A FORMAL MODEL FOR SUPPLY CHAIN NETWORKS –
CAPTURING SUPPLY CHAIN MANAGEMENT BY
A MATHEMATICAL ILLUSTRATION

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ABSTRACT

The purpose of this paper is to provide a formal framework for Supply Chain Management that takes into account the complexity of supply chain networks. This is achieved by developing a formal, mathematical representation that captures all involved supply chain network connections by using existing viewpoints and by adding additional aspects into a new emerging model. Integrating value-added elements into the model allows identification of generic implications. Furthermore, the model can be applied to specific supply chain networks. The practical value lies in the possibility of better evaluating a given supply chain network structure. The model supports the strategic decision making process in Supply Chain Management. This approach promises further potential by enhancing the model; it may be also used as a basis for economic modeling.

DEFINING SUPPLY CHAIN MANAGEMENT

In order to develop a formal framework for Supply Chain Management (SCM), we want to clarify some terminology issues. SCM is strongly connected with the terms “value chain”, “supply chain” and also “distribution chain”. With reference to Hammer and Champy’s (1994) framework that emphasizes a process view on companies, we have adopted a process view of SCM in order to analyze and structure a supply chain. With this, a specific product or service triggers all supply chain processes and therefore defines it. All participating companies in such a supply chain have three macro processes in common (Chopra & Meindl, 2004, p. 17): Supplier Relationship Management (SRM), Internal Supply Chain Management (ISCM), and Customer Relationship Management (CRM). For each company, all supply chain upstream activities of a company are connected to the respective SRM and constitute therefore a supply chain. All company-internal activities refer to the internal value creation process. Porter (1985) provides a well-known framework for this internal value chain. All supply chain downstream activities are
connected to the respective CRM process of a company and are therefore part of the distribution chain of this company. It should be noted here that depending on a company’s position within a certain supply chain, SRM, ISCM, and CRM consists of different characteristics and processes. If we see the entire supply chain as the sum of all activities to provide a product or service to the end consumer – which is ultimately always the case – the entire chain of a specific product or service can be seen as a supply chain from the end consumer’s point of view. Therefore, SCM aims at managing all physical, information, and financial flows of such a chain. Chopra and Meindl (2004, p.6) explicitly add the objective of maximizing total supply chain profitability.

Chopra and Meindl (2004, p.5) also note that the terms “supply network” or “supply web” might be a more accurate description since companies normally transform inputs from many suppliers into one or more outputs for often many different customers. Therefore, such constellations do not fit the term “chain”. McCormack and Johnson (2003) call such a constellation a “supply chain network”. But by adopting a process view of supply chains, it can be argued that it indeed does fit the description of a chain. Although there might be many suppliers of raw materials and modules throughout the supply process, in the end it is all merged into one product or service. This entire supply process can therefore be seen as a chain from a consumer’s perspective. Additionally, the physical flow of a supply chain routes through the system in only one, sequential direction.1 Accompanying information and financial flows, however, can go both

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1 We disregard here the increasing importance of reverse logistics or reverse SCM, since this does not essentially alter our considerations. The primary objective is to satisfy consumer demand. Without consumer demand, there would not be any reverse logistics necessary anyway. However, such considerations have implications on decisions within a supply chain without substantially altering its structure.

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ways, then again supporting the network view. But not only can the term “chain” be questioned, but also the term “supply” as it may not always be obvious.

Since the purpose of a supply chain is to provide value to the consumer, Porter (1985) describes such a system as *value system*. Ultimately, all activities and processes aim to bring value to the consumer, be it as a product or a service. The entire process is therefore a value creation process. In the end, value is expressed by the willingness of consumers to pay a certain price for the product or service. From a business process perspective, activities that do not add value are waste and should be eliminated. This is especially important in highly competitive markets. We also believe that the term “value” better describes the processes within the value creation process. However, there are also arguments that support the traditional and established terminology. In accordance with the explanation given above, i.e. that from a consumer perspective all activities are supply activities, all preceding activities and processes of the total value can also be seen as supply that contributes to this value.

