A Single Phase Unified Approach for Designing a Closed-Loop Supply Chain Network

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**ABSTRACT**

Strategic planning of a supply chain primarily deals with the design (what products should be processed/produced in what facilities etc) of the supply chain that is typically a long-range planning. Tactical planning involves the optimization of flow of goods and services across the supply chain and is typically a medium-range planning. In this paper, we present a single-phase unified approach, employing goal programming, for these two stages of planning of a Closed-Loop Supply Chain Network (CLSC). When solved, the model identifies simultaneously the most economical used-product to re-process in the supply chain, the efficient production facilities and the right mix and quantity of goods to be transported across the supply chain. A numerical example is considered to illustrate the methodology.

Keywords: Closed-Loop Supply Chain, Strategic Planning, Goal Programming.

**INTRODUCTION**

Both consumer and government concerns for the environment are driving many original equipment manufacturers (OEM) to engage in additional series of activities stemming from the reverse supply chain. As a result, economically feasible production and distribution systems are established that enable remanufacturing of used-products in conjunction with the manufacturing of new products [1], [2]. The combination of forward/traditional supply chain and reverse supply chain forms the closed-loop supply chain (CLSC). While this process is mandatory in Europe, it is still in its infancy in the United States.

Strategic, Tactical and Operational planning are the three important stages of planning in a Supply Chain. Strategic planning primarily deals with the design (what products should be processed/produced in what facilities etc) of the supply chain that is typically a long-range planning performed every few years when a supply chain needs to expand its capabilities. Tactical planning involves the optimization of flow of goods and services across the supply chain and is typically a medium-range planning performed on a monthly basis. Finally, Operational planning is a short-range planning that deals with the day-to-day production planning and inventory issues on the factory floor.

Much work is done in the area of designing forward and reverse supply chains (for example see [3], [4]). However, not many models deal with both the forward and reverse supply chains together. While the issue of environmental consciousness is not addressed in the forward supply chain models, the models dealing with reverse supply chain assume that each incoming used product is economical to re-process and each available production facility is efficient enough to re-process the incoming used products. As a result, there is a risk of re-processing uneconomical used products in inefficient facilities. Pochampally and Gupta [5] address these drawbacks in a reverse supply chain and propose a three phase mathematical programming approach for its strategic planning.

In this paper, we build on Pochampally and Gupta’s [5] work and concentrate on the strategic and tactical planning of a CLSC that ideally should involve the following phases:

1. Selection of the most economical product to reprocess in the CLSC
2. Identification of potential production facilities operating in the region
Transportation of the right mix and quantities of goods across the CLSC

In this paper, we propose a single phase unified mathematical model that addresses the three major issues, mentioned above, in the strategic and tactical planning of a CLSC.

**METHODOLOGY**

We employ pre-emptive goal programming technique [6] in solving the single phase mathematical model formulated that addresses the strategic and tactical planning stages of a CLSC. The model solves simultaneously for the most economical used-product to re-process in the closed-loop supply chain, the efficient production facilities that remanufacture used-products and/or produce new products and the right mix and quantity of goods to be transported across the supply chain. A numerical example is considered to illustrate the methodology.

We consider the following scenario in our model. Suppose that the manufacturer has incorporated a remanufacturing process for used products into her original production system, so that products can be manufactured directly from raw materials, or remanufactured from used-products. The final demand for the product is met either with new or remanufactured products.

