Periodic Review Inventory Control with Fluctuating Purchasing Costs

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Abstract

We consider the periodic review inventory control problem in which the purchasing cost of the product changes, in a Markovian fashion, from one period to the next. After establishing (with and without the non-speculation assumption) that an order upto policy is optimal, we develop an efficient recursive solution procedure to compute the optimal levels. In addition, we propose a measure for the magnitude of the cost fluctuations and show that this measure is an accurate indicator of the effectiveness of myopic heuristics. These results are validated using a computational study.

1 Introduction

We consider the periodic review inventory control problem with linear holding cost, \( h \) per unit per period, and sales price \( p \) per unit. The objective is to determine the optimal inventory levels in the presence of stochastic (with probability distribution function \( \phi(\cdot) \) and the cumulative distribution function \( \Phi(\cdot) \)) demand. The product purchasing cost changes from one period to the next as follows. In a period it takes one of \( k \) possible values \( \{c_1, c_2, \ldots, c_k\} \) where without loss of generality \( c_1 \geq 0 \), and \( c_{i+1} > c_i \ \forall \ 1 < i < k - 1 \). From one period to the next, it undergoes a Markovian transition with the probability transition matrix \( \Pi = [\pi_{ij}] \) where \( \pi_{ij} \) is the probability that the purchasing cost in the next period is \( c_j \) given that it is \( c_i \) in this period. We assume that the matrix \( \Pi \) is both regenerative and communicative with \( q_i \) the long term probability of being in state \( i \). We further assume that there are no setup costs, capacity restrictions, or leadtimes and that the unsatisfied demands are lost. In addition, the salvage costs are zero, but inventory cannot be salvaged except at the end of horizon.
We study this problem with and without the non-speculative assumption. The non-speculative assumption, which is common in the inventory control literature, implies that the change in the purchasing cost is always lower than the holding cost. Under this situation one never obtains excess inventory to take advantage of higher costs in the future. Here, the non-speculative assumption implies that, on the average, the change in the costs is at most $h$ (i.e. $\bar{c}_i - c_i \leq h \forall i$ where $\bar{c}_i = \sum_j c_{ij} \pi_{ij}$). Since the holding costs are usually around 20% of the purchasing costs, the fluctuations in the purchasing costs are severely restricted under the non-speculative assumption. Thus we also study this problem without this assumption. While the optimal solution procedure does not depend on whether this assumption holds or not, the heuristics need to be modified accordingly.

The main motivation for addressing this problem was derived from the recent trend, among many companies, towards globalization. In recent years many companies have started operating all over the world and their supply chains extend over many countries [2]. Due to the global nature of the business purchasing decisions must be made keeping in mind that the costs can and do change over time due to exchange rate fluctuations. There is a critical need to understand how the inventories must be managed in the presence of such fluctuations. For this situation, after establishing the structure of the optimal policies and detailing how to efficiently compute them, we also provide a measure for quantifying the severity of the fluctuations.

First we present a brief review of the related literature. Over the years, there have been a number of studies in inventory control. Many of these studies have focused on presence of setup or order costs [19], finite capacity [5, 6] [23], and non-stationary demands [13] [25]. These are only representative of the vast research in this field. There have been a few studies that have considered changes in purchasing price of the product. Zipkin [25] studied the problem in which the purchasing cost changes from one period to the next, but restricted his attention to cyclical changes. He presented an efficient solution procedure that computes the optimal order upto levels. However, he did not address general non-stationarities in the purchasing cost that we address here. Scheller-Wolf and Tayur [20] studied the situation when the purchasing price undergoes a Markovian transition in a capacitated setting. Their solution procedure, based on Infinitesimal Perturbation Analysis (IPA) (see [12]), is very time consuming. For the uncapacitated situation, solution procedures that are much more efficient (see [9]) can be developed.

The rest of this paper is organized as follows. In Section 2 we will prove that the
optimal policy is order upto for the single period, finite horizon, and the infinite horizon discounted cost criteria. However, the order upto levels depend on the current purchasing cost of the product and are not available in closed form. In Section 3, we will present an efficient solution procedure to compute these order upto levels. In Section 4, we will establish conditions under which, we can expect the myopic solution to be close to optimal. In section 7, we apply some of these results to commonly used models of exchange rate fluctuations.

