Reverse Bullwhip Effect in Pricing

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Abstract: This study analyzes the impact of procurement price variability to the retail prices. Procurement prices may fluctuate over time, for example when the supply chain players deploy auction type procurement mechanisms. Both simultaneous and sequential gaming scenarios are investigated here to show that there is an increase in retail price variability and a reverse bullwhip effect on prices under certain demand conditions.

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1 Introduction

Procurement accounts for a significant portion of an enterprise’s expenditures resulting in some cases up to 90% of the costs for direct inputs that go into the production of the final products [1]. Due to the potential cost savings and recent advances in e-business, companies have been re-evaluating their procurement policies [2] and looking into newer processes such as auctions and reverse auctions that are enabled through electronic marketplaces. It is estimated that physical goods exchange in an e-business environment will be over $3B in 2004 [3]. While mentioning the cost saving aspects of e-business processes, we would like to note here that a company buying materials through auctions and reverse auctions will face procurement price variability over time. As noted in the literature, price variations are one of the main causes of order information distortion in supply chains (a.k.a. the bullwhip effect [4]), which creates adverse effects such as excess inventories, backorders and resource inefficiencies.

Here our focus is on the impact of the supplier price variability possibly resulting from auction type of procurement mechanisms on the overall supply chain pricing. Our results indicate that variable purchase prices caused by auctions can increase the variability of the retailer prices which can be considered as a bullwhip effect in prices in the reverse direction from the upstream suppliers to the downstream retailers.

Next, we will continue introducing several models for single stage and two-stage supply chains. We will apply a game theoretic framework to capture the pricing mechanism be-
tween the supply chain players. Both simultaneous and sequential games will be discussed. The final section includes a summary of major findings and conclusions.

2 Model

2.1 Single Stage Supply Chain

Consider a retailer (R) subject to a deterministic demand curve \( q(p) \) that is dependent on retail price \( p \). \( q(p) \) is a decreasing function \(^3\) which basically shows that as the retail prices go up the customers will buy less and vice versa. Let us also define \( w \) as the wholesale price that the supplier charges to the retailer. The retailer identifies the optimal price to maximize its profit \( \Pi_R(p) \) which is defined as follows:

\[
\max_p \Pi_R(p) = (p - w)q(p)
\]  

(1)

Given an appropriate demand curve \( q(p) \), the solution to equation (1) is fairly straightforward and can be found solving the following optimality condition.

\[
\frac{d\Pi_R(p)}{dp} = q + (p - w)q' = 0
\]  

(2)

Here, our focus is on the impact of variability of \( w \) on \( p \). Reverse bullwhip effect in pricing is defined as the variability of \( p \) being larger than of \( w \) i.e. \( \sigma_p \geq \sigma_w \). As shown below in lemma 1, this happens when \( p \) grows faster relative to \( w \), i.e. \( \frac{dp}{dw} \geq 1 \).

Lemma 1. If \( \frac{dp}{dw} \geq 1 \) then \( \frac{\sigma_p}{\sigma_w} \geq 1 \).

Proof: The proof can be sketched as follows: \( p(w) \) can be represented as a combination of infinite many linear function approximations. If \( \frac{dp}{dw} \geq 1 \) then the variability of this function yields \( \sigma_{p(w)}^2 \geq \sigma_{w}^2 \). □

Our hypothesis is that the reverse bullwhip effect takes place for certain demand behaviours as expressed by the demand curve \( q(p) \). The following lemma describe the required condition for the reverse bullwhip in retail pricing in a single stage supply chain.

Proposition 1. For a single stage supply chain \( \sigma_p \geq \sigma_w \) iff \( 1 \leq \frac{qq''}{q'^2} \leq 2 \).