**Nomenclature used in the methodology**

- $A_{iuv}$ = decision variable representing number of used-products of type $i$ transported from collection center $u$ to remanufacturing facility $v$
- $B_{ivw}$ = decision variable representing number of used-products of type $i$ transported from production facility $v$ to demand center $w$
- $b_r$ = probability of breakage of product $i$
- $TA_{uv}$ = cost to transport 1 unit from collection center $u$ to remanufacturing facility $v$
- $TB_{vw}$ = cost to transport 1 unit from remanufacturing facility $v$ to demand center $w$
- $CC_u$ = cost per product retrieved at collection center $u$
- $CNP_v$ = cost to produce 1 unit of new product @ production facility $v$
- $CR_v$ = cost to remanufacture at production facility $v$
- $CD_i$ = disposal cost of product $i$
- $DI_y$ = disposal cost index of component $y$ in product $x$ ($0 = \text{lowest}, 10 = \text{highest}$)
- $DT_i$ = disassembly time for product $i$
- $DC$ = disassembly cost/unit time
- $i$ = product type
- $MINTPS$ = minimum through-put per supply
- $N_{i vw}$ = decision variable representing number of new products type $i$ transported from production facility $v$ to demand center $w$
- $Nd_{iw}$ = net demand for product type $i$ (remanufactured or new) at demand center $w$
- $PRC_i$ = % of recyclable contents by weight in product $i$
- $RCYR_i$ = total recycling revenue of product $i$
- $RSR_i$ = total resale revenue of product $i$
- $RCRI_y$ = recycling revenue index of component $y$ in product $x$
- $S_{1v}$ = storage capacity of remanufacturing facility $v$ for used products
- $S_{2v}$ = storage capacity of remanufacturing facility $v$ for remanufactured and new products
- $S_u$ = storage capacity of collection center $u$
- $SP_i$ = selling price of a unit of new product of type $i$
\( SU_{iu} \) = supply of used product \( i \) at collection center \( U \)
\( SF_v \) = supply of used products at production facility \( v \), different from \( SU_{iu} \); these are products that are fit for remanufacturing, after accounting for recycled and disposed products + new products
\( TP_v \) = through-put (considering only remanufactured products) of production facility \( v \)
\( U \) = collection center
\( V \) = remanufacturing facility
\( W \) = demand center
\( W_i \) = weight of product \( i \)
\( x_1 \) = space occupied by 1 unit of used product (square units per product)
\( x_2 \) = space occupied by 1 unit of remanufactured or new product (square units per product)
\( Y_v \) = decision variable signifying selection of production facility \( V \) (1 if selected, 0 if not)
\( Z_{iu} \) = decision variable representing number of units of product type \( i \) picked for remanufacturing at collection center \( u \) (\( SU_{iu} - Z_{iu} = \) recycled or disposed)
\( \delta_v \) = factor that accounts for un-assignable causes of variations at production facility \( v \)

GOAL PROGRAMMING

Goal programming (GP), generally applied to linear problems, deals with the achievement of specific targets/goals. This technique was first reported by Charnes and Cooper [7], [8] later extended in the 1960s and 1970s by Ijiri [9], Lee [10] and Ignizio [11]. The basic purpose of GP is to simultaneously satisfy several goals relevant to the decision-making situation. To this end, several criteria are to be considered in the problem situation on hand. For each criterion, a target value is determined. Next, the deviation variables are introduced which may be positive or negative (represented by \( \rho_k \) and \( \eta_k \) respectively). The negative deviation variable, \( \eta_k \), represents the under-achievement of the \( k \)th goal. Similarly, the positive deviation variable, \( \rho_k \), represents the over-achievement of the \( k \)th goal. Finally for each criterion, the desire to over-achieve (minimize \( \eta_k \)) or under-achieve (minimize \( \rho_k \)), or satisfy the target value exactly (minimize \( \rho_k + \eta_k \)) is articulated [12].

THE GOALS

We consider three goals in our GP model:
1. Maximize the total profit in the CLSC (TP)
2. Maximize the revenue from recycling (RR)
3. Minimize the number of disposed items (NDIS)

The first two goals involve minimizing the negative deviation from the respective target values while the third goal which has an “environmentally benign” character rather than a financial basis, involves minimizing the positive deviation from the target value.

TOTAL PROFIT AND RELATED TERMS

The total profit (TP) is the difference between all the revenues and all the costs considered in the model.

Revenues
1. Reuse Revenue (all product types together, taken 1 unit at a time)
\[
\sum_i \sum_u \{ Z_{iu} \ RSR_i \}
\]
2. Recycle Revenue (all product types together, taken 1 unit at a time)
\[ \sum_{i} \sum_{u}^{(SU_{iu} - Z_{iu})} RCY_{i} W_{i} PRC_{i} C_{df} \]

3. New Product Sale Revenue
\[ SP_{i} N_{iwv} \]

**Costs**

1. Collection/Retrieval Cost
\[ \sum_{u}^{CC_{u} SU_{iu}} \]

2. Processing Cost = Disassembly cost of used products + Remanufacturing cost of used products + New Products production cost in the forward supply chain
\[ DC \sum_{i}^{\sum_{u} \sum_{v} DT_{i} A_{iwv}} + \sum_{i}^{\sum_{u} \sum_{v} CR_{v} B_{ivw}} + \sum_{i}^{\sum_{u} \sum_{v} CNP_{v} N_{iwv}} \]

