Optimal Consolidation of Single Echelon Inventories

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Feb 2004

ABSTRACT

Effective management of inventory is vital in firm’s survival in today’s business environment where high level of customer service at reduced cost is expected. The effect of inventory consolidation across locations has received a considerable attention in the recent logistics literature. The impact of physical consolidation and risk pooling has been discussed. Previous studies have focused on the allocation decisions assuming that the consolidation locations are pre-determined. In this paper, we extend these analyses to include the selection of consolidation locations. We analyze the impact of ordering cost, inbound shipping and handling cost, outbound transportation costs, lead time variation and demand correlation on the overall cost reduction. We provide an optimization model for the inventory consolidation in the presence of variable lead-times, and demand correlations. As changes in demand quantity might impact the delivery lead times, our model also captures the correlation between demand and lead-time at each centralized location. Computational results will be presented to verify the sensitivity of the decisions to changes in cost parameters, and the impact of lead time variation and demand correlation between the potential consolidation locations.

The author would like to thank Professor Philip Evers at the University of Maryland, College Park for his insightful comments on earlier editions of this paper.
I. Introduction

It is well known that consolidation of stock keeping locations may reduce the safety stock inventory. The impact of consolidation and the extent that it is influenced by the demand characteristics at the centralized locations is investigated. Studies in the literature have assumed that the consolidation locations are pr-determined. Smykay (1973) and Maister (1976) derived square root laws. Zinn, Levy, and Bowersox (1989) showed that the Smykay square root law represents a special case of their portfolio effect model. They considered only the case where inventory locations are reduced to one. Evers and Beier (1993) developed a model of the square root law associated with safety stocks. Evers (1995) extended this model to include cycle stocks. Mahmoud (1992) considered the impact that safety stock centralization has on various cost factors and developed a method for determining the optimal consolidation scheme based on the portfolio effect model. Tallon (1993) addressed the issue of variable lead times. These square root laws are applicable to situations where certain assumptions hold. For the complete list of these assumptions and reference to their first introduction see Evers (1995).

In this paper we propose a cost optimization model for the selection of the centralized locations and the physical allocation of inventory to maximize the cost savings. Since demands are often correlated across geographical regions, and changes in demand might impact the delivery lead time, capturing the dependence relationships is necessary in modeling the reality. In this paper, we address the lead-time variability, correlated demands, and non-zero correlation between demand and lead-time at each location. The formulation we obtain is a mixed integer non-linear programming problem. Using this model we analyze the impact of changes in ordering cost, shipping and handling cost and outbound transportation costs. As the result of our experiments show these cost parameters impact the selection of the consolidating locations, the allocation of systems demand, and therefore the overall cost reduction. We also address the importance of factors such as lead time variation and demand correlation and their impact on the savings in safety stocks due to consolidation.

The remainder of this paper is organized as follows. In section II, we describe the methods used
to determine the required inventory levels at the stocking locations. We also describe the demand allocation and the formulas used for the computation of the safety stock requirements and cycle stock related cost after consolidation. The proposed optimization models are formulated in Section III. In Section IV we present our computational results. Our conclusions are provided in section V.

II. Background

Consider a single echelon distribution system consisting of n stocking locations. In what follows we make use of the following notations:

- $i$: Index for decentralized (before consolidation) locations $i \in \{1,2,\ldots,n\}$
- $D_i$: Mean aggregate daily demand at decentralized location $i$ during the planning period
- $L_i$: Mean lead time at decentralized location $i$ in days
- $\sigma_{D_i}$: std. for aggregate daily demand at decentralized location $i$
- $\sigma_{L_i}$: std. for lead time at decentralized location $i$
- $K$: Safety stock factor: for given fill rate $\alpha$, $K$ is standard Normal deviate such that $P(Z \leq K) = \alpha$
- $h$: Per unit holding cost during the period
- $\rho_{i,l}$: Coefficient for correlation between demand at locations $i$ and $l$
- $t_i$: Per unit cost of transportation for satisfying demand $D_i$ (from the stocking location $i$ to customers)

Before Consolidation:

We assume that the planners use the following formula to determine the level of safety stock required at any location.

$$K \sqrt{\sigma_{D_i}^2 L + \sigma_{L_i}^2 D_i^2 + \rho_{i,l} \sigma_{L_i} \sigma_{D_i}}$$  (1)

The planner would like to maintain a level of service defined by requiring a pre-specified
probability of no stock out during the replenishment lead time \((L, \sigma_L)\) which translate to the safety factor \((K)\). \((D, \sigma_D)\) characterize the distribution of demand for the location during the period. The last term under the square root, \(\tau \sigma_L \sigma_D\) represents the covariance of demand and lead time at the location with correlation coefficient \(\tau\). For more detail see Tersine (1994).

