Optimization of Mixed Logistics Network Considering Impedance Effect

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Abstract: Mixed logistics network has the advantages of Hub-and-Spoke network and PTP network. In order to solve the capacitated multiple allocation $p$-hub problem, this paper presents a non-linear model to optimize the network. Due to the fact that the current researches have some limitations, the capacity of materials disposal at hubs and the timeliness of transportation are added as constraints in the model based on the analysis of characteristics of mixed logistics network. In the process of optimization, an impedance function is introduced to balance material flow in logistics network in order to avoid local congestions. Additionally, we provide a genetic algorithm that finds satisfying solution within reasonable time. Then an example and simulation are given to verify the validity of the model. This research provides new and realistic insights into the mixed logistics network design problem.

Keywords: Mixed logistics network; Impedance function; Genetic algorithm; Numerical simulation
1 Introduction

As a special logistics network structure, Hub-and-Spoke (HS) network is widely used in actual production and transportation. However, pure HS network also has disadvantage in some conditions. For characterize of HS network, materials between demand nodes must be transported via hub nodes. Even if two demand nodes are adjacent, the materials between them also must be transported via hub nodes which are may be rather far away from them. Compared with direct delivery, indirect delivery will cause unnecessary increase of costs. A better network should allow partial direct delivery. So researches for mixed network have important signification.

Aykin (1995) considered one-hub-stop, two-hub-stop and, when permitted, direct services for each origin-destination pair in their hub location and routing problem[1]. Button etc. (1999) analyzed advantages and disadvantages of direct delivery and indirect delivery qualitatively[2]. Sung and Jin (2001) allowed non-stop (direct link) service when considering a hub network design problem and built a basic mixed logistics network optimal model[3]. Zäpfel and Wasner (2002) built a mixed hub-and-spoke transportation model and they applied it to a real case of an Australian parcel service provider[4]. Lijesen etc. (2002), Cheng and Yin (2003) respectively took international flights from Europe and the hub-and-spoke network between Taiwan area and China mainland for examples, validated direct delivery’s importance for improving the whole network through numerical calculate[5,6]. Liu etc. (2003) studied a mixed truck delivery system that allows both hub-and-spoke and direct shipment delivery modes[7]. In order to maximize the total profit of a network, Bai and Zhu
(2006) investigated the effects of the network model parameters on flight frequency, traffic and single flight occupancy of fully-connected and hub-and-spoke airline networks\cite{8}. Brandão (2009) considered the fleet size and mix vehicle routing problem which consists of defining the type and the number of vehicles of each type\cite{9}.

In actual logistics process, every hub node has certain material disposal ability. On another hand, as an economy activity, timeliness is an important factor of logistics. However, there are less researches both considering the limitation of capacity and timeliness in our collected literatures. Besides, current researches mainly focus on the goal of reducing the building and the operation costs of a logistics network. The caused result is that they paid much attention on improving economic benefits of the logistics network itself while neglecting the possible decrease of the performance of the network caused in the process of pursuits for economic benefits. Therefore, we try to balance the materials flow on mixed logistics network considering the limitation of hub’s capacity and timeliness of transportation in order to avoid local congestion.

The paper is organized as follows: In the next section we provide the description of the hub-and-spoke logistics network optimization problem and present our nonlinear integer programming model. In Section 3 we design a genetic algorithm to solve this model. We also give an example and simulation of the model in Section 4. Finally we make some conclusions in Section 5.

2 Problem description and mathematical model

As the above paragraph expatiates, we assume that $N$ is the set of $n$ demand nodes,
\( i, j \in N \). \( M \) is the set of \( m \) candidate hub nodes and \( H \) is the set of selected hub nodes, \( H \subset M, k, l \in H. \) If hub node \( k \) is selected, \( H_k=1; \) otherwise, \( H_k=0. \) \( p \) is the number of hub nodes. Let \( d_{ik} \) is the distance between demand node \( i \) and hub node \( k \) as well as \( d_{kl} \) is the distance between hub node \( k \) and \( l. \) \( W_{ij} \) is the materials quantity to be transported from demand node \( i \) to demand node \( j \). For the network we considered is asymmetric, \( W_{ij} \) is not required to equal to \( W_{ji}. \) \( F_k \) is the disposal and transfer capacity of materials at hub node \( k \). \( T_L \) is the time limitation of transportation. \( X_{ijkl} \) and \( Z_{ij} \) are decision variables. If indirect delivery is chosen and material flow \( W_{ij} \) starts from demand node \( i \) and ends at demand node \( j \) via hub node \( k \) and \( j \) by order, \( X_{ijkl}=1 \) representing whether the route of \( i-k-l-j \) is selected; if else, \( X_{ijkl}=0. \) If direct delivery is chosen and material flow is transported from demand node \( i \) to \( j, Z_{ij}=1; \) if else, \( Z_{ij}=0. \)

