Newsvendor Selling to Loss Averse Consumers with Stochastic Reference Points

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Abstract
We study a newsvendor who sells a perishable asset over repeated sales seasons to loss averse consumers. We identify conditions under which the expected price can be increasing in the consumer loss aversion level, and numerically show that the firm can prefer low and moderate levels of demand variability over no demand uncertainty. Moreover, we obtain a set of counterintuitive insights on how consumers loss aversion affects the firms optimal operations policies.

Key words: Loss Aversion, Contingent Pricing, Reference Points.

1. Introduction
Experimental studies indicate that consumers often evaluate economic outcomes, e.g., price, relative to a distribution of reference levels, and not merely in terms of absolute levels (see, e.g., Blinder 1998). Such an effect is particularly significant over repeated purchase interactions, as consumers tend to draw past experiences as benchmarks: Consumers form ideas about what a firm usually charges and judge the value of a product based on the difference between what is being charged and what prices have been charged. Consumers evaluate changes from reference levels differently depending on whether the changes are gains or losses. In particular, there is significant empirical evidence indicating that consumers are loss averse, i.e., they weigh losses more heavily than equally-sized gains (see, e.g., Kahneman et al. 1990, Hardie et al. 1993). Moreover, intuitively, the reference points should be endogenized within consumers themselves, rather than exogenously given (e.g., the selection of the average historic prices as a reference price seems arbitrary): If a consumer decides not to buy a product at all, her feelings should not be affected by any price change. This view has gained traction in the behavioral literature and was formalized by Koszegi and Rabin (2006) with many follow-up works.
Behavioral economics literature (e.g. Heidhues and Koszegi 2005, Koszegi and Rabin 2006) has identified two opposite effects of running sales when consumers are loss averse with endogenized stochastic reference points: the comparison effect and the attachment effect. The comparison effect means that higher sales frequency increases the weight of sale prices in the loss-averse consumers’ reference distribution, making consumers get used to the sales price and harder to purchase at the full price. The attachment effect, on the contrary, means that higher sales frequency increases consumers’ attachment to forming a plan of purchasing. Hence, to avoid the pain of not getting the product in the event of observing a full price, consumers are willing to pay the full price that can be higher than their intrinsic valuations.

Motivated by grocers’ markdown practice over repeated sales horizons, we are interested in the following research questions in a repeated newsvendor setting. First, how should the newsvendor optimally make its ordering and pricing decisions over repeated sales horizons? The answer to this question will provide an alternative justification to the contingent markdown pricing practice. Second, is it possible or under what circumstances, the newsvendor may benefit from consumers’ loss averse behavior? The answer to this question will provide insights on how firms may improve profitability in the presence of loss averse consumers. Third, like in a typical newsvendor problem, there exists probability of running out of stock. The stockout events critically factor into loss-averse consumers’ decision making over repeated interactions. What is the effect of product unavailability on the firm’s profitability when consumers are loss averse and how should the firm optimally respond? The answer to this question will further help understand the effect of stocking decisions on firms’ profitability.

This paper makes three main contributions as follows: First, we are the first to explicitly take into account consumers’ loss aversion with stochastic reference points in firm’s operational decisions, such as the order quantity and contingent pricing policy. An important aspect of this study is the effect of the possibility of unavailable inventory on the willingness-to-pay of loss-averse consumers, which leads to many counterintuitive insights. Second, we demonstrate that contingent pricing policies allow the firm not only to efficiently match supply with demand, but also to profitably manipulate consumers’ stochastic reference points. In line with this finding, we show numerically that the firm may prefer small or moderate levels of demand variability to no demand variability under the optimal contingent pricing strategy we consider. This is an observation that is in
stark contrast to the results in a classic newsvendor setting. Third, we show how consumers’ loss aversion affects the firm’s optimal operations decisions, with insights significantly different from those in a classic newsvendor setting (see, e.g., Petruzzi and Dada 1999). Based on our results, we caution that it is critical to take into account consumers’ loss aversion behavior when designing markdown pricing algorithms; otherwise, the suggested solutions by algorithms can be significantly sub-optimal.