2 Optimal Policy

In this section we will establish that order upto policies are optimal for finite horizon and discounted infinite horizon criteria. The optimal order upto level in each period will depend on the state of the current period. We will start by analyzing the one period problem. For ease of exposition, we will say that the problem is in state $i$ if the purchasing cost in that period is $c_i$ per unit. Although some of the proofs follow standard arguments from inventory theory, they have been detailed here as a stepping stone for understanding the arguments and the solution procedures presented later in the paper.

**Property 1**  For the one period problem with starting state $i$, an order upto policy is optimal. The optimal one period cost function is concave.

**Proof**  Suppose we have $x$ units on hand at the beginning of the period. We can place an order and bring the level to $y$ such that the one period profit $V_{i1}^i(x)$ is maximized. That is

$$V_{i1}^i(x) = \max_{y \geq x} \{ -c_i(y - x) + L(y) \}$$

where $L(y)$, the one period expected cost when the inventory is $y$, is computed as follows:

$$L(y) = \int_0^y [pt - h(y - t)]d\Phi(t) + py \int_y^\infty d\Phi(t)$$

$L(y)$ is concave in $y$ and $V_{i1}^i(x)$ is maximized at $y_{i1}^i$ that satisfies the relation $\Phi_i(y_{i1}^i) = \frac{p - c_i}{p + h}$. The optimal policy is to order $y_{i1}^i - x$ units if $x < y_{i1}^i$ and to order nothing if $x \geq y_{i1}^i$.

The first and second order derivatives of the optimal one-period cost with respect to the initial inventory are (we use $', ''$ is to denote first and second derivatives of a function
respectively):

\[
V_i'(x) = c_i \quad \text{if} \quad x < y_i^i \\
= L'(x) < c_i \quad \text{if} \quad x \geq y_i^i \\
V_i''(x) = 0 \quad \text{if} \quad x < y_i^i \\
= L''(x) < 0 \quad \text{if} \quad x \geq y_i^i
\]

Since \( V_i''(x) \leq 0 \) for all values of \( x \), \( V_i'(x) \) is concave in \( x \). \( \square \)

Next, we consider an \( n \)-period problem with initial state \( i \) and initial inventory \( x \). Let \( \xi_n \) be the random demand in a period \( n \). The optimal \( n \)-period profit \( V_i^n(x) = \min_{y \geq x} \{-c_i(y-x) + J_i^n(y)\} \) where

\[
J_i^n(y) = L(y) + \sum_{j=1}^{k} \pi_{ij}(E_{\xi}[V_j^{n-1}(y-\xi)])
\]

Since there are no salvage costs, \( V_0^i(x) \equiv 0 \) for all \( i, x \).

**Property 2** For all values of \( n \):

1. \( J_i^n(y) \) is concave in \( y \);

2. \( V_i^n(x) \) is concave in \( x \); and

3. An order upto policy is optimal.

**Proof** These statements are proved using induction from the boundary condition \( V_i^0(x) \equiv 0 \) and is concave in \( x \). Assume that \( V_j^{n-1}(x) \) is concave in \( x \) for all values of \( j \). After taking two derivatives using Leibnitz’s rule and properly accounting for the discontinuity resulting from \( (y-\xi)^+ \), it can be easily established that \( J_i^n(y) \) is concave in \( y \) implying that \( c_i(y-x) + J_i^n(y) \) is concave in \( y \). Based on arguments similar to those in Property 1, \( V_i^n(x) \) is concave in \( x \). By induction for every \( n \), \( J_i^n(y) \) and \( V_i^n(x) \) are concave in \( y \) and \( x \) respectively. So, there exists an optimal order upto level \( y_i^n \) for the \( n \)-period problem with the initial state \( i \). We should note here that our assumption of zero leadtime enables us to characterize the optimal solution in this lost sales inventory model. \( \square \)

\( y_i^n \) satisfies the relation:

\[
p - c_i - (h+p)\Phi(y_i^n) + \sum_{j=1}^{k} \pi_{ij}(\int_{0}^{y_i^n} [V_j^{n-1}(y-t)]\phi(t)dt) = 0
\]
The derivative of the optimal \( n \)-period cost with respect to the initial inventory is:

\[
V'_n(x) = \begin{cases} 
  c_i & \text{if } x < y_n^i \\
  L'(x) + \sum_{j=1}^{k} \pi_{ij} \left( \int_{0}^{y} [V'_{n-1}(y-t)] \phi(t) dt \right) < c_i & \text{if } x \geq y_n^i
\end{cases}
\]

When the purchasing cost is constant over time, say at \( c \) per unit, then the optimal order up to level is a solution to the equation

\[
p - c - (h + p) \Phi(y_n^i) + \int_{0}^{y} [V'_{n-1}(y-t)] \phi(t) dt = 0.
\]

Since \( V'_{n-1}(y-t) = c \) for all values of \( t \), the optimal order up to level can be expressed in closed form as \( \Phi^{-1}(\frac{p-c}{p+h-c}) \). However, when the purchasing cost fluctuates over time, the integral, \( \int_{0}^{y} [V'_{n-1}(y-t)] \phi(t) dt \), cannot be easily evaluated and the order up to level cannot be expressed in closed form.

