Consequences of order crossover in (s, Q) inventory systems under stochastic environment

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Abstract - This paper discusses the issue of order crossover and its effects in order point, lot size (s, Q) inventory systems. Orders are likely to cross when their lead times are stochastic. Two inventory systems are considered to examine the consequences of order crossover. The first inventory system considers a fast moving item having deterministic demand and stochastic lead time. It is assumed lead time demand follows Normal distribution. The second inventory system considers a slow moving inventory item having stochastic lead time and demand during lead time follows a Laplace distribution. The order crossover phenomenon is studied in both systems. Numerical problems are also considered to demonstrate the results. A sensitivity analysis is done to examine the effect of order crossover with change in variance of lead time demand on total cost, lot size and safety stock factor in both systems.

Keywords - inventory; stochastic; lead time; sensitivity analysis; order crossover

1. Introduction

The phenomena of order crossover happen when orders reach in dissimilar sequence in which they were placed. The order crossover was first recognized by Hadley and Whitin (1963). The presence of different suppliers at different locations making use of different modes of transportation enhances the likelihood that orders cross. The orders usually cross due to stochastic lead time. However, Hadley and Whiten (1963) suggested the probability of order crossover is negligible and can be ignored. In the past, many authors circumvented order crossover like Libertone (1979), Song (1994) and Zipkin (2000) considering the assumption of non-interchangeable demand. However, inventory literature shows recently many authors have considered order crossover in their work like Reizebos (2006), Hayya and Harrison (2010), Srinivasan et al. (2011), Wensigh and Kuhn (2014) and Bischak et al. (2014).

The literature suggests most of authors examined order crossover for fast moving items like Hayya et al. (2008), Hayya and Harrison (2010), Srivastav and Agrawal (2015). There are very few authors that have studied order crossover for slow moving items like Srivastav and Agrawal (2013). They studied order crossover for backorder inventory model for slow moving inventory item.

In this paper, we have studied order crossover for real world situations and have developed inventory models considering time proportional backorders with lost sales. Other authors that considered time proportional backorders with lost sales are Park (1982), Kim and Park (1985) and Shore (1986). These authors have not considered order crossover in their model. There has been no paper in the literature that examines order crossover for both fast and slow moving inventory item. This paper not only studies order crossover for both fast and slow moving inventory item but also compares the issues and benefits of both models.
2. Notations

\[ C(Q, z_0) = \text{Total cost} \]

\[ A = \text{Set up cost per order} \]

\[ D = \text{Demand per unit time} \]

\[ h = \text{Holding cost per unit time} \]

\[ q = \text{Fraction of shortages backordered} \]

\[ \pi_0 = \text{Lost sales cost on per unit lost sales} \]

\[ \Pi = \text{Backorder cost per unit per unit time} \]

\[ Q = \text{Order quantity} \]

\[ P(z>z_0) = \text{Stockout probability} \]

\[ \sigma_x = \text{Standard deviation of demand during lead time} \]

\[ \sigma_{ELT} = \text{Standard deviation of demand during effective lead time} \]

\[ z_0 = \text{Safety stock factor} \]

\[ G(z_0) = \text{Special function used to find expected shortage per replenishment cycle} \]

\[ a, b = \text{Regression coefficients} \]

3. Fast Moving Inventory System

We have considered a continuous review inventory system and formulated the cost equation for fast moving inventory system as below:

\[
C(Q, z_0) = \frac{AD}{Q} + h\left(\frac{Q}{2} + z_0\sigma_x\right) + \left[h(1-q) + \frac{D}{Q}\left[\pi_0(1-q)\right]\right]\sigma_x G(z_0) + \frac{q(\pi + h)\sigma_x^2 (a_2 G(z_0))^2 + b_2 G(z_0))}{Q}
\]  

(1)

The cost expression is obtained by making fixed backorder cost zero in the total cost expression of Srivastav and Agrawal (forthcoming). Here \(a_2 = 0.479706\) and \(b_2 = 0.443603\) are coefficients used for estimation of time weighted backorder component of total cost.

Differentiating the above total cost equation with respect to order quantity and safety stock factor gives the below expression for optimal order quantity and stockout risk respectively for sequential approach.
To consider order crossover, authors considered approach of effective lead time given by Hayya et al. (2008). The effective lead time is the time between first arrival and first placement of order, irrespective of the order in which they were placed.

The below regression equation is used to determine the standard deviation during effective lead time.

\[ \sigma_{ELT} = a + b \left( \frac{Q}{D} \right) \]  

(4)

Based on the experiments conducted with changing the time between orders (from 0.25 to 2), regression coefficients obtained are as \( a = 0.4645 \) and \( b = 0.5481 \) respectively. The lead time is considered as exponential with mean 2.5.