In our cost optimization model in section III among other costs, we need to capture the total cost related to the required cycle stock at each location. We will compute the total cost of holding, ordering, shipping and handling the cycle stock during the planning period. This cost depends on the number of order cycles for each location \(i\). The cost of ordering, shipping, handling, and holding the cycle stock, as a function of the number of replenishment orders \(N_i\), would be:

\[
C(N_i) = O_i N_i + h\bar{D}_i / 2N_i + f_i N_i + v_i \bar{D}_i, \tag{2}
\]

where \(\bar{D}\) is the mean demand during the planning period. Taking the first derivative of the cost function (1) and solving for \(N_i\) as in (3);

\[
O_i + f_i - h\bar{D}_i / 2N_i^2 = 0
\]

\[
N_i = \sqrt{h\bar{D}_i / 2(O_i + f_i)} \tag{3}
\]

The total cost related to ordering, holding, and shipping and handling the required cycle stock \(C_i\) for location \(i\) can be obtained by substituting the optimal \(N_i\) from (3) into (2) as follows:

\[
C_i = \sqrt{2(O_i + f_i)h\bar{D}_i + v_i \bar{D}_i} \tag{4}
\]

**After Consolidation:**

When the number of stocking locations are reduced, \(m < n\) locations are selected to hold the systems inventory and satisfy the total demand. In what follows we make use of these additional notations:

- \(j\) : Index for centralized (after consolidation) locations
\( L_j \) : Mean lead time at centralized location \( j \)
\( \sigma_{L_j} \) : std. for lead time at centralized location \( j \)
\( \tau_j \) : Coefficient for correlation between demand and lead time at location \( j \)
\( W_{ij} \) : Proportion of demand for location \( i \) (before consolidation) assigned to centralized location \( j \) (due to consolidation).

\( e_{ij} \) : The extra per unit cost of transportation due to satisfying location \( i \) demand by location \( j \)
\( O_j \) : Fixed order cost.
\( F_j \) : Fixed cost of operation at location \( j \) if holding the item inventory.

\( f_j \) : Fixed shipping and handling cost from supplier to the location \( j \).
\( v_j \) : Variable shipping and handling cost from supplier to the location \( j \).

The demand for the \( n \) stocking locations need to be partially or fully directed (allocated) to the \( m \) centralized locations. For any location \( j \) selected to hold consolidated inventory, the effective demand (centralized demand) will be the sum of the demands from locations \( 1, \ldots, n \) allocated to \( j \). For the purpose of this definition only, we denote the mean effective demand at centralized location \( j \), by \( D_j \) and let \( \sigma_{D_j} \) denote the standard deviation of effective demand at centralized location \( j \) after consolidation. Then the centralized demand seen by location \( j \), will have the following statistics:

\[
D_j = \sum_{i=1}^{n} W_{ij} D_i,
\]

and

\[
\sigma_{D_j}^2 = \sum_{i=1}^{n} W_{ij}^2 \sigma_{D_i}^2 + 2 \sum_{i=2}^{n} \sum_{l=1}^{i-1} W_{ij} W_{il} \rho_{ij} \sigma_{D_i} \sigma_{D_l},
\]

where \( W_{ij} \) is the proportion of demand for location \( i \) demand allocated to location \( j \). The allocation derives the order size, and the level of safety and cycle stocks. Incorporating the above representations in (1) and using \( \sigma_{D_{ij}} = \rho_{ij} \sigma_{D_i} \sigma_{D_j} \) we obtain the required safety stock \( SS_j \) at each centralized location \( j \) as a function of the allocation \( W_{ij} \) as follows:
Birjandi and Golovashkin (1998) showed that the terms under the big square root are convex and treated this as the square root of a convex function. Equation (5) indicates that, the planned safety stock inventory at each centralized location \( j \) after consolidation depends on proportion of mean demand during the period directed from each decentralized location \((W_{i,j})\), the safety stock factor \((K)\), mean and standard deviation of lead time \((L\& \sigma_{L})\), mean and standard deviation of demand \((D_{i}\& \sigma_{D_{i}})\), demand correlation coefficients \((\rho_{i,j})\) and the coefficients of correlation between demand and lead time \((\tau_{j})\).