As we can see, SCM has many facets and there are many good reasons to question the terminology. The terms supply chain and SCM, however, are well established. It should be accepted that they cover all the aspects described above. We choose to use supply chain and supply chain network as synonyms. Using different terms beyond this – within the framework described above – is likely to cause confusion without adding any additional insight.

Although we do not want to start or continue terminological discussions – and by that follow Popper’s recommendation to avoid terminological discussions because of their lack of contribution to the increase in the body of knowledge (see Popper, 1992a, p.41; Popper 1992b, p.15) – the clarifications above seem to be necessary.
THE SUPPLY CHAIN NETWORK FRAMEWORK

For developing our formal model, we use the supply chain network framework provided by Walker (2003). Under this approach, first the main thread in a supply chain should be identified. Mostly, the main thread constitutes the physical flow that carries the most added value (Walker, 2003, pp. 104-105). In case of a service supply chain network, it might be an information flow as well. The emphasis is put on the value added. However, it can also be a crucially important link within the network without adding much monetary value. The rest of the network builds around the main thread (see figure 1). The main thread indicates a sequential throughput. This is important for our formal model later on as it allows the division of the supply chain network into layers. There is always at least one main thread position in each layer and they are all connected to each other, i.e. a main thread position in a supply chain network cannot exist without being connected to all other main thread positions through at least one main thread position.

Figure 1: Simplified supply chain network, identifying the “main thread” (see Walker, 2003, p. 105)
Another important factor is to identify the supply chain driver, or, as Walker (2003) and McCormack and Johnson (2003) put it, an orchestrator. The supply chain driver is the member that “owns” the key value to the supply chain. For example, if a manufacturer holds a patent for a certain technology that defines the product or service, then this manufacturer possesses the most power within the supply chain. Consequently, he is responsible for taking the lead in defining the overall supply chain strategy. In case access to the customer is crucial, companies holding the key to the customers constitute a relatively high control power over the supply chain, as it is, e.g., for mail-order companies, department stores, discounters, hardware stores, or electronics superstores. It could also be that the supply chain driver is not even part of the physical supply chain, as in the case of Red Bull. Red Bull only possesses its marketing power and drink recipe. The bottling and selling is done by supply chain network members that are independent from Red Bull. Still, Red Bull drives this supply chain. Being aware of the supply chain driver is important when it comes to determining the key player in aligning and coordinating the supply chain to maximize total supply chain profitability.

The last key aspect of Walker’s framework (2003) is the explicit modeling of physical, information, and financial flows as “planes” (p. 110-111). These “planes” represent an explicit view of physical, information, and financial flows by separating them visually into three horizontal layers. He shows that by looking at them individually from a business process perspective, the linkages become clear while still being distinct. Within a company, they affect each other but between companies, each “plane” links with the corresponding one at the connected company. Figure 2 illustrates this for our simplified supply chain network. This

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2 Red Bull is a energy drink company based in Austria. Their sales grew in 2004 by 32.3% to 1,668 Mio. EUR.
network is aligned according to the main (physical) thread. However, information and financial flows can spread among all supply chain members, depending on the policies to which they are exposed. This is omitted at this point, but the possibility is considered later in the formal model.

**Figure 2: Simplified supply chain network, illustrating physical, information and financial flows**

**A FORMAL REPRESENTATION FOR SUPPLY CHAIN NETWORKS**

Based on the presented framework above, we can now develop a formal model that helps identify all supply chain network elements and define an objective formulation for overall supply chain profitability. We derive a general framework that has to be adapted to the requirements of the supply chain network under consideration. When applied, the model design depends highly on the individual supply chain network. Therefore, one should follow such a contingency approach when applying it. Consequently, as with any model, it is of crucial importance that the
A supply chain network is defined by its main product or service. Ultimately, each and every individual product or service has its own supply chain. Obviously, it is not feasible to model such a view. Even considering an aggregate view of a single product might be too narrow a focus. It seems to be suitable rather to build a homogeneous product group that generally shares the same supply chain. For example, there are many different VW Golf models available, e.g., with diesel or gasoline engine, convertible, station wagon or sedan and so forth. It is appropriate to build a homogeneous product group “Golf” instead of a group for each model variation.\(^3\) The main factor that should be considered when including several model variations is the implied changes necessary within a supply chain. If the supply chain network structure changes significantly by including more product variations, they should not be included. The total value created by a supply chain network over a period of time can generally be described as follows:

\[
X_{Pa} = s_{Pa} - v_{Pa}
\]

with \(X_{Pa}\) = total value created in period \(a\) by the supply chain network of product \(P\) that defines the supply chain network