3. Inventory Cost: Assuming inventory carrying cost @ collection center (used products) is 20% of collection cost (CC_u); inventory carrying cost @ production facility (remanufactured or new products) is 25% of remanufacturing or new product production cost, whichever is applicable.
\[ \sum_{i}^{\sum_{u} \sum_{v} (CC_{u} / 5) A_{iwv}} + \left( \sum_{i}^{\sum_{u} \sum_{v} ((CR_{v} / 4) B_{ivw}) + (CNP_{v} / 4) N_{iwv}} \right) \]

4. Transportation Costs: Used products from collection centers to production facility + Remanufactured and New Products from production facilities to demand centers.
\[ TA_{iwv} \sum_{i}^{\sum_{u} \sum_{v} A_{iwv}} + TB_{ivw} \sum_{i}^{\sum_{u} \sum_{v} (B_{ivw} + N_{iwv})} \]

5. Disposal Cost: Products that can’t be remanufactured or recycled (all product types together, taken 1 unit at a time)
\[ \sum_{i}^{\sum_{u} (SU_{iu} - Z_{iu}) DI_{i} W_{i} (1 - PRC_{i})} C_{df} \]

**System Constraints**

1. The number of used-products sent to all production facilities from a collection center \( u \) must be equal to the number of used-products picked for remanufacturing at that collection center.
\[ \sum_{v} A_{iwv} = Z_{iu} \]

2. Demand at each center \( w \) must be met either by new or remanufactured goods.
\[ \sum_{v} (B_{iwv} + N_{iwv}) = Nd_{iw} \forall w \]

3. Number of remanufactured products transported from a production facility \( v \) to a demand center \( w = (\text{Number of used products fit for remanufacturing, transported from collection center} u \text{ to that production facility}) \cdot \delta_{v} \) i.e., no loss of products in the supply chain due to reasons other than common cause variations, over which there’s no control. \( \delta_{v} \) is a factor that accounts for the un-assignable causes of variation at the production facility \( v \).
\[ \sum_{w} B_{iwv} = \sum_{u} A_{iwv} \cdot \delta_{v} \forall v \]
4. Total number of used products of type \( i \) picked for remanufacturing @ \( u \) must be at most equal to the total # of used products fit for remanufacturing.
   \[ Z_{iu} \leq SU_{iu} (1 - b_i) \]

5. Total number of used products of all types collected @ all collection centers must be at least equal to the net demand.
   \[ \sum_{i} \sum_{u} SU_{iu} \geq \sum_{i} \sum_{w} Nd_{iw} \]

6. Number of remanufactured products must be at most equal to the net demand; this is to avoid excess remanufacturing.
   \[ \sum_{i} \sum_{u} Z_{iu} \leq \sum_{i} \sum_{w} Nd_{iw} \]

Facilities Space Constraints

7. Space constraints for used products at production facility \( v \),
   \[ x_1 \sum_{i} \sum_{u} A_{iu} \leq S_{1v} y_v \]

8. Space constraint for new and remanufactured products at production facility \( v \), assuming new and remanufactured products occupy the same space.
   \[ \sum_{i} \sum_{w} x_2 (B_{iw} + N_{iw}) \leq S_{2v} y_v \]

9. Space constraint for used products at collection center
   \[ x_1 \sum_{i} \sum_{v} a_{iv} \leq S_{iu} \]

Production Facility’s Potentiality Constraints, valid only for remanufactured products:

10. \( (TP_v / SF_v) y_v \geq \text{MINTPS} \) (MINTPS = minimum through-put per supply)

Non-Negativity Constraints:
   \[ A_{iu}, B_{ivw}, N_{iw}, Z_{iu}, Z_{iu} \geq 0, \quad \forall \ u, v, w \]
   \[ y_v \in [0,1] \quad \forall \ v, \ 0 \text{ if facility } v \text{ not selected, } 1 \text{ if selected} \]

Procedure to solve the GP model

The following steps are used to solve the GP model:

**Step 1:** Read in all the relevant data, set the first goal as the current goal.

**Step 2:** Obtain a linear programming (LP) solution with the current goal as the objective function.

**Step 3:** If the current goal is the last goal, set it equal to the LP objective function value found in Step 2, STOP. Else, go to Step 4.

**Step 4:** If the current goal is just achieved or over-achieved, set it equal to its aspiration level and add this equation to the constraint set, go to Step 5. Else, if the value of the current goal is under-achieved, set the aspiration level of the current goal to the LP objective function value found in Step 2, go to Step 5.