2. The Model
We consider a single risk-neutral profit maximizing firm selling a single perishable product over a short sales horizon on a repeated basis, e.g., a grocery store sells fruit plates with a sales horizon measured in days. The firm orders $q$ units of the product at cost $c$ per unit and sells to customers who request a single unit of the product.

**Consumer.** There is a random number, $X$, of consumers, where $X \geq 0$ has cumulative distribution function (cdf) $F(\cdot)$ and has an expected value $E(X) < \infty$. The consumers have a known intrinsic valuation $v > c$ of the product. Consumers are *loss-averse*. Our loss aversion model, to be discussed in further detail shortly, extends the one in Koszegi and Rabin (2006) to account for operational issues such as the impact of limited inventory on consumers’ purchase decisions.

**Sale Price.** We assume that there is a sale price $s < c$ at which the firm can clear on-hand inventory. We fix the sale price and allow the full price to be determined. Same qualitative insights will be obtained if one fixes the full price but allows the sale price to vary.

**Inventory Availability.** A feature of our model is to consider product availability in the framework of stochastic reference points. We define *fill rate* as the long-run probability that the product is successfully procured when consumers are willing to buy it. Consumers can observe the fill rate through repeated interactions.

**Contingent Pricing Scheme.** Let $\bar{p} > s$ denote the higher of the two prices, namely, the *full price*. Let $\Omega$ be the set of demand realizations under which the price is set at $s$, then the contingent pricing scheme is

$$p = \begin{cases} 
  s & \text{if } x \in \Omega, \\
  \bar{p} & \text{if } x \in \Omega^c,
\end{cases}$$

where $\Omega^c = \mathbb{R}^+ \setminus \Omega$. (We will characterize the optimal set $\Omega$ in Section 4.)
Figure 1 Sequence of Events

Details of the game are elaborated as follows. First, the firm commits and announces the fill rate $\phi$ and the full/sale price distribution generated by the contingent pricing scheme (1). According to the contingent pricing scheme (1), the p.m.f. of the price is

$$g(p) = \begin{cases} \int_{x \in \Omega_c} dF(x) & \text{if } p = s, \\ \int_{x \in \Omega} dF(x) & \text{if } p = \bar{p}. \end{cases}$$

Based on $g(p)$ and $\phi$, consumers form their reference levels and a credible plan for purchase that will be discussed shortly in §3.1.

Next, we consider the events occurring in each sales season. At the beginning of each period, the firm purchases $q$ units of inventory, then observes the realized number of consumers $x$, and afterwards, prices the product according to the contingent pricing policy (1). Finally, consumers follow their purchase plan to make purchases: If the product is offered with a price at which the consumers have planned to buy, they will purchase the product; Otherwise, they will not. If there are more consumers willing to purchase the product than the available inventory, units are randomly rationed among consumers.

2.1. Consumer’s Problem

The consumers’ problem is to decide whether or not to purchase a single unit of the product, given that it is priced at $p$. A consumer’s expected utility is the sum of her expected consumption utility and her expected gain-and-loss utility. Let the binary variable $b \in \{0, 1\}$ denote consumer’s purchase outcome, where $b = 1$ indicates that the consumer successfully procures the product and
b = 0 otherwise. A consumer’s utility function has two components: product and money. Denote the consumption outcome by \( k = (k^v, k^p) \), where \( k^v = vb \) is the valuation drawn from a purchase outcome, and \( k^p = -pb \) is the out-of-pocket cost incurred for a purchase outcome, so the combined consumption utility is

\[
C(k) = k^v + k^p = (v - p)b.
\]

In addition, a loss averse consumer compares her *actual* consumption outcome \( k = (k^v, k^p) \) to a *possible* consumption outcome \( r = (r^v, r^p) \) (i.e., a reference point) in her reference point distribution, where \( r^v \) is the reference product valuation and \( r^p \) is the reference out-of-pocket cost. Comparing her actual consumption outcome to a reference point, the consumer obtains a *gain-and-loss utility* along both dimensions of product and money:

\[
W(k|r) = \eta(k^v - r^v)^+ + \eta\lambda(k^v - r^v)^- + \eta(k^p - r^p)^+ + \eta\lambda(k^p - r^p)^-,
\]

where \( \eta > 0 \), \( \lambda > 1 \), \( a^+ = \max\{a, 0\} \) and \( a^- = \min\{a, 0\} \) for any real number \( a \). Note that \( \lambda \geq 1 \) implies that the consumer feels losses stronger than equally sized gains. Therefore, customer’s total utility of a consumption outcome \( k \) conditional on a reference point \( r \) is

\[
u(k|r) = C(k) + W(k|r).
\]