\(s_{Pa}\) = total sales revenue of product \(P\) in period \(a\)

\(v_{Pa}\) = cost of material and services procured from outside the supply chain network in period \(a\)

\(^3\) In the remainder of the paper, we refer for simplicity reasons to homogeneous product and service groups as products.
In order to determine the span of value creation a supply chain network covers, we can calculate a supply chain depth indicator $X_{Pa}$ in the following way:

\[
(2): \quad X_{Pa} = 1 - \frac{V_{Pa}}{s_{Pa}}
\]

According to our supply chain network model, we can identify $i = 1,\ldots,n$ layers between the source and the end of the supply chain. The layers are aligned according to the main thread. Each layer $i$ holds $j = 1,\ldots,m_i$ positions for companies that contribute and add value to this supply chain network (see figure 3).

The total value creation $X_{Pa}$ of a supply chain network is therefore the sum of the added values of all positions and can be calculated as follows:

\[
\text{Figure 3: Supply chain network structure including position notions}
\]
(3): \( X_{Pa} = \sum_{i=1}^{n} \sum_{j=1}^{m_i} (X_{Pa})_{ij} \)

with \( X_{Pa} = \) total value created by the supply chain network of product \( P \) that defines the supply chain network in period \( a \)

\( (X_{Pa})_{ij} = \) incremental added value of position \( ij \) for product \( P \) in period \( a \)

\( i = \) supply chain network layer, \( i = 1(\text{source}), \ldots, n \) (terminal layer)

\( j = \) position within layer \( i, j = 1, \ldots, m_i \)

\( (X^{*}_{Pa})_{ij} = \) indicates a main thread position

Mostly, the (physical) main thread routes through some kind of Original Equipment Manufacturer (OEM). This OEM can serve as an orientation point for a supply chain network since the network “funnels” through this layer. Usually, the OEM layer consists only of the OEM position. Figure 4 illustrates this view.

![Figure 4: Layer depiction of a supply chain network](image)

The start of the supply chain network depends on the specific supply chain constellation. The (functioning and efficient) markets for raw materials are often a good starting point. The raw material layer should only be included if it represents a raw material that is of particular relevance for the supply chain. Commodity products can be excluded from the supply chain...
network perspective if an efficient supply is ensured and / or it is not particularly important or critical to the product. In some instances, it can make sense to go beyond these market boundaries in order to secure supply. From formula (1), it becomes clear that in the extreme case of including all companies connected to a supply chain, we would achieve total integration with \( v_a \) being zero since there would no longer be any outsiders.

From this we now derive the objective function for supply chain networks. One important aspect of SCM is to maximize total supply chain profitability (Chopra & Meindl, 2004, p.6). However, profitability can have many different meanings. In our model, we assume that the long-term profitability of the supply chain network should be maximized. While there are various profitability performance measures possible, we have chosen the Net Discounted Cash Flow (NDCF) for \( k \) periods – the planning horizon – as an appropriate profitability measure. However, other profitability or performance measures could also be used to determine supply chain objectives. The objective here, therefore, is to maximize the sum \( Z_P \) of all supply chain network participants’ NDCFs.

\[
(4): Z_P = \sum_{i=1}^{n} \sum_{j=1}^{m} (NDCF_{ij}^P) \rightarrow \text{max! for } k \text{ periods}
\]

with \((NDCF_{ij}^P)\) = Net Discounted Cash Flows of position \(ij\) in the supply chain network of product \(P\), with a planning horizon of \(k\) periods

