, guarantees the existence of equilibria between the buyer and both sellers under broader conditions.

If seller 1 is chosen as the leader and seller 2 the follower, then, for each price quote $Q_1$ of seller 1, let $\psi_2(Q_1)$ be the best response of seller 2, i.e. $\psi_2(Q_1) = \arg \max_{Q_2} V_2(q^*(0), Q_1, Q_2)$. The left continuity of $V_2(q^*(0), Q_1, ..)$ ensures that $\psi_2(Q_1)$ always exists. Since seller 1 is the leader, it will quote a price $Q_1^*$ satisfying

$$Q_1^* = \arg \max_{Q_1} V_1(q^*(0), Q_1, \psi_2(Q_1)).$$

If (4.5) has no solution, i.e. $Q_1^*$ does not exist, then seller 1 is unable to quote a price and is therefore not in the market. Hence, the problem reduces to the case where the buyer negotiates with only a single seller, i.e. seller 2 in the foreign market. Similarly, if seller 2 is chosen as the leader and seller 1 the follower, then for each price quote $Q_2$ of seller 2, let $\psi_1(Q_2)$ be the best response of seller 1, i.e. $\psi_1(Q_2) = \arg \max_{Q_1} V_1(q^*(0), Q_1, Q_2)$. The left continuity of
\( V_1(q^*(0),\nu_1,\nu_2) \) ensures that \( \psi_1(\nu_2) \) exists. Since seller 2 is the leader, it will quote a price \( Q_2^* \) satisfying

\[
(4.6) \quad Q_2^* = \arg \max_{Q_2} V_2(q^*(0),\psi_1(\nu_2),\nu_2).
\]

If \( Q_2^* \) does not exist, seller 2 is unable to quote a price so that seller 1 is the only seller in the market and we are again in the single seller scenario discussed earlier. The buyer will choose seller 1 (seller 2) as the leader and seller 2 (seller 1) as the follower in equilibrium if and only if

\[
V(q^*(0),Q_1^*,\psi_2(\nu_1)) > (>)V(q^*(0),\psi_1(\nu_2),Q_2^*).
\]

**Equilibria between the Buyer and Sellers**

The following result provides *sufficient* conditions for both sellers to co-exist in any possible equilibrium with the buyer or for either seller to capture the market.

**Proposition 4.1** Suppose a solution to either (4.5) or (4.6) exists.

a) If \((\lambda_{\min},\lambda_{\max})\) is non-empty and \( \lambda_{\min} < C_1 / C_2 < \lambda_{\max} \), then the capture of the market by either seller cannot be an equilibrium outcome, that is, the sellers must co-exist in any possible equilibrium.

b) Suppose \((\lambda_{\min},\lambda_{\max})\) is empty so that \( \lambda_{\min} = \lambda_{\max} = \lambda^* \). If \( C_1 / C_2 > \lambda^* \), seller 2 captures the market in any equilibrium. If \( C_1 / C_2 < \lambda^* \), seller 1 captures the market in any equilibrium. If \( C_1 / C_2 = \lambda^* \), the sellers quote prices equal to their marginal costs in the unique equilibrium, and the buyer is indifferent between them.

**Proof.** In the Appendix.

We argued previously that for the sellers to co-exist in equilibrium, the switching interval \((\lambda_{\min},\lambda_{\max})\) must be nonempty and the ratio of the prices they quote must lie in the interval. The result of Proposition 4.1 a) says that if the costs of the sellers are "aligned" so that their ratio lies
within $\left( \lambda_{\text{min}}, \lambda_{\text{max}} \right)$, then the ratio of the prices they quote in equilibrium must also lie within $\left( \lambda_{\text{min}}, \lambda_{\text{max}} \right)$. On the other hand, if $\left( \lambda_{\text{min}}, \lambda_{\text{max}} \right)$ is empty, then the seller with the comparative advantage (depending on whether $C_1 / C_2 > \lambda^*$ or $C_1 / C_2 < \lambda^*$) captures the market. However, as the following elementary result (whose proof we omit for the sake of brevity) shows, the level of prices when seller 1 captures the market may be very different from the level when seller 2 captures it.