As a consumer’s reference is her *probabilistic beliefs* about the outcomes, we use \( \Gamma(\cdot) \) to denote the probability distribution over \( r \). We call \( \Gamma(\cdot) \) the customer’s *reference distribution* in order to distinguish it from a deterministic reference point. Therefore, the expected utility of a consumption outcome \( k \) conditional on the customer’s reference distribution is

\[
U(k|\Gamma) = \sum_r u(k|r) \cdot \Gamma(r), \tag{3}
\]

Heidhues and Koszegi (2013) show that the consumer’s purchase plan follows a cut-off structure: the consumer chooses to buy at any price lower than or equal to the cut-off price, and not to buy at any price higher. Then to induce consumers to make a purchase, the full price \( p \) must be the cut-off price (we will use them interchangeably) and satisfy:

\[
U((v, -p)|\Gamma) = U((0, 0)|\Gamma), \tag{4}
\]

where the random reference distribution \( \Gamma \) is endogenously induced by the purchase plan with the cut-off price \( \bar{p} \) (see Heidhues and Koszegi 2013, Definition 1).
Given the firm’s price distribution \( g(p) \) and fill rate \( \phi \), consumer’s purchase plan determines the probability distribution of the consumption outcomes, which serve as her random reference points. In equilibrium, consumers have no incentive to deviate from a purchase plan with the reference distribution \( \Gamma(\cdot) \) induced by the same purchase plan. Given that consumers choose a purchase plan \( \overline{p} \) that leads to purchase at both the full and the sale price, the discrete reference distribution \( \Gamma(\cdot) \) can be explicitly written as
\[
\Gamma(r; g(\cdot), \phi, \overline{p}) = \begin{cases} 
\phi g(s) & \text{if } r = (v, -s), \\
\phi g(\overline{p}) & \text{if } r = (v, -\overline{p}), \\
1 - \phi g(s) - \phi g(\overline{p}) & \text{if } r = (0,0).
\end{cases}
\]
(5)

**Lemma 1** Given pricing scheme (1) and inventory, \( q \), the threshold (cut-off) price \( \overline{p} \) satisfies
\[
(v - \overline{p}) + (\eta v - \eta \lambda \overline{p}) + \eta (\lambda - 1) \sum_{p=\overline{p}}^{s} (v + p) \cdot \phi g(p) = 0,
\]
so the cut-off price \( \overline{p} \) can be written as
\[
\overline{p} = v + \frac{(v + s)\phi g(s) - v + 2v\phi g(\overline{p})}{1 + \eta(\lambda - \phi(\lambda - 1)g(\overline{p}))}(\lambda - 1)\eta.
\]
(6)

### 2.2. Firm’s Problem

Let \( \xi(p, q, x) \) denote the availability of product to consumers when the full price is \( p \), the order quantity is \( q \) and the demand is realized as \( x \). Given consumers’ purchase plan \( \overline{p} \), the firm will optimally set the full price at \( \overline{p} \) and need to further decide the order quantity \( q \), the set of consumer demand realizations, \( \Omega \), that triggers the sales, and product availability \( \xi(\overline{p}, q, x) \) at each demand realization to maximize its expected profit. Since the fill rate is dependent on both consumers’ purchase plan and the inventory level, we use \( \phi(\overline{p}, q) \) to emphasize such dependency. The fill rate can be written as
\[
\phi(\overline{p}, q) = \int_{0}^{\infty} \xi(\overline{p}, q, x)dF(x).
\]
(7)

Accordingly, the firm’s expected profit is
\[
\Pi(\overline{p}, q, \Omega, \xi(\overline{p}, q, x)) = sq \int_{x \in \Omega} dF(x) + \overline{p} \int_{x \in \Omega} \xi(\overline{p}, q, x)xdF(x) - cq.
\]
(8)