Also incorporating the above allocation in (4), derives the total cost related to the required cycle stock \( C_{j} \) at each centralized location \( j \) and the average cycle stock held at the location during the replenishment cycle \( CS_{j} \)

\[
SS_{j} = K \sqrt{\left( \sum_{i=1}^{n} W_{i,j}^{2} \sigma_{D_{i}}^{2} + 2 \sum_{j=2}^{i=1} W_{i,j} \sigma_{D_{i,j}} \right) L_{j} + \sigma_{L_{j}}^{2} \left( \sum_{i=1}^{n} W_{i,j} D_{i} \right)^{2}} + \tau_{j} \sigma_{L_{j}} \sqrt{\left( \sum_{i=1}^{n} W_{i,j}^{2} \sigma_{D_{i}}^{2} + 2 \sum_{i=2}^{j=1} W_{i,j} \sigma_{D_{i,j}} \right)}
\]

As indicated in (7) the fixed order cost \((O_{j})\), the fixed shipping and handling cost \((f_{j})\), and the holding cost \((h)\), all impact the planned inventory level at location \( j \).

III. The optimization Models:

In this section we first provide an optimization model (model 1) that deals with the allocation of safety stock capturing the supply lead-time, demand correlations, and the correlation between demand and lead times. Assuming that the selection decision is made a
prio, \( m \) locations are selected to hold the inventory. We experiment with model 1 and analyze the impact of lead-time variations, demand correlations, and the correlation between demand and lead times on the savings in safety stocks. We then present the proposed cost minimization model (model 2) that deals with the selection decisions and the allocation decisions simultaneously to maximize the overall cost savings due to consolidation while a specified level of service is maintained.

**The allocation decision:**

Consider the case where there are \( n \) locations \( i = 1, 2, \ldots, n \) and without loss of generality, assume that locations \( j = 1, 2, \ldots, m \) with known expansion capacities \( \eta_j \) have been selected to hold the consolidated inventory. In order to maximize the consolidation effect, we use the following nonlinear program:

**Model 1**

Minimize: 

\[
\sum_{j=1}^{m} SS_j
\]  

Subject to:

\[\sum_{j=1}^{m} W_{i,j} = 1 \quad \forall i = 1, \ldots, n \]  \hspace{1cm} (9)

\[\sum_{i=1}^{n} W_{i,j}D_i \leq \eta_jD_j \quad \forall j = 1, \ldots, m \]  \hspace{1cm} (10)

\[W_{j,j} = 1 \quad \forall j = 1, \ldots, n \]  \hspace{1cm} (11)

\[0 \leq W_{i,j} \leq 1 \quad \forall i = 1, \ldots, n, j = 1, \ldots, m, i \neq j \]  \hspace{1cm} (12)

Where \( SS_j \) is the aggregate safety stock allocated to location \( j \) as defined in (5) and the objective is to minimize the total safety stock after consolidation. The constraint set (9) ensures that the total system demand will be covered after consolidation. Note that when some of the assumptions mentioned above are relaxed and when there are no limitation on capacity expansions, the non linear model of Evers (1995) will be equivalent to this model.

**The selection and allocation decisions:**

We now develop a cost optimization model to optimally select the locations and
optimally allocate the demands among the selected locations. As described in Section II, 5) and 6) capture the overall ordering, shipping & handling, and holding cost for each centralized location after consolidation. Other cost components we need to consider include the cost of in-bound transportation and out-bound transportation. The total out-bound transportation cost (13) needs to include the extra cost $e_{i,j}$ associated with location $j$ serving the demand originated at location $i$ after consolidation. The increase in transportation cost is due to the increase in the average delivery distance from the stocking location to the demand source. The inbound transportation is also influenced by the number and choice of the locations selected. In our study we capture this by assigning a constant percentage increase for any additional location to a base value as shown in (14) (e.g. $v = 5\%$).

\[
OT_j = \sum_{i=1}^{n} (t_i + e_{i,j}) W_{i,j} \bar{D}_i
\]

\[
V = v(\sum_{j=1}^{n} X_j)(\sum_{i=1}^{n} \nu_i \bar{D}_i)
\]

The objective of optimally selecting a number of locations amongst $1, 2, \ldots n$ to continue as stocking locations and satisfy the demand for all the locations at minimum cost can be defined as in (15). Constraints (16) through (20) describe the desired allocation.