**Proposition 4.2** Suppose a solution to either (4.5) or (4.6) exists. If seller 1 captures the market in equilibrium, then the equilibrium price $Q_1^*$ must satisfy $C_1 < Q_1^* \leq \lambda_{\text{min}} C_2$. If seller 2 captures the market, then the equilibrium price $Q_2^*$ must satisfy $C_2 < Q_2^* \leq \frac{C_1}{\lambda_{\text{max}}} < C_1$.

From the above proposition, we see that if seller 2 captures the market so that $C_2 < C_1$, it does so by quoting a price lower than seller 1’s variable cost $C_1$. In the absence of switching costs, the usual result of Bertrand competition would lead to seller 2 capturing the market by quoting a price $C_1$. Therefore, the presence of switching costs leads to lower prices. On the other hand, if $C_1 \leq C_2$ and seller 1 captures the market, it does so by quoting a price that is, typically higher than seller 2’s variable cost $C_2$; the price it would quote to capture the market in the absence of switching costs. Therefore, switching costs may either raise or lower the level of prices depending on which seller captures the market.

It is interesting to combine the results of the previous two propositions and the result that the buyer’s switching option has zero value for all possible seller prices when the exchange rate volatility is above a critical level. In this scenario, seller 1 may capture the market if it has a comparative advantage (that is, $C_1 / C_2 < \lambda^*$) by “exploiting” the buyer, that is, by quoting prices
higher than it would quote in the absence of switching costs. On the other hand, if seller 2 has the comparative advantage (that is, \( C_1 / C_2 > \lambda^* \)), it captures the market by engaging in predation, that is, by quoting prices lower than it would quote in the absence of switching costs. Therefore, the exchange rate volatility plays an important role in determining whether switching costs raise or lower competition among the sellers and whether the overall level of prices is higher or lower.

**Figure 4** shows the variation of the equilibrium prices quoted by the sellers with the ratio of the costs of the two sellers. The figure depicts all the three possible equilibria between the buyer and the sellers: co-existence or market capture by either seller. Consistent with the result of Proposition 4.1, we note that both sellers co-exist when \( \lambda_{\min} = 1.04 < C_1 / C_2 < 1.09 = \lambda_{\max} \).

However, as the figure indicates, the sellers may co-exist in equilibrium even when \( \lambda \) does not lie in \((\lambda_{\min}, \lambda_{\max})\) since the condition of Proposition 4.1 is a sufficient but not necessary condition for co-existence. We notice that the level of prices quoted by seller 1 when it captures the market is higher than the level of prices quoted by seller 2 when it captures the market. This is consistent with the result of Proposition 4.2.

**Strategic Relationships with Switching Costs and Exchange Rate Uncertainty**

We now combine all the results we have obtained to describe the effects of the interplay between exchange rate uncertainty and switching costs on the strategic relationships between the respective players. In the absence of exchange rate uncertainty, we see that the seller with the more favorable combination of its variable costs and buyer switching costs, captures the market. In the presence of “low” uncertainty, we may have equilibria where either seller captures the market or both sellers co-exist (as Figure 4 indicates) depending on the relative magnitudes of their respective variable costs and the switching costs of the buyer. However, if the costs of the sellers are “aligned” as in the hypothesis of Proposition 4.1, the sellers must co-exist in
equilibrium with the buyer. Therefore, the presence of switching costs and exogenous uncertainty may allow both sellers to co-exist in equilibrium.

When the switching costs of the buyer vis-à-vis the sellers are different, the switching option of the buyer may have positive value provided the exchange rate volatility is below a critical threshold (Proposition 3.2). In general, either seller may be able to quote a price that captures the market or both may choose to quote prices in equilibrium that allow their co-existence. However, as Proposition 3.2 and Figure 3 illustrate, above the critical volatility level where the buyer’s switching option has zero value, both sellers cannot co-exist in equilibrium regardless of their costs. By the results of part b) of Proposition 4.1 and Proposition 4.2, if seller 1 captures the market, it does so by quoting a price that is typically higher than it would quote without switching costs. On the other hand, if seller 2 captures the market, it does so by quoting a price that is lower than it would quote without switching costs. Therefore, the presence of switching costs and exchange rate uncertainty may either increase or decrease the level of prices. The complexity of the equilibrium dynamics between the buyer and the sellers described above indicates that the presence of both differing switching costs and exchange rate uncertainty, causes very significant changes in the strategic relationships between the respective players.