Proof: From equation (2), we obtain \( p = w - \frac{q}{q'} \). Identifying the condition for \( \frac{dp}{dw} \geq 1 \) yields the desired result based on Lemma 1. □

Example:

Let \( q(p) = ap^{-k}, k \geq 1 \). It is easy to verify that \( 1 < \frac{qq''}{(q')^2} = \frac{k+1}{k} \leq 2, \forall k, k \geq 1 \)

2.2 Two-Stage Supply Chain

Simultaneous Games:

Next, the price variability in a two-stage supply chain is analyzed when the supplier and retailer determine their prices simultaneously. This case requires the supplier (S) to solve the following objective function

\(^3\)Throughout the paper, \( q \) is used as a shorthand for \( q(p) \), and \( q' \) and \( q'' \) to denote the first and second derivatives in order to ease the readability of the manuscript whenever the expressions tend to get complicated.
where \( c \) is the cost of the supplier and \( q(p) \) is the amount of material the retailer purchases from the supplier to meet the end customer demand. In this case, the retailer solves equation (1) as before, but the dependency of supplier demand to the retail price and retail price to the wholesale price creates a gaming situation. Supplier’s optimality condition can be expressed as:

\[
0 = q + (w - c) \frac{dq(p)}{dw}\]

and similarly for the retailer the following optimality condition is obtained:

\[
0 = (1 - \frac{dw}{dp})q + (p - w)q'\]

Based on these conditions the following lemma can be driven

**Proposition 2.** In a two-stage supply chain with simultaneous pricing game, the retailer’s optimality condition can be equivalently expressed in the same structure as in the one-stage case as given in equation (2).

**Proof:** By rearranging equation (4), we obtain \( \frac{dw}{dp} = -\frac{(w - c)q'}{q} \). Substituting this expression into the retailer’s optimality condition given in equation (5) yields the desired result. \( \square \)

Based on propositions 1 and 2 the following corollary is derived:

**Corollary 1.** For two-stage supply chains with simultaneous pricing game, \( \sigma_p \geq \sigma_c \) iff \( 1 \leq \frac{q^\prime}{(q^\prime)^2} \leq 2. \)

**Sequential Games:**

In a two-stage supply chain, a sequential game involves two steps. In step 1, the dominant supplier or the “Stackelberg Leader” sets his price. Then in step 2, the retailer sets the retail price for the end customer based on the supplier’s decision. The objective functions for the retailer is the same as before as given in equation (1). In step 1, the supplier anticipates the retailer’s reaction function and uses this information when making pricing decisions. The reaction function for the retailer can be defined as follows:

\[
r(w) = \{p | q + (p - w)q' = 0\}\]

Next, the supplier identifies the corresponding demand as \( q(r(w)) \) and defines its objective function

\[
\max_w \Pi_S(w) = (w - c)q(r(w))\]

which yields the following optimality condition for \( w \):

\[
0 = q(r(w)) + (w - c) \frac{dq(r(w))}{dw}\]

Finally, in step 2, the retailer identifies the optimal price \( p \) using the reaction function given in (6). The following lemma summarizes price variability results for a sequential game in a two-stage supply chain.

**Proposition 3.** In a two-stage supply chain with sequential pricing games, \( \sigma_p \geq \sigma_c \) iff

\[
\left[2 - \frac{q^\prime}{(q^\prime)^2}\right]^{-1} \left[2 - \frac{q^\prime q(r(w))/dw^2}{(dq(r(w))/dw)^2}\right]^{-1} \geq 1.\]
Proof: The proof requires deriving $\frac{dp}{dc} = \frac{dp}{dw} \frac{dw}{dc}$. □

The above result indicates that the effect of sequential games on pricing variability is multiplicative. Hence, when the variability is amplified from one stage to another a reverse bullwhip effect on pricing is created. This observation is summarized in the following corollary.

**Corollary 2.** In a two-stage supply chain with sequential pricing games, a reverse bullwhip effect takes place with amplified variability when $1 < \frac{d^2}{dp^2} < 2$ and $1 < \frac{qdw^2}{dqw^2} < 2$.

**Example:**
Let $q(p) = ap^{-k}$. It is each to verify that $p = r(w) = \frac{k}{k-1}w$ and accordingly, $\frac{dp}{dw} = \frac{k}{k-1}$. Based on the corresponding demand function $q(r(w)) = a (\frac{k}{k-1}w)^{-k}$ the supplier identifies its optimal price as $w = \frac{k}{k-1}c$. Accordingly, $\frac{dw}{dc} = \frac{k}{k-1}$ and $\frac{dp}{dc} = \frac{dp}{dw} \frac{dw}{dc} = (\frac{k}{k-1})^2$. Since $\frac{k}{k-1} > 1$, $\forall k, k \geq 1$ a reverse bullwhip effect takes place.

**References**