**Step 5:** Set the next goal as the current goal, go to Step 2.
NUMERICAL EXAMPLE

We consider a Closed-Loop Supply Chain with three collection centers, two production facilities to choose from, two demand centers to be served and three brands of similar products.

The example data we take to implement the GP model are:

\[
\begin{align*}
CCu &= 0.01; \quad SU_{11}=50; \quad SU_{12}=45; \quad SU_{21}=25; \quad SU_{22}=35; \quad SU_{31}=38; \quad SU_{32}=22; \quad SU_{33}=30; \quad SU_{34}=35; \\
SU_{35} &= 28; \quad DC=0.05; \quad DT_{1}=10; \quad DT_{2}=12; \quad DT_{3}=9; \quad CR_{1}=13; \quad CR_{2}=10; \quad CNP_{1}=60; \quad CNP_{2}=45; \\
TA_{11} &= 0.01; \quad TA_{12}=0.09; \quad TA_{21}=0.5; \quad TA_{22}=0.1; \quad TA_{31}=0.02; \quad TA_{32}=0.04; \quad TB_{11}=0.04; \quad TB_{12}=0.03; \\
TB_{21} &= 0.09; \quad TB_{22}=0.05; \quad DI_{1}=4; \quad DI_{2}=6; \quad DI_{3}=5; \quad W_{1}=0.8; \quad W_{2}=1.0; \quad W_{3}=0.9; \quad PRC_{1}=0.5; \quad PRC_{2}=0.6; \\
P_{RC_{3}} &= 0.75; \quad Cd_{1}=0.2; \quad Cd_{2}=0.5; \quad Cd_{3}=0.3; \quad RSR_{1}=30; \quad RSR_{2}=40; \quad RSR_{3}=45; \quad RCY_{1}=1.5; \\
RCY_{2} &= 2; \quad RCY_{3}=2.5; \quad RCR_{1}=7; \quad RCR_{2}=4; \quad RCR_{3}=5; \quad SP_{1}=65; \quad SP_{2}=55; \quad SP_{3}=60; \quad ND_{11}=20; \\
ND_{12} &= 15; \quad ND_{21}=16; \quad ND_{22}=22; \quad ND_{31}=25; \quad ND_{32}=20; \quad \delta_{1}=0.4; \quad \delta_{2}=0.6; \quad b_{1}=0.2; \quad b_{2}=0.4; \quad b_{3}=0.3; \quad X_{1}=0.7; \\
X_{11} &= 400; \quad S_{12}=400; \quad S_{1}=150; \quad S_{2}=150; \quad S_{3}=150; \quad X_{2}=0.7; \quad S_{21}=500; \quad S_{22}=500; \quad MNTPS=0.25.
\end{align*}
\]

Upon solving the GP model using LINGO, we get the following optimal solution:

\[
\begin{align*}
TP &= 3945 \text{ (Target=2500)}; \quad RR=951 \text{ (Target=750)}; \quad NDIS=74 \text{ (Target=50)}; \quad Z_{12}=35; \quad Z_{21}=2; \quad Z_{22}=23; \\
Z_{23} &= 13; \quad Z_{31}=20; \quad Z_{32}=12; \quad Z_{33}=12; \quad N_{11}=3; \quad N_{12}=5; \quad N_{21}=8; \quad N_{22}=1; \quad N_{23}=3; \quad N_{31}=14; \quad A_{12}=15; \\
A_{122} &= 20; \quad A_{212}=2; \quad A_{221}=5; \quad A_{222}=18; \quad A_{31}=5; \quad A_{32}=8; \quad A_{33}=11; \quad A_{34}=9; \quad A_{35}=12; \quad A_{36}=3; \quad B_{11}=8; \\
B_{112} &= 4; \quad B_{211}=5; \quad B_{212}=3; \quad B_{311}=15; \quad B_{312}=6; \quad B_{12}=9; \quad B_{22}=6; \quad B_{221}=3; \quad B_{222}=18; \quad B_{321}=7; \quad Y_{1}=1; \quad Y_{2}=2.
\end{align*}
\]

It is obvious from the above solution that both the production facilities were chosen for the network design.

CONCLUSIONS

In this paper, we formulated a unified single-phase mathematical model that was solved using goal programming technique, to address the critical issues in the strategic and tactical planning stages of a closed-loop supply chain. The model when solved identifies the most economical used-products and their quantities to re-process in the CLSC, the efficient production facilities and the right mix and quantities of goods to be transported across the supply chain. A numerical example is considered to illustrate the methodology.

REFERENCES