**Property 3** If \( \pi_{ij} = \tau_j \forall i \), i.e. the price in a period is independent of the price in the previous period, then the order up to levels are ordered. i.e. \( y_n^i \geq y_n^{(i+1)} \) \( \forall 1 < i < k - 1 \).

**Proof** \( y_n^i \) and \( y_n^{(i+1)} \) are solution to the equations respectively:

\[
L'(y) + \sum_{j=1}^{k} \tau_j \left( \int_{0}^{y} [V'_{n-1}(y-t)] \phi(t) dt \right) = c_i
\]

\[
L'(y) + \sum_{j=1}^{k} \tau_j \left( \int_{0}^{y} [V'_{n-1}(y-t)] \phi(t) dt \right) = c_{i+1}
\]

Notice that both the equations above have the same left hand side for every value of \( y \). Furthermore, the left hand side is a decreasing function in \( y \) due to the concavity established in Property 2. Since the right hand side of the first equation is smaller (due to our assumption on \( \{c_i, i = 1, \ldots, k\} \)), it follows that \( y_n^i \) will be bigger than \( y_n^{(i+1)} \).

This monotonicity result can be extended to situations when two conditions hold:

1. The transition matrix is totally positive. It is probable that the price of the product will be small (big) in the next period if the price in the current period is small (big).

   Let \( P_{ij} = \sum_{l=0}^{j} \pi_{il} \) be the probability that the state of the next period is at most \( j \) given that the state of this period is \( i \). It is reasonable to expect that \( P_{ij} \geq P_{(i+1)j} \forall 0 < i < k - 1 \). That means, it is probable that price of the product will be small (big)
in the next period if the price in the current period is small (big). This property is satisfied by the Markovian transitions if Π is a Totally Positive matrix.

2. The price speculation in state \( i + 1 \) is at most that in state \( i \). Price speculation, the predicted increase in the purchasing cost, is usually a major reason for carrying inventory. The speculation in state \( i \) is \( \sum_{j=1}^{n} [\pi_{ij} c_j] - c_i \). In state \( i \), it must hold that
\[
\sum_{j=1}^{n} [\pi_{(i+1)j} c_j] - c_{i+1} \leq \sum_{j=1}^{n} [\pi_{ij} c_j] - c_i
\]

Under these conditions, the monotonicity result of property 3 still holds. That is, the order upto level in state \( i + 1 \) will be lower than that in state \( i \). This is established in Property 4 below.

**Property 4** If \( \sum_{j=1}^{n} [\pi_{(i+1)j} c_j] - c_{i+1} \leq \sum_{j=1}^{n} [\pi_{ij} c_j] - c_i \), i.e. the speculation in state \( i + 1 \) is at most that in state \( i \), and the probability transition matrix is totally positive (\( P_{ij} \geq P_{(i+1)j} \forall 0 < i < k - 1 \) \( \forall j \)), then

1. The order upto levels are ordered. i.e. \( y_n^i \geq y_n^{i+1} \forall 1 < i < k - 1 \).

2. For all values of \( x \), \( c_{i+1} - V_n^{i+1}(x) \geq c_i - V_n^i(x) \).

**Proof** Under the assumption of no salvage cost (i.e. \( V_0^i(x) = 0 \)), the properties (1) and (2) are trivially true for the base case \( n = 0 \). Assume that they are true for \( n - 1 \). Since \( P_{ij} \geq P_{(i+1)j} \) and \( c_j - V_n^j(x) \) is an increasing function of \( j \) we have that
\[
\sum_{j=1}^{k} \pi_{(i+1)j} (c_j - E_\xi [V_n^{i+1}(y - \xi)^+]) \geq \sum_{j=1}^{k} \pi_{ij} (c_j - E_\xi [V_n^i(y - \xi)^+])
\]