**MINIMIZE:**

\[
\left[ \sum_{j=1}^{n} X_j (C_j + h SS_j + OT_j + F_j) \right] + V
\]

\[
\sum_{j=1}^{n} X_j W_{i,j} = 1 \quad \forall i = 1, \ldots, n
\]

\[
\sum_{i=1}^{n} W_{i,j} D_i \leq X_j \eta_j D_j \quad \forall j = 1, \ldots, n
\]

\[
0 \leq W_{i,j} \leq 1 \quad \forall i = 1, \ldots, n,j = 1, \ldots, n
\]

\[
W_{j,j} = X_j \quad \forall j = 1, \ldots, n
\]

\[
X_j \in \{0,1\} \quad \forall j = 1, \ldots, n
\]

However the non-linear terms due to multiplication of two variables in the objective function and
in the constraint set (15) adds to the complexity. We remove these non-linear terms by redefinition of the boundary condition for $W_{i,j}$ and modification of the objective function as follows:

**Model 2**

\[
\text{MINIMIZE} \quad \sum_{j=1}^{n} C_j + h \sum_{i=1}^{n} S_j + OT_j + X_j F_j + V \\
\text{SUBJECT TO:} \\
\sum_{j=1}^{n} W_{i,j} = 1 \quad \forall i = 1, \ldots, n \\
\sum_{i=1}^{n} W_{i,j} D_i \leq X_j \eta_j D_j \quad \forall j = 1, \ldots, n \\
0 \leq W_{i,j} \leq X_j \quad \forall i = 1, \ldots, n, j = 1, \ldots, n \\
X_j \in \{0,1\} \quad \forall j = 1, \ldots, n
\] (15')

If $X_j=1$, location $j$ is selected for centralization. The constraint set (17) describes the limitation on possible capacity expansion imposed through the input data $\eta_j$ for the locations if they are selected to hold the inventory. Constraint set (18') ensures that the proportion of demand from any location $i$ allocated to location $j$, can be positive only if location $j$ is selected. We removed the constraint set (19) as the increase in outbound transportation cost enforces that the locations serve their own demand if they are selected. This constraint can be added if the increase in outbound transportation cost is not significant and the allocation of demand originated at some selected locations to other locations is not desirable for other managerial reasons.

**IV. Experimental Results**

In this section, we outline the result of the computational tests with the two models. We experimented with a program written in GAMS. The nonlinear solvers exploit the convexity of the terms under the big square root and solve the problems to optimality in a fraction of a
For analysis in the ten cases listed below, we solved problems consisting of seven decentralized locations \((n = 7)\) to be considered for consolidation. In experimentation with model 1, four locations are pre-selected \((m = 4\) locations). No cost is explicitly involved and we will examine the impact of demand and supply characteristics on the safety stock consolidation. In experimenting with model 2, the optimal number \((m)\) will be determined by the optimization and the centralized locations are selected to hold the optimally allocated inventory. Many cost tradeoffs and demand and supply parameters collectively influence the selection and allocation decisions and impact the overall cost saving. It is very difficult to measure and report on the impacts in all the combinations. We will define a base case for each experiment by setting equal levels for almost all the parameters at all locations and create seven cases: three cases for model 1 and four cases for model 2, by variation of one parameter of interest at a time to see the impact. The equal settings for all the locations might diminish the magnitude of improvements due to consolidation. The purpose of these tests is not to show the magnitude of improvement as much as it is the sensitivity of the decisions to change in the parameters. Table 1 and Table 2 will summarize the results for an instance in each case.

**Model 1 (The allocation problem):**

**Base case:** In our base case for location \(i\), we set the mean daily demands equal to \(200+10\ i\) and the mean lead times equal to \(18 + i\) days. We assume the demand standard deviations are equal to 10\% of mean daily demands. The standard deviations in lead time are also assumed to be equal to 10\% of mean lead times. The safety stock factor of 2.0 is required for all the centralized locations. The coefficients for the demand correlation were generated randomly from \(U(-0.4, 0.3)\). We used a C program to test the positive semi-definiteness of our input correlation matrices making sure they are symmetric positive semi definite and all diagonal elements are equal to 1. The coefficients of correlation between demand and lead time are set equal to 0.1 for all the locations. Finally, we assumed that capacity of each location if selected can be expanded to support twice its original demand \((\eta_j = 3\) for all \(j=1,...,n\)). The locations selected after solving the base case, are 1, 2, 3, and 4. The optimal allocation is provided below.
### Case 1:

In this case we examine the impact of increased variation in replenishment lead times by changing the standard deviation for all the locations from 10% of mean lead time to 30% of mean lead time. The optimal overall safety stock level increases from 6541 for the base case to 19431 units. The safety stock reduction due to consolidation in this case is equal to 11% which is 1% more than the base case. The allocation is changed and location 1 serves more demand from 5, 6, and 7 as before 1, 2, 3, 4, also serve their own demands.