### 3. Market Equilibrium

In the market equilibrium, neither the firm nor consumers have incentives to deviate from their decisions. Specifically, given the firm’s decisions on contingent pricing, inventory, and fill rate, consumers’ purchase plan is a personal equilibrium; in turn, given consumers’ purchase plan, the firm’s decisions maximize its expected profit.
3.1. Fixed Order Quantity

This section characterizes the firm’s contingent pricing scheme for a given order quantity level $q$. (We study the optimal choice of $q$ in §3.2.) Additionally, we study how the pricing decision changes with respect to the loss aversion level $\lambda$ and order quantity $q$. To streamline the analysis, we make the following assumption.

Assumption 1

$$ \frac{v}{s} \geq \frac{1 + \eta \lambda}{1 + \eta}.$$  

Assumption 1 requires that consumers’ valuation of the product is sufficiently higher than the sale price $s$ in proportion that is dictated by their degree of loss aversion. The following results (Propositions 1 to 7) are only applicable within the range where the optimal sales frequency is nonzero.

Proposition 1 (Optimal Contingent Pricing Policy) Given an initial order quantity $q$, the optimal contingent pricing policy must have the following structure, which induces consumers to purchase at both prices.

(i) The pricing scheme is of a threshold form:

$$ p = \begin{cases} s & \text{if } x \leq \tau^*, \\ \bar{p}^* & \text{if } x > \tau^*, \end{cases} $$

where

$$ \bar{p}^* = v + \frac{\phi^* sF(\tau^*) - [1 - \phi^*(2 - F(\tau^*))]v}{1 + \eta[\lambda - \phi^*(\lambda - 1)(1 - F(\tau^*))]}(\lambda - 1)\eta > s, $$

$F(\tau^*)$ is the likelihood of running a sales at the price $s$, or sales frequency, and

$$ \phi^* = \int_0^\infty \xi(\bar{p}^*, q, x)dF(x) = \int_0^\infty \frac{\min(x, q)}{x}dF(x) $$

is the optimal fill rate given order quantity $q$.

(ii) The nonzero optimal sales threshold $\tau^* \leq q$ is the solution to

$$ sq - \bar{p}^* + \frac{\partial \bar{p}^*}{\partial F} \int_\tau^\infty \min\{x, q\}dF(x) = 0, $$

where $\partial \bar{p}^*/\partial F$ is the derivative of the optimal full price with respect to sales likelihood $F(\tau^*)$. 

When consumers are loss averse and demand is uncertain, the optimal full price can be manipulated, and the firm may price the product at a level higher than consumers’ intrinsic valuation. By (9), the full price $p^*$ is greater than $v$ if and only if the numerator of the second term in the right hand side of (9) is positive. A straightforward summary of this finding is as follows.

**Proposition 2 (Attachment Effect and Fill Rate)** Under the optimal contingent pricing policy, consumers pay a full price higher than their valuation, if and only if, the sales frequency $F(\tau^*)$ and the fill rate $\phi^*$ simultaneously satisfy the following two conditions:

\begin{align*}
\phi^* &> \frac{1}{2}, \\
F(\tau^*) &\leq \frac{2\phi^* - 1}{v - s} = \widehat{F}.
\end{align*}

Next, we investigate how the optimal full price $p^*$, the optimal sales threshold $\tau^*$, and the average price changes with respect to the fill rate $\phi^*$ (or equivalently by (10), the order quantity level $q$) and the consumer loss aversion parameter $\lambda$.

**Proposition 3 (Higher Availability, Larger Attachment Effect)** The optimal full price $p^*$ is increasing and the optimal sales threshold $\tau^*$ is decreasing, in the optimal fill rate $\phi^*$. Therefore, the expected price $\widehat{p}^* = sF(\tau^*) + p^*(1 - F(\tau^*))$ is increasing in the optimal fill rate $\phi^*$.