This, along with our assumption on speculation that \( c_{i+1} - \sum_{j=1}^{k} \pi_{(i+1)j} c_j \geq c_i - \sum_{j=1}^{k} \pi_{ij} c_j \), implies that
\[
c_{i+1} - \sum_{j=1}^{k} \pi_{(i+1)j} c_j + \sum_{j=1}^{k} \pi_{(i+1)j} (c_j - E_\xi [V_n^{i+1}(y - \xi)^+]) \geq c_i - \sum_{j=1}^{k} \pi_{ij} c_j + \sum_{j=1}^{k} \pi_{ij} (c_j - E_\xi [V_n^i(y - \xi)^+])
\]

Simplifying both sides of the inequality above results in the inequality
\[
c_{i+1} - \sum_{j=1}^{k} \pi_{(i+1)j} (E_\xi [V_n^{i+1}(y - \xi)^+]) \geq c_i - \sum_{j=1}^{k} \pi_{ij} (E_\xi [V_n^i(y - \xi)^+])
\]
which we will utilize in case 2 below. Incorporating this inequality into corresponding equations (as shown in Property 3) for $y_n^i$ and $y_n^{(i+1)}$ implies that $y_n^i \geq y_n^{(i+1)}$. Thus part (1) holds for $n$.

Notice that $c_{i+1} - V_n^{(i+1)}(x)$ and $c_i - V_n^{(i)}(x)$ are defined as follows:

$$c_{i+1} - V_n^{(i+1)}(x) = 0 \text{ if } x < y_n^{(i+1)}$$
$$= c_{i+1} - L'(x) - \sum_{j=1}^{k} \pi_{i(j+1)}(E_x[V_n^{j-1}(y - \xi)]) \text{ if } x \geq y_n^{(i+1)}$$

$$c_i - V_n^{(i)}(x) = 0 \text{ if } x < y_n^i$$
$$= c_i - L'(x) - \sum_{j=1}^{k} \pi_{ij}(E_x[V_n^{j-1}(y - \xi)]) \text{ if } x \geq y_n^i$$

To establish part (2) for $n$, we will consider three cases that cover the whole range for $x$.

**Case 1** $x \leq y_n^{(i+1)}$: In this case $c_{i+1} - V_n^{(i+1)}(x) = c_i - V_n^{(i)}(x) = 0$ implying that part (2) is true for $n$.

**Case 2** $x \geq y_n^i$: Here $c_{i+1} - V_n^{(i+1)}(x) = c_{i+1} - L'(x) - \sum_{j=1}^{k} \pi_{i(j+1)}(E_x[V_n^{j-1}(y - \xi)])$ and $c_i - V_n^{(i)}(x) = c_i - L'(x) - \sum_{j=1}^{k} \pi_{ij}(E_x[V_n^{j-1}(y - \xi)])$. Part (2) holds due to the inequality above.

**Case 3** $y_n^{(i+1)} < x < y_n^i$: Here $c_{i+1} - V_n^{(i+1)}(x) = c_{i+1} - L'(x) - \sum_{j=1}^{k} \pi_{i(j+1)}(E_x[V_n^{j-1}(y - \xi)])$ which is greater than zero due to concavity established in Property 2 above and $c_i - V_n^{(i)}(x) = 0$ implying that part (2) holds for $n$. \hfill \square

Next we consider the infinite horizon problem under the discounted cost criterion. Under this criterion, the costs in the future are discounted by a factor $0 < \beta < 1$. The finite horizon recursive relation for $V_n^i(x)$ is modified as below to include $\beta$.

$$V_n^i(x) = \min_{y \geq x} \{c_i(y - x) + J_n^i(y)\}$$
$$J_n^i(y) = L(y) + \beta \sum_{j=1}^{k} \pi_{ij}(E_x[V_n^{j}(y - \xi)])$$

Let us define $V_n^i(x) = \lim_{n \to \infty} V_n^i(x)$; the objective is to minimize $V_n^i(x)$. Based on the results in Bertsekas [3], it can be easily shown that the optimal policy exits and is order upto.
3 Solution Procedure

In this section we will describe an efficient recursive solution procedure for computing the optimal order up to levels. As noted earlier, due to the presence of an integral term that is difficult to evaluate, there exists no closed form solution for these order up to levels. In addition, the solution procedures detailed in Karlin [13], Zipkin [25], and Song and Zipkin [22] are not applicable here due to their restrictive assumptions. Specifically Karlin [13] and Zipkin [25] assumed that the costs vary in a cyclic manner while Song and Zipkin [22] assumed exponential demands in continuous time. Our approach, similar to the one detailed in Gavirneni and Tayur [8] is based on recursively estimating the derivative of the infinite horizon cost function. In order for this recursion to work, we need to assume that the demand in any period is non-zero. Since we will be working with a discretized demand distribution we will assume that the demand is at least one unit and that $\tau_\rho$ is the probability that the demand is $\rho$ units.