5. Summary and Conclusions

We propose and investigate a continuous-time framework to analyze strategic relationships between a representative buyer and two sellers in different countries exposed to exchange rate uncertainty. Our objective is to analyze the impact of the interplay between switching costs incurred by the buyer vis-à-vis the two sellers, and exchange rate uncertainty, on the relative bargaining power of the respective players and the resulting effect on the nature of their strategic relationships. We show that the presence of switching costs may lead to switching
equilibria where the sellers co-exist or no-switching equilibria where either seller may capture the market. In the presence of exchange rate uncertainty, switching costs may either raise or lower the level of long-term prices. In particular, we show that, beyond a critical level of the exchange rate volatility, the two sellers cannot co-exist in any equilibrium. The interplay between switching costs and exchange rate uncertainty, therefore, has a significant impact on buyer-seller relationships.

Our results are qualitatively robust to modifications of the basic framework. First, we can alter the model to examine the scenario wherein the buyer is myopic, that is, its decision horizon ends at the time it exits the foreign market from a relationship with either seller. The buyer renegotiates prices with the two sellers before each re-entry into the foreign market. The sellers, in turn, quote prices by rationally anticipating the buyer’s myopic policy. Since each “round” of this dynamic renegotiation game is independent of other rounds, we can show that our results are not affected qualitatively. Second, our analysis can also be extended to the scenario wherein the buyer can renegotiate its contracts with the sellers each time the exchange rate reaches an exogenously specified level. Such an assumption is realistic, for example, in the scenario wherein the buyer only contemplates sourcing from the foreign suppliers, and therefore negotiating contracts with them, when the exchange rate is at a favorable level. Given the process (2.1) for the exchange rate, it can be shown that equilibria of this repeated game are renegotiation-proof, that is, the buyer's optimal policy is such that the sellers quoted prices are never renegotiated. However, the equilibrium prices the sellers quote in this game are, in general, different from the prices we obtain within the framework assumed in the current paper.

Consistent with seminal studies in the “switching costs” literature that we primarily relate to (Von Weizsacker 1984, Klemperer 1987, Dixit 1989, Farrell and Shapiro 1988, Beggs and
Klemperer 1992, Klemperer 1995, Padilla 1995), we assumed that the buyer cannot adopt “inventory management” strategies where it meets its demand for the product at future dates by building up an excess inventory at an earlier date when prices are more favorable (this assumption is realistic, for example, if the product is perishable, or inventory/storage costs are prohibitively high). While a rigorous examination of the consequences of relaxing this assumption is undoubtedly important, it would complicate the analysis considerably, and it is unclear whether the resulting model would be tractable. Our primary objective in this article is to contribute to the debate initiated by the above-mentioned studies by examining the impact of the interplay between switching costs and dynamic uncertainty on buyer-seller relationships in a comparable, analytically tractable framework.

In future research, it is important to extend the model to incorporate the possibility of more general, dynamic renegotiation of prices, and inventory management by the buyer. This presents a significant conceptual as well as analytical challenge since the buyer’s sourcing strategies and the sellers’ pricing strategies would have to be determined in an equilibrium of a complex, dynamic game. Moreover, the presence of switching costs would introduce non-trivial path dependencies in this game that would further complicate this analysis. Therefore, it is likely that the more general model would only be amenable to numerical analysis.
Appendix

Proof of Proposition 3.1: We begin by noting that \( z^i_1(p_0) \) is a monotonically decreasing continuous function of \( \lambda \). For \( \lambda = 1 \), it is clearly optimal for the buyer to use seller 1 alone. Therefore, \( z^i_1(p_0) = v^i_1(p_0) > v^i_2(p_0) \). Moreover, \( \lim_{\lambda \to 1} z^i_1(p_0) = 0 < v^i_2(p_0) \). Hence, \( \lambda_{\text{max}} \) exists and \( \lambda_{\text{max}} > 1 \). Since, \( z^j_1(p_0) = v^j_1(p_0) \) for \( \lambda \leq 1 \), we must have \( \lambda_0 > 1 \). Therefore, \( \lambda_{\text{min}} \geq 1 \).