Next, we consider the impact of consumers’ loss aversion on the firm’s optimal sales frequency and profitability. Note that given a fixed order quantity $q$, the optimal fill rate $\phi^*$ is not affected by consumers’ loss aversion $\lambda$ (see part (i) of Proposition 1). But the optimal sales threshold $\tau^*$ does depend on $\lambda$ (see part (ii) of Proposition 1), and so does the optimal sales frequency $F(\tau^*)$. Hence, we use the notation $\tau^*(\lambda)$ and $F(\tau^*(\lambda))$ to emphasize such dependence on loss aversion levels.

**Proposition 4 (Comparative Statics on the Loss Aversion Level)** Given a fixed order quantity $q$, if $F(\tau^*(\lambda)) < \widehat{F}$, where $\widehat{F}$ is defined in (13), the optimal sales threshold $\tau^*(\lambda)$ is decreasing in $\lambda$, and moreover, both the regular price $p^*$ and hence the expected price $\widehat{p}^*$ are increasing in $\lambda$.

The next proposition discusses how the firm’s optimal profit changes with respect to consumers’ loss aversion levels. Given a fixed inventory $q$, the firm’s revenue under the optimal contingent pricing policy is

\begin{equation}
\Pi^* = \Pi(p^*, q, \tau^*, \xi(p^*, q, x)) = sqF(\tau^*) + p^* \int_{\tau^*}^{\infty} \min\{x, q\} dF(x).
\end{equation}
We can see that the profitability depends on the loss aversion levels, through the optimal sales threshold. Let \( \tau^*(1) \) be the value of the sales threshold \( \tau^*(\lambda) \) at \( \lambda = 1 \) and \( \tau^*(\infty) = \lim_{\lambda \to \infty} \tau^*(\lambda) \), then we have the following results.

**Proposition 5 (When Loss Aversion Benefits)** Given a fixed order quantity \( q \),

(i) If \( F(\tau^*(1)) \leq \tilde{F} \), the firm’s profit \( \Pi^* \) is strictly increasing in \( \lambda \);

(ii) If \( F(\tau^*(\infty)) \geq \tilde{F} \), the firm’s profit \( \Pi^* \) is strictly decreasing in \( \lambda \);

(iii) Otherwise, the firm’s profit \( \Pi^* \) has a U-shape in \( \lambda \), with a unique minimum \( \lambda_{\text{min}} \) such that \( F(\tau^*(\lambda_{\text{min}})) = \tilde{F} \).

### 3.2. Optimal Order Quantity

In this section, we discuss the firm’s choice of order quantity, given that the firm implements the optimal contingent pricing scheme specified in Proposition 1 for each order quantity. Given a fixed order quantity \( q \), the optimal sales threshold \( \tau^* \) should satisfy the first order condition (11). Given a fixed sales threshold \( \tau \), the partial derivative of the expected profit function \( \Pi \) with respect to \( q \) is

\[
\frac{\partial \Pi(q, \tau)}{\partial q} = sF(\tau) + p'(1 - F(q)) + \frac{\partial p'}{\partial \phi^*} \frac{\partial \phi^*}{\partial q} \int_{\tau}^{\infty} \min\{x, q\} dF(x) - c. \tag{15}
\]

By examining the solutions to these two equations, we have the following results on comparative statics of optimal order quantity \( q^* \) with respect to loss aversion level \( \lambda \) and procurement cost \( c \).

**Proposition 6 (More Loss Aversion, Larger Initial Order)** One of the following two scenarios must prevail:

(i) The firm’s optimal order quantity \( q^* \) is increasing in \( \lambda \);

(ii) The firm’s optimal order quantity \( q^* \) has a U-shape in \( \lambda \).

Next, we consider how the firm’s optimal order quantity decisions may change with respect to the procurement cost. One may expect that the firm would sell the product at a higher price if the procurement cost is more expensive. This is true for a market with non-loss-averse consumers. However, if the firm sells to loss-averse consumers with stochastic reference points, we find that the optimal full price becomes lower when the procurement cost increases. The following proposition summarizes these findings.
**Proposition 7 (Higher Cost, More Sales and Lower Full Price)** The following statements hold.

(i) The optimal order quantity $q^*$ is decreasing in the procurement cost $c$;
(ii) The optimal sales threshold $\tau^*$ is increasing in the procurement cost $c$;
(iii) Both the optimal full price $p^*$ and the expected price are decreasing in the procurement cost $c$.

**References**