For ease of presentation, we will denote $V^i_s(x)$ by $W^i(x)$. Based on the arguments from the previous section, we can say that

$$W^i(x) = c_i \quad \text{if} \quad x < y^i_s$$

$$= L'(x) + \beta \sum_{j=1}^{k} \pi_{ij} (\sum_{1}^{y} [W'^j(x - t \rho_t) \tau_t]) \quad \text{if} \quad x \geq y^i_s$$

where $y^i_s$ is the infinite horizon order up to level in state $i$. From these equations, we know that if the current inventory level is below the optimal level, then the derivative of the infinite horizon cost at that level can be set equal to the purchasing cost in that period. However if the inventory level is above the order up to level, then the derivative is equal to the derivative at the inventory level which can be computed by evaluating $L'(x) + \beta \sum_{j=1}^{k} \pi_{ij} (\sum_{1}^{y} [W'^j(x - t \rho_t) \tau_t])$ if the derivatives at all the inventory levels below it are known. Further we can recognize that the current inventory level is below the optimal level and set its derivative equal to $c_i$, if $L'(x) + \beta \sum_{j=1}^{k} \pi_{ij} (\sum_{1}^{y} [W'^j(x - t \rho_t) \tau_t])$ turns out to be greater than $c_i$. We can start with the boundary condition $W^i(0) = c_i$ and recursively estimate, incrementing by one unit each time, the derivative at all the inventory levels for all states. Suppose at an inventory level for a state, the derivative is greater than the purchasing cost, then we know that optimal order up to level is above that inventory level and thus the derivative must equal the purchasing cost. Proceeding in this manner we recursively estimate $W^i(x)$ at all values of $x$ and $i$. Once we know the derivative of the
infinite horizon cost function, the optimal level in state $i$ is the largest inventory level at which the derivative is equal to $c_i$. The algorithm is described more formally below.

**Step 1** Set $W^i(0) = c_i$ for all values of $i$.

**Step 2** Repeat the following recursion for values of $x$ starting at 1 and increasing by 1 and noting that $L'(x) = p - (h + p)\Phi(x)$.

$$W^i(x) = L'_i(x) + \sum_{\rho=1}^{x} \tau_\rho \sum_{j=1}^{k} [W^j(x - \rho)]\pi_{ij} \quad i \in [1, \ldots, k]$$

At any point during the recursion, if $W^i(x)$ is greater than $c_i$, recognizing that $x$ is below the optimal solution we set it equal to $c_i$.

**Step 3** For each state $i$, find the order up-to level $y^i_i = \inf\{x|W^i(x) < c_i\}$.

The stopping condition in step 2 is when all the order upto levels have been determined (i.e. $\{W^i(x) < c_i \forall i\}$). If desired these recursions can be continued to compute the derivatives at inventory levels above the optimal order upto levels.

In some situations, the cost (state) in the next period may be a function of the demand in this period. This relationship can be modelled by defining $\pi_{ij}$ as a function of the demand $\rho$. This procedure can still be utilized by modifying step 2 by replacing $\pi_{ij}$ with $\pi_{ij}(\rho)$ in its right hand side.

### 4 Myopic Heuristics

A myopic inventory policy (heuristic) is obtained as follows:

1. Truncate the problem at the point of arrival of the next order, after the one currently being considered.

2. Set the salvage value for left-over inventory at the end equal to the (linear) variable part of the ordering cost.

3. Solve the resulting problem.
This definition of a myopic solution is similar to the one used by Sobel [21] and Veinott [24]. Myopic solutions while easy to compute, are generally not optimal for non-stationary inventory control problems such as this one. Myopic policies are now known to be optimal, or nearly optimal for a wide variety of inventory problems [1], [4], [11], [14], [16], [17]. In this section we will propose some myopic heuristics for this problem with and without the non-speculative assumption. After commenting on the possibility of them being close to optimal we will conduct a computational study to evaluate their effectiveness.

Determining the myopic solution as described above is very similar to our earlier analysis of the one period problem with the only difference being that the excess inventory can be salvaged at the purchasing cost of the next period. Thus, if $\overline{c}_i = \sum_{j=1}^{k} c_j \pi_{ij}$, then the myopic order upto level in state $i$ is equal to $\Phi^{-1}(\frac{p-c_i}{p+h-\overline{c}_i})$. In addition if $\overline{c}_i = \overline{c}$ for all values of $i$, then the myopic solution in state $i$ can be simplified to $\Phi^{-1}(\frac{p-c_i}{p+h-c})$.