The total cost through order crossover approach can be found by substituting \( \sigma_{x} = D\sigma_{ELT} \) in cost equation (1).

3.1 Numerical Problem

Consider the case of fast moving item with

\( D = 1500 \) units/year, \( A = $8 \), exponential lead time with mean 2.5, \( h = $1 \), \( \pi_{0} = $25 \), \( \bar{\pi} = $30 \), \( q = 0.85 \). Determine the total cost considering sequential approach and order crossover respectively.

3.2 Results obtained

<table>
<thead>
<tr>
<th>Approach</th>
<th>Parameters</th>
<th>Cost (in $)</th>
<th>Order Quantity</th>
<th>Safety stock factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequential</td>
<td></td>
<td>11957.20</td>
<td>363.76</td>
<td>3.09</td>
</tr>
<tr>
<td>Order crossover</td>
<td></td>
<td>2436.31</td>
<td>155</td>
<td>2.86</td>
</tr>
</tbody>
</table>

4. Slow Moving Inventory System

We have considered a continuous review inventory system and formulated the cost equation for slow moving inventory system as below:

\[ C(Q, z_{0}) = \frac{AD}{Q} + h \left( \frac{Q}{2} + z_{0}\sigma_{x} \right) + \left( h(1-q) + \pi_{0}(1-q) \frac{D}{Q} \right) \sigma_{x} \frac{1}{2\sqrt{2}} e^{-\sqrt{2}z_{0}} + (\bar{\pi} + h)q \frac{\sigma_{x}^2}{4Q} e^{-\sqrt{2}z_{0}} \]  

(5)

The cost expression is obtained as a mixture of lost sales and time weighted backorders. This equation is developed from the motivation of the work of Muckstadt and Sapra (2009) who had developed total cost expressions for slow moving items considering backorders.

Differentiating the above total cost equation with respect to order quantity and safety stock factor gives the below expression for optimal order quantity and stockout risk respectively for sequential approach.
The total cost considering order crossover can be calculated using equation (4) as given in above section.

\[ Q = \sqrt{\frac{2AD + \left( \frac{\pi_0(1-q)D\sigma_x e^{-\sqrt{2}z_0}}{2\sqrt{2}} \right) + \left( \frac{(\pi + h)q\sigma_x^2 e^{-\sqrt{2}z_0}}{2} \right)}} \]  

\[ z_0 = -\frac{1}{\sqrt{2}} \ln \left[ \frac{2h}{h(1-q) + \pi_0(1-q)\frac{D}{Q} + (\pi + h)q\frac{\sigma_x}{\sqrt{2Q}}} \right] \]  

The total cost considering order crossover can be calculated using equation (4) as given in above section.

4.1 Numerical Problem

Consider the case of slow moving item with same data as used for fast moving item except D as below:

\( D = 20 \) units/year, \( A = $8 \), exponential lead time with mean 2.5, \( h = $1 \), \( \pi_0 = $25 \), \( \pi = $30 \), \( q = 0.85 \). Determine the total cost considering sequential approach and order crossover respectively.

4.2 Results obtained

Table 2. Results for slow moving inventory system

<table>
<thead>
<tr>
<th>Approach</th>
<th>Parameters</th>
<th>Cost (in $)</th>
<th>Order Quantity</th>
<th>Safety stock factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequential</td>
<td></td>
<td>142.77</td>
<td>79.31</td>
<td>1.31</td>
</tr>
<tr>
<td>Order crossover</td>
<td></td>
<td>61.22</td>
<td>11</td>
<td>1.98</td>
</tr>
</tbody>
</table>

The results show that cost, safety stock factor and order quantity are sensitive to variance of lead time. As considering order crossover variance of lead time reduces the cost and order quantity decreases and safety factor increases.

5. Conclusions

We have considered two inventory systems with shortages are calculated as combination of time weighted backorders and lost sales. We have developed total cost expressions for both systems. It is found the consideration of order crossover in inventory systems leads to the significant reduction in inventory cost. The order crossover reduces cost by 79% in fast moving inventory system in comparison to sequential approach. Similarly, in slow moving inventory system benefit of 57% is observed in cost saving. It is observed order quantity also reduces in both models. The safety factor is also improved in slow moving inventory system and remain nearly identical in fast moving inventory system. Therefore, irrespective of type of item, industries have to consider order crossover in the calculation of total cost.

References