Commonly acknowledged is the fact that each independent supply chain network member aims to maximize its own profits independently. Furthermore, companies often participate in more than one supply chain. Their profitability function \(Z_U\) can be described as follows (in accordance with above, for \(k\) periods):

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(5): \( Z_U = \sum_{P=p_1}^{p_n} \sum_{i=1}^{m_i} (NDCF_p)_{ij} \cdot (U_p)_{ij} \rightarrow \text{max! for } k \text{ periods} \)

with \( (U_p)_{ij} = 1 \), if company \( U \) fills position \( ij \) in product \( P \)'s supply chain network, 0 otherwise

\( P = \text{product or product group } P \) with \( P = p_1, \ldots, p_{lU} \) (i.e. products company \( U \) is involved in)

As we can see, formulae (4) and (5) are only partly linked to each other. Only parts of the objective function of company \( U \) are also elements of the objective function of the supply chain network for a specific product \( P \) if company \( U \) is involved in more than one supply chain network. Without a perfect incentive policy, solutions are likely to be sub-optimal.

So far, we have not considered the fact that the structure of a supply chain influences the effectiveness and efficiency of a supply chain network significantly. Even if all participants of a supply chain network acted in the best interest of the entire supply chain, profitability might be limited due to the structure, i.e. the linkages between supply chain members. Changing the structure of a given network can move the “efficient frontier” (see Chopra & van Mieghem, 2000) outwards, i.e. expanding the effectiveness and efficiency potential of this network. Therefore, this structure has to be formally represented as well. To achieve this, we identify connections of physical, information, and financial flows as follows:

(6): \( M_{(Y_p)_{ij}(Y_p)_{i'j'}} := 1 \), if physical flow between \( (Y_p)_{ij} \) and \( (Y_p)_{i'j'} \) exists, 0 otherwise

with \( i \neq i' \), if \( j = j' \) and \( j \neq j' \), if \( i = i' \)
(7): \( I_{(Y_p)_{ij}(Y_p)_{i'j'}} := 1 \), if information flow between \((Y_p)_{ij}\) and \((Y_p)_{i'j'}\) exists, 0 otherwise

\[
\text{with } i \neq i', \text{ if } j = j' \text{ and } j \neq j', \text{ if } i = i'
\]

(8): \( F_{(Y_p)_{ij}(Y_p)_{i'j'}} := 1 \), if financial flow between \((Y_p)_{ij}\) and \((Y_p)_{i'j'}\) exists, 0 otherwise

\[
\text{with } i \neq i', \text{ if } j = j' \text{ and } j \neq j', \text{ if } i = i'
\]

In our model, the variables \(M, I,\) and \(F\) for physical, information, and financial flows merely define whether a flow exists or not. In case a flow exists, it is also important to define the characteristics of this flow both quantitatively and qualitatively. Additionally, structural changes may be established by changing connections within the existing supply chain network members or by adding/removing positions to/from the network. Furthermore, it may be considered to exchanging companies occupying certain network positions.

**SELECTED ISSUES ARISING FROM THE FORMAL MODEL**

As we noted above, maximizing the monetary overall supply chain profitability with network members that belong to more than one supply chain network is problematic due to conflicting objective functions. But there are also other implications attached to this constellation. Different supply chains are also often aligned differently, with different policies and requirements. Some of these differences have consequences for all supply chains as it is difficult to totally separate them internally within a company. *Table 1* shows scenarios and possible generic implications.
This shows that dealing with companies that are not solely dedicated to one supply chain network complicates supply chain alignment matters further.

Even in the absence of such alignment problems (e.g., a company is only dedicated to one supply chain or only involved in complementary products), supply chain improvement initiatives might face unanticipated problems. As Sterman, Repenning and Kofman (1997) showed, improvement initiatives already prove to be difficult even when only applied internally. When more than one company is involved, we can suspect that similar problems are likely to be even worse and more complicated.