**Property 6**  The myopic order upto level is an upperbound on the optimal level.

**Proof** From the previous section, we know that the optimal order upto level is a solution to the equation

$$L'(y) + \sum_{j=1}^{k} \pi_{ij}(\int_0^y [V_j^*(y-t)]\phi(t)dt) = c_i$$

where as the myopic level is a solution to the equation

$$L'(y) + \overline{c}_i \Phi(y) = c_i.$$

Comparing these two equations and using the facts that (1) $V_j^*(y-t) \leq c_j$ and (2) both the left hand sides are decreasing in $y$, it is easily seen that the myopic solution is no smaller than the optimal solution. \(\square\)

But, how effective is the myopic heuristic? To answer that question, we use the concept of $\mathcal{P}^{(1-P)}$-myopic described in Gavirneni et al. [7]. A discrete time multi-period stochastic inventory problem is said to be $\mathcal{P}^{(1-P)}$-myopic when at the beginning of a period, if the inventory level is at the myopic level, and the probability that the inventory level at the beginning of the next period (i.e. after satisfying demand in this period and before production in the next period) is greater than the myopic level for that period, is less than $P$. The argument was that the value of $(1 - P)$ will be a good indicator of the closeness of the myopic solution to the optimal solution.
Table 1: $(1 - P)$ value as an indicator for the effectiveness of the myopic solution

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</table>

For this problem, the value of $(1 - P)$ for the myopic solution can be computed using the relation

$$1 - P = \sum_{i=1}^{k} q_i \sum_{j=1}^{k} 1 - \Phi((\Phi^{-1}(\frac{p - c_i}{p + h - c_i}) - \Phi^{-1}(\frac{p - c_j}{p + h - c_j}))^+)$$

This $(1 - P)$ value predicts, on the average, the probability that we will be at the myopic level in a period when using the myopic policy. Since the myopic level is defined to be good on the short term, if there is a higher chance that we can reach the myopic level in every period, then the myopic solution may be a good approximation to the optimal solution.

To test this hypothesis (i.e. whether the value of $(1 - P)$ is really a good indicator of the effectiveness of the myopic solution), we performed a computational study with three demand distributions, namely Unif[1,200], exp(100), and exp(100) truncated at 300. The demands were discretized with a gap of one unit. The sales price was held constant at 20 dollars while the purchasing cost was allowed to fluctuate, with equal probability, between
the values of 11, 12, 13, 14, and 15. The holding cost was made to vary from 2.1 to 4.9 in increments of 0.2. Notice that under these conditions, the non-speculative assumption was satisfied. This setup gave a wide range of \((1 - P)\) values from 0.75 to 0.99. The percentage difference (between the optimal profit and the myopic profit) along with the \((1 - P)\) value for all the cases are given in table 2.

Figure 1 contains the plot of the percentage difference versus the \((1 - P)\) value. Notice that when the \((1 - P)\) value was higher, the percentage difference was lower indicating that the myopic heuristic was very effective. However as the \((1 - P)\) value decreased, the effectiveness of the myopic heuristic worsened and the myopic heuristic had particularly poor performance when the \((1 - P)\) value was smaller than 0.80. Figure 2 contains the output from a linear regression performed between the logarithm of the percentage difference and the \((1 - P)\) value. Notice that the \(R^2\) value was greater than 0.98 and the regression was significant with an extremely low \(p\)-value. From these results we conclude that the \((1 - P)\) value is a very good indicator for the effectiveness of the myopic heuristic. From table 2 we also observe that the myopic solution was especially effective for the uniform distribution. For the exponential demands which have considerably larger variances, the myopic solution was a little less effective especially when the holding cost was low.

The effect that the fluctuations have on the inventory levels is largely dependent on the relative values of the holding cost and sales price. For example, the same fluctuations
SUMMARY OUTPUT

Regression Statistics

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ANOVA

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<tr>
<td>F</td>
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Coefficients

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<td>Lower 95%</td>
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<td>Upper 95%</td>
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<td>Intercept</td>
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<td>0.24032</td>
<td>-49.8409</td>
<td>1.12E-39</td>
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</table>

Figure 2: Regression output for log of % difference versus the $(1 - P)$ value

will have a smaller effect on the inventories at higher sales price. The $(1 - P)$ value will be, by definition, a good indicator to judge the severity of the effect of these fluctuations. If the fluctuations are judged to be significant, then there is considerable justification for pursuing a sophisticated solution for combating these fluctuations. On the other hand if these fluctuations are judged to be not significant then using a myopic heuristic such as the one proposed above might suffice.