Obviously, the more frequent the material circulation on a section, the more likely congestion even paralysis will emerge on the section. It is necessary to describe quantitatively the relationship between material flow and congestion degree on the section. In traffic control system theory, Davidson function and BPR function are the most famous impedance function that expresses the relationship between traffic flow and section costs. Compared with Davidson function, BPR function is strictly monotone increasing. Because of its good characteristic it is widely used practically. For traffic transportation is carrier of logistics, the material flow has the same characteristic of traffic flow. So we utilize the classic section impedance function—BPR function to describe the change of transit time on section. BPR function is formulated as follows:
In the above formulation, $t_a$ represents the impedance on section $a$; $t_0$ is the impedance on section $a$ when there is no traffic flow; $q_a$ is the traffic flow on section $a$; $c_a$ is the capacity of section $a$; $\alpha, \beta$ are retardance parameters. In traffic flow distribution program of US Highway Administration Bureau, $\alpha=0.15$ and $\beta=4$. The curve of BPR function is shown in Figure 1.

![Figure 1. Curve of BPR function](image)

As shown in the curve in Figure 1, when traffic flow closes to the capacity of section, the impedance on section will increase rapidly. So the transit time dose.

Combining characteristics of logistics network, we transform the BPR function as follows. $v_{kl}$ represents the speed of transportation on between hub node $k$ and $l$.

$$t_{kl} = \frac{d_{kl}}{v_{kl}} [1 + \alpha \left( \frac{q_{kl}}{c_{kl}} \right)^{\beta}]$$

(2)

Based on the above research, we build a nonlinear integer programming model which is shown as follows:
The objective of this model is to minimize the operation costs of mixed logistics network, as formulation (3) shows, where $\rho$ is a discount factor. Formulation (4) regulates that each pair of demand nodes has to select either direct or indirect delivery. Formulation (5) ensures that the number of hub nodes is $p$. Formulation (6) and (7) ensure that node $k$ and $l$ must be chosen if route $i-j-k-l$ exists. Formulation (8) and (9) add the constraints of disposal capacity of hub nodes and the timeliness of transportation. Formulation (10) regulates 0-1 value of decision variables.

3 Genetic algorithm

For the model is nonlinear, we present a genetic algorithm. Coding rule, population constructing and genetic operation are key parts in our algorithm.

3.1 Coding rule

A $2n^2$ dimension vector is used as a chromosome. We randomly select a hub node which is open as the value for each element in the vector. In the vector, two
adjacent elements represent a route. Figure 2 gives a $2n^2$ dimension vector. From left to right, every 2n elements belong to a demand node which is as the start. In the 2n elements, every 2 elements belong to a demand node which is as the end. In the 2 elements, the left one represents the first hub and the right one represents the second hub. So we can represent a whole route.

![Figure 2. Structure of a chromosome](image)

It should be noticed that, because direct connections are allowed between any two demand nodes, the available Hub should not only be chosen from $H$. In order to integrate direct connections, we introduce $H' = H \cup \{0\}$ and select the value of each element in the chromosome from $H'$. The following requirement should be satisfied when we choose the value of each element in the chromosome. First, due to the definition, the values of the left one and right one in a chromosome, which are mentioned in the above, should not be the same. Second, if the left (right) one takes the value of 0, the right (left) one must be 0 too.

3.2 Population constructing

The structure of initial population has rather influence of the performance of the algorithm. Given the characteristic of the coding rule, the genes of a chromosome are plentiful. In order to improve the efficiency of the initial population and the superiority of the modes, we use the following method. For each demand node $i$, we
calculate the distances from it to hub nodes in $H$. Then we confirm the probability for the hub node that be chosen by demand node $i$ as route according to the rule that the shorter distance is, the bigger probability is.

### 3.3 Genetic operation

In order to guarantee the best mode not to be destroyed, we preserve the chromosome with the best fitness value. So it can get into the next generation without genetic operation.

1. **Selection operator.** Our algorithm uses the classical roulette selection to choose proper chromosomes. At first, we should calculate the selection probability of each chromosome based on its fitness value. If $\text{Fitvalue}(i)$ is the fitness value of chromosome $i$, its selection probability is $(1 - \text{Fitvalue}(i)/\text{total})/s$. Variable $\text{total}$ represents the sum of the chromosomes in population and $s$ represents the dimension of the population.