One way to solve coordination and integration problems with independent companies is to aim for a high vertical integration. Recently, the Spanish clothing company Zara proved to be highly successful by having “…five fingers touching the factory and five touching the customer.” (Ferdows, Lewis & Machuca, 2004, p. 106). By not including many external companies, Zara is able to apply full control over most of its supply chain (Ferdows, Lewis & Machuca, 2004). Even in this case our model can still be applied. In fact, it is much easier to impose policies that ensure overall supply chain profitability. There is no private information or differences in objectives. However, such a setup is not suitable for all supply chains, especially if certain

<table>
<thead>
<tr>
<th>supplies also</th>
<th>independent products</th>
<th>same product, competing supply chain network</th>
<th>competing products</th>
<th>substitute products</th>
<th>complementary products</th>
</tr>
</thead>
<tbody>
<tr>
<td>implication</td>
<td>Problem of overhead cost allocation. Maybe different strategic orientation and competitive situation of different supply chains.</td>
<td>Hard to establish a trust-based relationship. Protective attitude of supplier, likely to hold back information. Hard to achieve advantage over other supply chain.</td>
<td>Difficult to share confidential information. Supplier might tend to benefit the stronger customer.</td>
<td>Difficult to share confidential information. Supplier is likely to shift focus.</td>
<td>Little friction. Likely to follow similar objectives. Still competition for incentive allocation. Maybe different supply chain requirements.</td>
</tr>
</tbody>
</table>
supply chain functions do not reach a critical mass to integrate them or if there are know-how, other capabilities, or resources missing to perform the function.

Determining each position \((Y_{ij})\) in a supply chain network in terms of monetary added value is not sufficient, as it might be implied above. In order to align and improve the supply chain network processes, there have to be functions and purposes attached to each position in order to better understand the entire network. Many layers are actually intermediate layers between a manufacturer of a product (or the source of a service in a service supply chain) and the end consumer. One framework that can be used to determine functions and purposes of such intermediaries incorporates ten dimensions of intermediation functions based on the framework provided by Kotler et al. (1999). The following dimensions can classify the function of intermediaries in a supply chain: (1) sales and promotion, (2) procurement and offering (also of technology), (3) volume adjustment, (4) inventory and immediacy, (5) speed and transportation, (6) financing, (7) risk, (8) market information, (9) management support, and (10) integrating and balancing customer and supplier interests. Companies should fulfill one or more of these functions in order to add value to the network. These dimensions can then be attached to each position \((Y_{ij})\). The value in reviewing these dimensions lies in the better understanding of the function of each supply chain network position. Based on this understanding, the analysis of supply chain coordination and integration practices can be improved.

**CONCLUSION AND FUTURE RESEARCH**

In this paper, we suggest a formal representation of supply chain networks. The formal model we developed can be used for maximizing supply chain network performance by:

- formulating an objective function,
- defining the structure of a supply chain network,
- identifying the relationships within the supply chain network, and
- improving the supply chain network processes according to the objective function.

It can then serve as a framework to better structure supply chain issues, design supply chain interfaces, and choose appropriate analysis tools, and quantitative methods, especially due to the segmentation into layers. Furthermore, the number of layers of a supply chain can be used to evaluate the degree of vertical integration of a supply chain besides just the monetary measure $X_{PA}$.

Such an explicit, formal description supports the identification of all elements of a supply chain network. Based on the formal structure obtained, SCM attributes can be identified and an appropriate objective function can be derived. The framework then supports the improvement of supply chain flows as well as the effectiveness and efficiency of the overall value creation process. However, problems arising in internal improvement initiatives (see e.g. Sterman, Repenning & Kofman, 1997) are likely to be even more severe when such initiatives aim to span over an entire network.

In future research, it can be investigated if techniques and concepts used to address internal improvement initiative problems can be extended to supply chain network improvement initiatives. Furthermore, our model can be used as a basis for economic models that investigate the effectiveness and efficiency of supply chain network structures and designs. Also, since an optimization based on the mathematical representation might not be feasible or even not be possible due to the complexity and dynamics of behavior, the mathematical framework described above might serve as input in the process of developing a simulation model. Such a simulation model can be used to evaluate the dynamic consequences of different policies and structures of a supply chain.
REFERENCES