For the myopic heuristic described above, it is necessary that $\frac{p - c_i}{p + h - c_i} < 1$ which translates into a condition $\bar{c}_i - c_i < h$ for all $i$. This condition is clearly not satisfied when the non-speculative assumption is not imposed on the movement of the purchasing cost. Thus under that situation, we need to look at some other approach to computing heuristic order up to levels. Gavirneni and Morton [8] presented an effective heuristic for inventory control under speculation but restricted their attention to just one change in the purchasing cost. We will modify that heuristic to this situation. Let $y^i_h$ be the heuristic level in state $i$. If $\bar{c}_i - c_i < h$, we will set it equal to the myopic level. However, if $\bar{c}_i - c_i \geq h$, we will compute $y^i_h$ using the formula

$$y^i_h = \Phi^{-1}\left(\frac{p - c_i}{p + h - c_i}\right) + \mu \ast K$$

where $\mu$ is the mean demand in a period and $K = \min\left\{\frac{\bar{c}_i - c_i}{h}, \frac{1}{q_i}\right\}$. The intuition behind these calculations is as follows. The inventory level if the price did not change would be $\Phi^{-1}\left(\frac{p - c_i}{p + h - c_i}\right)$. However, we should obtain more inventory due to the predicted increase in
prices since $\bar{c}_i - c_i$ is greater than $h$. Clearly it does not make sense to stock up more inventory than will be demanded in $\frac{\bar{c}_i - c_i}{h}$ periods. In addition since this state will, on the average, occur once in every $\frac{1}{q_i}$ periods, it again does not make sense to accrue more inventory than that will be demanded in $\frac{1}{q_i}$ periods. Thus, the argument that the heuristic level should be the myopic level plus the demand during $K$ periods, where $K = \min\{\frac{\bar{c}_i - c_i}{h}, \frac{1}{q_i}\}$. In addition we could impose ordering in the order upto levels if the conditions of property 3 were satisfied.

We performed a computational study to observe the effectiveness of this heuristic. The setup was similar to the one in section 3. The percentage difference between the optimal and the heuristic profits is given in Table 3 for various values of the holding cost for the three distributions.

<table>
<thead>
<tr>
<th>h</th>
<th>Exp(100)</th>
<th>Trunc. Exp(100)</th>
<th>Uniform[1,200]</th>
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<td>0.1</td>
<td>9.85</td>
<td>7.40</td>
<td>5.15</td>
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<tr>
<td>0.3</td>
<td>2.18</td>
<td>5.53</td>
<td>4.72</td>
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<td>0.5</td>
<td>0.71</td>
<td>2.88</td>
<td>2.72</td>
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<td>0.7</td>
<td>0.42</td>
<td>1.83</td>
<td>2.10</td>
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<td>0.38</td>
<td>1.33</td>
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<td>1.1</td>
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<td>1.5</td>
<td>1.52</td>
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<td>1.7</td>
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<tr>
<td>1.9</td>
<td>0.76</td>
<td>0.85</td>
<td>1.58</td>
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</table>

Table 2: Effectiveness (percentage away from optimal) of the heuristic without the non-speculative assumption

From table 3, it is clear that the heuristic is very effective with an average difference of 2.24% between the optimal and heuristic profits. The heuristic exhibits poor performance when the holding cost is very small compared to the fluctuations in the purchasing cost. That is because under that condition, the heuristic overestimates the inventory levels. For reasonable values of the holding cost, the heuristic performs consistently well and the difference between the optimal and heuristic profits is at most 3%. These observations are consistent with the observations in Gavirneni and Morton [8].
5 Exchange Rate Fluctuations

In this section we consider some commonly used models for exchange rate fluctuations and apply the insights gained in the previous sections. Even though there is no single universally accepted model for exchange rate fluctuations, random walk (RW) model, mean reverting (MR) model, and momentum (MO) model are the ones that are most popular (see [18]). Scheller-Wolf and Tayur [20] used these models in their computational testing. We will use their experimental setup to illustrate the results of this paper.