Since \( v^i_2(p_0) \) does not depend on \( \lambda \), it follows by the definition of \( \lambda_{\text{max}} \) that it is always optimal for the buyer to enter the market with seller 1 for \( \lambda < \lambda_{\text{max}} \) and use seller 2 alone for \( \lambda > \lambda_{\text{max}} \) with \( \lambda_{\text{max}} \) being the point of indifference between these two policies.

a) Since, by definition, \( \lambda_{\text{min}} \leq \lambda_{\text{max}} \), it follows from the proof of part a) that for \( \lambda < \lambda_{\text{min}} \), it is optimal for the buyer to enter the market with seller 1. Moreover, by the definition of \( \lambda_{\text{min}} \) in the statement of the proposition, it is optimal for the buyer to use seller 1 alone for \( \lambda < \lambda_{\text{min}} \). It remains to prove that if \( \lambda_{\text{min}} = \lambda_0 < \lambda_{\text{max}} \) and \( \lambda_{\text{min}} < \lambda < \lambda_{\text{max}} \), it is never optimal for the buyer to use seller 1 alone, i.e. the buyer will always enter the market with seller 1 and optimally switch to seller 2 when the price falls further. We can prove the result by the following arguments.

By the definition of \( \lambda_0 \) and the fact that \( z^i_1(p_0), v^i_1(p_0) \) are continuous functions of \( \lambda \), \( z^i_{\lambda}(p_0) = v^i_{\lambda}(p_0) \), that is, for \( \lambda = \lambda_0 \), the buyer is indifferent between the policy of using seller 1 alone and the policy of entering the market with seller 1 and optimally switching to seller 2 if the price falls further. Let the optimal entry and exit points for state 2 when \( \lambda = \lambda_0 \) be given by \( p_{12}, p_{20} \) respectively. Then \( \lambda_0 \) is the indifference point if and only if

\[
E_{p_{12}} \int_0^{\tau_{20}} \exp(-\beta s)(1-p(s))ds - E_{p_{12}} \int_0^{\tau_{20}} \exp(-\beta s)[1_{p(s) < \frac{1}{\lambda_0}}(1-\lambda_0 p(s))]ds = k_2
\]
In the above, the first term on the left is the expected value (conditional on the current price being \( p_{12} \)) of switching to seller 2 and continuing with seller 2 until the exit trigger \( p_{20} \) is reached and the second term on the left hand side is the corresponding expected value if the buyer were to instead follow the policy of entering and exiting with seller 1 at the threshold \( 1/\lambda_0 \) without ever switching to seller 2. These arguments would repeat for each subsequent re-entry of the buyer into the foreign market after exiting it from a relationship with seller 2. Suppose it were optimal to use seller 1 alone for some \( \lambda > \lambda_0 \). Since \( \lambda > \lambda_0, 1/\lambda < 1/\lambda_0 \), we see that

\[
E_{p_{12}} \int_{0}^{\tau_{p_{20}}} \exp(-\beta s) \frac{1}{\lambda_0} (1-\lambda_0 p(s)) ds > E_{p_{12}} \int_{0}^{\tau_{p_{20}}} \exp(-\beta s) \frac{1}{\lambda_0} (1-\lambda_0 p(s)) ds.
\]

Therefore, we see that

\[
E_{p_{12}} \int_{0}^{\tau_{p_{20}}} \exp(-\beta s)(1-p(s)) ds - E_{p_{12}} \int_{0}^{\tau_{p_{20}}} \exp(-\beta s) \frac{1}{\lambda_0} (1-\lambda_0 p(s)) ds > k_2,
\]

Therefore, the policy of switching to seller 2 at \( p_{12} \) and exiting at \( p_{20} \) has strictly greater value than the policy of using seller 1 alone and the policy of using seller 1 alone cannot be optimal.