2. **Crossover operator.** In the population we select two chromosomes randomly. Then choose a certain number of demand node pairs to exchange their routes. First, find the elements representing the route between a node pairs in two chromosomes. Second, exchange the values. Then we can obtain two new chromosomes.

3. **Mutation operator.** When a new population generate, we do not mutate its chromosomes so to make them cross adequately. But if no better chromosomes emerge in continuous several generations, in order to avoid prematurity convergence, we should introduce new modes by mutating the population. After many time’s trials, we think that it is proper to mutate the population when the number of continuous
generations in which no better chromosomes emerge reach to 10% of total iterations.

4 Example and simulation

In order to verify the validity of the model and the algorithm we designed in the above paragraph, we give an example in this section.

We design an initial network including 15 demand nodes and 6 candidate hub nodes. In the 6 candidate hub nodes, we will select 4 hubs. The materials quantity from demand node $i$ to demand node $j$ and the distances between demand nodes and hub nodes are shown in Table 1. Table 2 gives the distances and the capacities of the section between hub nodes. For simplicity, we set $\rho=0.5$, $v_{kl}=13$ and $T_l=12$, $k, l \in H$.

TABLE 1. The material quantity and distances between demand nodes and hub nodes

<table>
<thead>
<tr>
<th>Demand Node</th>
<th>Materials Quantity/Distance between Demand Node</th>
<th>Distance to Hub Node</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0/ 0 23/22 31/65 40/110 53/130 24/180 19/150 22/180 20/190 32/150 34/90 50/100 29/72 15/44 25/41</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>28/22 0/ 0 22/50 31/100 28/120 41/170 38/150 47/170 27/190 22/160 34/100 14/120 32/94 18/66 23/60</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>24/65 28/50 0/ 0 31/50 19/70 25/120 29/99 36/140 40/160 39/140 27/110 32/140 24/130 27/100 23/80</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>30/110 23/100 26/50 0/ 0 18/28 20/ 0 77 32/ 64 24/100 35/140 21/140 33/140 27/180 29/170 35/150 25/120</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>25/130 22/120 32/70 31/28 0/ 0 45/52 21/36 38/72 34/110 28/120 32/140 28/180 20/180 23/160 31/130</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>26/180 21/170 35/120 26/77 21/ 52 0/ 0 18/37 38/51 51/21/100 25/130 30/170 28/210 23/230 21/200 42/180</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>24/150 23/150 31/99 21/64 37/36 43/37 0/ 0 20/36 24/81 22/98 32/140 19/180 25/190 23/170 30/140</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>36/180 32/170 29/140 19/100 23/72 22/51 35/36 0/ 0 42/51 34/84 29/140 38/190 40/210 31/190 30/160</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>34/190 22/190 18/160 32/140 22/110 32/100 36/81 27/ 51 0/ 0 38/51 37/130 41/170 29/200 28/190 25/160</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>24/150 28/160 28/140 34/140 27/120 31/130 28/98 21/84 32/51 0/ 0 34/85 35/120 21/160 22/150 29/120</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>27/ 90 32/100 33/110 37/140 28/140 23/170 25/140 32/140 31/130 20/85 0/ 0 32/42 25/75 34/65 43/49</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>31/100 21/120 32/140 36/180 32/180 29/210 23/180 21/190 19/170 23/120 22/ 42 0/ 0 22/50 26/61 39/64</td>
<td></td>
</tr>
</tbody>
</table>
For the number of candidate hub nodes are 6 and we must select 4 hubs in them, we have 15 different combining forms of hub nodes for $H$ to choose. For each $H$, we set a population which contains 100 chromosomes in order to obtain a rather better result. In addition, we let each population iterates for 20000 times.

After the simulation, we can obtain the results which are shown in Figure 3. In the figure, we can see all results for every combination of $H$. Obviously the lowest cost belongs to the fifth $H$ among all of the combinations. Then we focus on the fifth $H$ which is \{1, 2, 4, 6\}. In Figure 4 we give the change trend of the best fitness value in the population which is for the fifth $H$. From the curve of the change trend, we can see that the fitness value converges at 628193.5 after about 16000 iterations. This verifies that the genetic algorithm we design is effective. After obtaining the
chromosome matching along with fifth $H$, we transform the chromosome basing on the coding rule and get the best solution.

5 Conclusions

This paper considers many actual factors and selects the time limit of transportation and capacity of materials disposal at hub nodes as constraints. When we are building a new optimization model to design a mixed logistics network, we pay attention on balancing the materials flow in the network. So we introduce a classical BPR function, into our model in order to avoid local congestion. Through numerical simulation, we verify the effectiveness of our model and algorithm.

References