The purchasing cost was allowed to take values \{16, 18, 20, 22, 24\}, the sales price was 40 and the holding cost was 5. The probability transition matrices were

\[
\begin{pmatrix}
0.5 & 0.5 & 0.0 & 0.0 & 0.0 \\
0.5 & 0.0 & 0.5 & 0.0 & 0.0 \\
0.0 & 0.5 & 0.0 & 0.5 & 0.0 \\
0.0 & 0.0 & 0.5 & 0.0 & 0.5 \\
0.0 & 0.0 & 0.0 & 0.5 & 0.5 \\
\end{pmatrix}
\begin{pmatrix}
0.2 & 0.8 & 0.0 & 0.0 & 0.0 \\
0.1 & 0.3 & 0.6 & 0.0 & 0.0 \\
0.0 & 0.1 & 0.8 & 0.1 & 0.0 \\
0.0 & 0.0 & 0.6 & 0.3 & 0.1 \\
0.0 & 0.0 & 0.0 & 0.8 & 0.2 \\
\end{pmatrix}
\begin{pmatrix}
0.8 & 0.2 & 0.0 & 0.0 & 0.0 \\
0.6 & 0.3 & 0.1 & 0.0 & 0.0 \\
0.0 & 0.5 & 0.0 & 0.5 & 0.0 \\
0.0 & 0.0 & 0.1 & 0.3 & 0.6 \\
0.0 & 0.0 & 0.0 & 0.2 & 0.8 \\
\end{pmatrix}
\]

for the random walk, mean reverting, and momentum models respectively. We used three distributions, namely Uniform[1,200], exp(100), and exp(100) truncated at 300 to model the demand. Since the nonspeculative assumption was satisfied we decided to analyze the myopic heuristic only. For each state the myopic order up to level, \(y^i_m\), was computed using the relation

\[
y^i_m = \Phi^{-1}(\frac{p - c_i}{p + h - c_i}).
\]

In addition the \(1 - P\) values as explained in the previous section were computed using the relation

\[
1 - P = \sum_{i=1}^{k} q_i \sum_{j=1}^{k} 1 - \Phi((\Phi^{-1}(\frac{p - c_i}{p + h - c_i}) - \Phi^{-1}(\frac{p - c_j}{p + h - c_j}))^+)
\]

It is worthwhile noting that despite their appearance, these functions are easily evaluated in a spreadsheet for most common distributions. In addition, as evidenced in table 1, the \(1-P\) value can be a very good indicator on the severity of the fluctuations and effectiveness of the myopic solution. The \(1-P\) values calculated for the three distributions are given in table 3 below.

Notice the \(1-P\) values were greater than 0.93 in all the situations. For the uniform demand distribution, the values were around 0.987. They were slightly lower for the other two distributions. Relating these values back to figure 1 indicates that myopic solution will
be quite effective in these situations. This was confirmed by our computational study which showed virtually no difference between the optimal and myopic profits. The large \((1 - P)\) values also indicate that the fluctuations in the costs are not significant and thus it is not necessary to look for sophisticated approaches to manage them.

For these three models, we computed the optimal and the myopic order upto policies and their corresponding profits. As mentioned earlier the profits were virtually indistinguishable between the two situations. The inventory levels were also very close to each other considering the fact that the optimal solution procedure required some discretization. The optimal and the myopic order upto levels for the exp(100) demand distribution are given in table 4. The characteristics were representative of the other two distributions.

Notice that in all the cases the myopic level was within one or two units of the optimal solutions. Also the optimal order upto levels were monotonic in purchasing cost for the random walk and the mean reverting models where as they were not for the momentum model. This was due to the fact that the random walk and the mean reverting models satisfy the assumptions in property 4 where as the momentum model violates them. These observations were also true for the other two demand distributions.

### 6 Conclusions

We addressed the inventory control problem in which the purchasing price of the product undergoes a Markovian transition from one period to the next. Analysis of this problem enables us to understand the effects of exchange rate fluctuations on inventory control decisions. For this problem, we established the structure of optimal policies and presented a solution procedure that computes the optimal parameters efficiently. In addition, we have also shown that myopic solutions are very effective under the non-speculative assumption. When the non-speculative assumption is not satisfied, we proposed a heuristic that was

<table>
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<th>trunc. exp(100)</th>
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</tr>
<tr>
<td>Mean Reverting</td>
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<td>0.934</td>
<td>0.963</td>
</tr>
<tr>
<td>Momentum</td>
<td>0.988</td>
<td>0.948</td>
<td>0.970</td>
</tr>
</tbody>
</table>

Table 3: \((1 - P)\) values for the three distributions under the three models
Table 4: The optimal \( y^*_i \) and the myopic \( y^m_i \) up to levels for \( \exp(100) \) shown to be very effective.

References