This completes the proof of the proposition. ♦

**Proof of Proposition 3.2**: For analytical simplicity, recall that we consider the case where \( k_1 = 0, k_2 > 0 \). We have previously argued that if it is optimal for the buyer to enter the foreign market with seller 2, it will never switch to seller 1 so that its switching option trivially has zero value. Therefore, it suffices for us to show the existence of \( \sigma_r \) such that for \( \sigma > \sigma_r \), if the buyer enters the market with seller 1, it will never subsequently switch to seller 2 for any value of \( \lambda \).
**Step 1** Characterization of Buyer’s Additional Value from Switching. The buyer enters the market with seller 1 when the price process $p(.) = 1/\lambda$. We may assume that $\lambda > 1$ since it is optimal for the buyer to use seller 1 alone if $\lambda \leq 1$. We characterize the maximum possible additional value $w(p_e, \lambda, \sigma)$ the buyer obtains from switching to seller 2 at some $p_e \leq 1/\lambda$ and continuing with seller 2 until it exits the market at some $p_q(\lambda, \sigma) > 1$. The additional value from switching and the optimal exit point both depend on $\lambda$ and $\sigma$. The function $w(p_e, \lambda, \sigma)$ can be expressed as follows:

$$w(p_e, \lambda, \sigma) = E_{p_e} \left\{ \int_0^\beta e^{-\beta k} [(1 - p(s)) - 1_{p(s) \leq 1/\lambda} (1 - \lambda p(s))] ds \right\} - k_2 = x(p_e, \lambda, \sigma) - k_2$$

The first term in the integrand above is the discounted profit from switching to seller 2 and the second term is the discounted profit from continuing with the policy of entering and exiting the market with seller 1 at $1/\lambda$, until the exit point $p_q(\lambda, \sigma)$ is reached. We can use Ito’s lemma to see that $x(p, \lambda, \sigma)$ satisfies the following differential equation for each $\lambda, \sigma$:

$$\frac{1}{2} \sigma^2 p^2 \frac{\partial^2 x}{\partial p^2} + \mu p \frac{\partial x}{\partial p} - \beta x + (1 - p) - 1_{p \leq 1/\lambda}(1 - \lambda p) = 0$$

with the boundary conditions $x(p_q(\lambda, \sigma), \lambda, \sigma) = 0$, $\frac{\partial x(p_e, \lambda, \sigma)}{\partial p} \bigg|_{p = p_q(\lambda, \sigma)} = 0$. These arise from the fact that $x(p, \lambda, \sigma)$ attains its maximum possible value at $p = p_q(\lambda, \sigma)$. From the above, we see that $x(p, \lambda, \sigma)$ has the following functional form:

$$x(p, \lambda, \sigma) = A(\lambda, \sigma)p^{\eta_1(\sigma)} + \frac{(\lambda - 1)p}{\beta - \mu}; \text{ for } p < 1/\lambda$$

$$= B(\lambda, \sigma)p^{\eta_1(\sigma)} + C(\lambda, \sigma)p^{\eta_1(\sigma)} + \frac{1}{\beta} - \frac{p}{\beta - \mu}; \text{ for } 1/\lambda \leq p < p_q(\lambda, \sigma)$$

$$= 0; \text{ for } p \geq p_q(\lambda, \sigma)$$
In the above, \( \eta_i^+(\sigma), \eta_i^- (\sigma) \) are the positive and negative roots of the quadratic equation (2.13) where we have indicated their explicit dependence on the exchange rate volatility \( \sigma \).

**Step 2 Coefficients and Exit Point** \( p_q(\lambda, \sigma) \)

The coefficients above and the exit point \( p_q(\lambda, \sigma) \) are obtained from value matching (continuity) and smooth pasting (differentiability) conditions at the boundaries of the different regions. We can solve these equations after some tedious algebra to obtain the following equation for \( p_q(\lambda, \sigma) \):

\[
(A3) \quad (\mu \eta_i^+(\sigma) - \beta)(\lambda p_q(\lambda, \sigma))^{\eta_i^+(\sigma)} + \beta p_q(\lambda, \sigma)(1 - \eta_i^+(\sigma)) = (\mu - \beta)\eta_i^+(\sigma)
\]

The root of the above equation that is greater than 1 is the required exit point.

**Step 3 Properties of roots** \( \eta_i^+(\sigma), \eta_i^- (\sigma) \) and coefficients \( A(\lambda, \sigma), B(\lambda, \sigma) \)

The coefficients \( B(\lambda, \sigma), A(\lambda, \sigma) \) in (A2) are given by

\[
(A4) \quad B(\lambda, \sigma) = \frac{\eta_i^-(\sigma)}{\beta(\eta_i^+(\sigma) - \eta_i^-(\sigma))} (p_q(\lambda, \sigma))^{-\eta_i^+(\sigma)} + \frac{1 - \eta_i^-(\sigma)}{(\beta - \mu)(\eta_i^+(\sigma) - \eta_i^-(\sigma))} (p_q(\lambda, \sigma))^{1-\eta_i^+(\sigma)}
\]

\[
A(\lambda, \sigma) = B(\lambda, \sigma) + \frac{\lambda \eta_i^+(\sigma)}{(\eta_i^+(\sigma) - \eta_i^-(\sigma))} \left( \frac{\mu \eta_i^-(\sigma) - \beta}{\beta(\beta - \mu)} \right)
\]

We may easily check that the roots \( \eta_i^+(\sigma), \eta_i^- (\sigma) \) of equation (2.13) have the following properties:

\[
(A5) \quad \eta_i^+(\sigma) > 1, \quad \eta_i^- (\sigma) < 0, \quad \lim_{\sigma \to \infty} \eta_i^+(\sigma) = 1, \quad \lim_{\sigma \to \infty} \eta_i^- (\sigma) = 0
\]

From (A4) we see that

\[
(A6) \quad |B(\lambda, \sigma)| \leq \frac{\eta_i^-(\sigma)}{\beta(\eta_i^+(\sigma) - \eta_i^-(\sigma))} ||(p_q(\lambda, \sigma))^{-\eta_i^+(\sigma)}|| + \frac{1 - \eta_i^-(\sigma)}{(\beta - \mu)(\eta_i^+(\sigma) - \eta_i^-(\sigma))} ||(p_q(\lambda, \sigma))^{1-\eta_i^+(\sigma)}||
\]

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Since \( p_q(\lambda, \sigma) > 1 \) and \( \lambda > 1 \) and \( \eta^+(\sigma) > 1 \), we see that

\[
(p_q(\lambda, \sigma))^{-\eta^+(\sigma)} < 1 \quad \text{and} \quad (p_q(\lambda, \sigma))^{1-\eta^+(\sigma)} < 1.
\]

Hence, from (A5) and (A6), we see that

\[
(A7) \quad \lim_{\sigma \to \infty} |B(\lambda, \sigma)| \leq \frac{1}{\beta - \mu}
\]

where the inequality above holds uniformly in \( \lambda \). We see from (A1), (A2), (A4), (A5), (A7),

\[
l_{\sigma \rightarrow \infty} w(p_e, \lambda, \sigma) = \lim_{\sigma \to \infty} x(p_e, \lambda, \sigma) - k_2 = \lim_{\sigma \to \infty} A(\lambda, \sigma)(p_e)^{\eta^+(\sigma)} + \frac{(\lambda - 1)p_e}{\beta - \mu} - k_2
\]

\[
= \lim_{\sigma \to \infty} B(\lambda, \sigma)(p_e)^{\eta^+(\sigma)} + \frac{(\lambda - 1)p_e}{\beta(\beta - \mu)} - k_2
\]

\[
= (1 - \lambda)p_e + \frac{(\lambda - 1)p_e}{\beta - \mu} - k_2 < 0
\]

Since \( p_e \leq \frac{1}{\lambda} \) so that \( \lambda p_e \leq 1 \), we see that the inequality above holds uniformly in \( p_e \in [0,1] \) and

\( \lambda \in [1, \frac{1}{p_e}] \). Since \( w(p_e, \lambda, \sigma) \) is a continuous function of its arguments, this clearly implies the existence of an exchange rate volatility level \( \sigma_T \) beyond which the additional value of switching to seller 2 from seller 1 is non-positive for all \( \lambda \) and all possible switching points \( p_e \leq 1/\lambda \).

Since, as we have noted earlier, it is sub-optimal for the buyer to switch to seller 1 after entering the market with seller 2, it follows that the buyer’s switching option has zero value for \( \sigma > \sigma_T \).

This completes the proof. ♦

**Proof of Proposition 4.1**: For the sake of concreteness, we will assume that (4.5) has a solution but (4.6) may or may not have a solution. The arguments are similar for the other case.

a) The proof proceeds by contradiction. Suppose seller 2 captures the market in some equilibrium and let the equilibrium price quoted by seller 2 be \( Q_2 > C_2 \). Seller 2 may capture
the market if it were chosen as the leader or the follower by the buyer. Suppose first that it were
chosen as the follower. Since $\lambda_{\min} < \frac{C_1}{C_2} < \lambda_{\max}$, seller 1 can always guarantee itself strictly
positive expected profits by quoting a price in the interval $(C_1, \lambda_{\max} C_2)$. Since a solution to (4.5)
exists by hypothesis, capture of the market by seller 2 cannot be an equilibrium outcome when
seller 1 is chosen as the leader and seller 2 the follower. Suppose seller 2 captures the market
being chosen as the leader. This can only occur if a solution to (4.6) exists. In this case, the
previous argument applies to show that seller 1 may always obtain strictly positive expected
profits by quoting a price in the interval $(C_1, \lambda_{\max} C_2)$. Hence, conditions (4.5) ensure that seller
1 has an optimal response $\psi_1(Q_2)$ at which it obtains strictly positive expected profits. Hence,
the capture of the market by seller 2 cannot be an equilibrium outcome.

On the other hand, suppose seller 1 captures the market in some equilibrium and let the
equilibrium price quoted by seller 1 be $Q_1 > C_1$. Suppose first that seller 1 was chosen as the
leader and seller 2 the follower. Since $C_1 > \lambda_{\min} C_2$ by hypothesis, in response to the price $Q_1$
quoted by seller 1, seller 2 may always guarantee itself strictly positive expected profits by
quoting a price in the interval $(C_2, C_1 / \lambda_{\min})$. Hence, conditions (4.6) ensure that seller 2 has an
optimal response $\psi_2(Q_1)$ at which it obtains strictly positive expected profits. Since a solution to
(4.5) exists by hypothesis, capture of the market by seller 1 cannot be an equilibrium outcome if
it were chosen as the leader and seller 2 the follower. On the other hand, suppose seller 2 were
chosen as the leader and seller 1 the follower. If (4.6) has a solution, then the argument above
applies to show that seller 1 cannot capture the market in equilibrium. If (4.6) does not have a
solution, then seller 2 is unable to quote a price so that seller 1 captures the market. However,
the buyer's value function in this case is clearly lower than its value function from choosing

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seller 1 as the leader and seller 2 as the follower (for which an equilibrium exists by hypothesis) since the buyer benefits from competition. Since we have already shown that both sellers must co-exist in such an equilibrium outcome, we have shown that the capture of the market by seller 1 cannot be an equilibrium outcome.

b) The proof basically follows from the fact that both sellers must follow markup pricing policies and the buyer’s optimal policy involves the use of either seller 1 over time or seller 2 over time depending on the ratio of the prices they quote. We omit the details here for the sake of brevity.

References


Figure 1a: Variation of Value Functions with Lambda
(drift = 0, beta = 0.025, initial price = 1.5, k1 = 0.01, k2 = 1.0, sigma = 0.1)

Figure 1b: Variation of Value Functions with Lambda
(drift = 0, beta = 0.025, initial price = 1.5, k1 = 0.1, k2 = 1.0, sigma = 0.1)
Figure 2a: Variation of Value Functions with $k_2$
($\mu = 0$, $\beta = 0.025$, $k_1 = 0.01$, initial price = 1.5, $\sigma = 0.15$, $\lambda = 1.1$)

$k_2$

$0.5$ $0.7$ $0.9$ $1.1$ $1.3$ $1.5$

$k_{min} = 0.5$, $k_{max} = 1.2$

Figure 2b: Variation of Value Functions with $k_2$
($\mu = 0$, $\beta = 0.025$, $k_1 = 0.01$, initial price = 1.5, $\sigma = 0.15$, $\lambda = 1.7$)

$k_2$

$5$ $6$ $7$ $8$ $9$

Degenerate Switching Region
**Fig. 3:** Variation of \( \lambda_{\text{min}} \) and \( \lambda_{\text{max}} \) with \( \sigma \)

\( k_1 = 0.01, k_2 = 0.25, \mu = 0, \beta = 0.025 \)

\( \sigma^T = 0.84 \)

**Figure 4:** Variation of Equilibrium Prices with \( C_1/C_2 \)

\( \mu = 0, \beta = \beta' = 0.025, \sigma = 0.1, k_1 = 0.01, k_2 = 0.5, C_1 = 0.8 \)

\( \lambda_{\text{min}} = 1.04, \lambda_{\text{max}} = 1.09 \)