<table>
<thead>
<tr>
<th></th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.522</td>
<td>0.522</td>
<td>0.569</td>
</tr>
<tr>
<td>2</td>
<td>0.478</td>
<td>0.478</td>
<td>0.431</td>
</tr>
</tbody>
</table>

### Case 2:

In this case we examine the impact of increased correlations between demand and lead time at each location. For the base case we used 0.1 at all the location and 10% lead time deviation. For this case we used the high correlation coefficient 0.9 at all the locations and applied the 30% deviation. Comparing with case 1, the same locations are selected but the allocation is changed and the extra saving due to high lead time deviations is reduced back to provide 10.1% improvement as in case 1.

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<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.478</td>
<td>0.478</td>
<td>0.431</td>
</tr>
<tr>
<td>2</td>
<td>0.522</td>
<td>0.522</td>
<td>0.569</td>
</tr>
</tbody>
</table>
Case 3:
In this case we investigate the impact of increased demand correlations. For the base case and the instance of case 3, we randomly generated and checked the following coefficient matrices for positive semi definiteness.
As we can see in Table 1, the high correlation between demands is increasing the safety stock requirement at the centralized location and the saving in safety stock has decreased by 18%.

<table>
<thead>
<tr>
<th>Test Cases</th>
<th>Decentralized Safety Stock</th>
<th>Centralized Safety Stock</th>
<th>Percent Reduction In Safety Stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>7278</td>
<td>6541</td>
<td>10.1</td>
</tr>
<tr>
<td>Case 1</td>
<td>21834</td>
<td>19431</td>
<td>11</td>
</tr>
<tr>
<td>Case 2</td>
<td>21392</td>
<td>19226</td>
<td>10.1</td>
</tr>
<tr>
<td>Case 3</td>
<td>7278</td>
<td>6672</td>
<td>8.32</td>
</tr>
</tbody>
</table>

Table 1: The result of experiment with Model 1

**Model 2 (The selection and allocation problem):**

**Base case:** For the cost optimization model, in addition to the parameters described for the Model 1 base case, we set the cost parameters as follows:

For all the locations, the per unit out bound transportation cost is equally set to $10 with a $2 per unit increase due to satisfying the demand originated to $i$ by location $j$ ($e_{ij} = 2$ & $e_{jj} = 0$). The base variable in bound transportation cost is set equal to $10 and increases by a factor of 0.05($m$). We also equally assigned fixed location cost of $2000, fixed per order cost of $50, fixed shipping & handling cost of $30 ($f = 30$), and carrying charge of $50 (25% of the inventory value for annual holding cost) for all the locations.

The optimal selection is 1, 2, and 7 and the overall cost improvement is 48%. These locations serve their own inventory additional allocation from the rest of the locations is shown bellow:
Case 4:
In this case we examine the impact of increased the inventory value on the selection and allocation decisions. We increase the per unit item value from $200 to $2000 while holding the other cost parameters constant. As in the base case the 25% annual carrying rate was used, the per unit holding cost increases from $50 to $500. To allow the optimization for taking this impact we increased the capacity expansion factor for all the locations to from 3 to 4. The optimal cost information is provided in table 3. Note that the optimal locations selected is changed from 1,2, and 7 to 1 and 2. The allocation below shows that in addition to its own inventory location 1 serves the demand for location 6 and 7 at 100% and covers 40 % of demand at location 5. Location 2 covers for its own and the demand at 3,4 at 100% it also serve 60% of demand at 5.

<table>
<thead>
<tr>
<th>i</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>j</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.125</td>
<td>1</td>
<td>0.538</td>
</tr>
<tr>
<td>2</td>
<td>0.875</td>
<td></td>
<td>0.462</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Case 5:
In this case we examine the impact of out bound transportation cost increase as a result of demand allocation. We increase the additional cost from $2 to $50 while holding the other cost parameters constant. The optimal cost information is provided in table 3. In comparison with the base case result, the number of location has increased and Locations 1,2,3,5, and 7 are selected to hold the inventory. As seen in the optimal allocation below, the increase in transportation cost

<table>
<thead>
<tr>
<th>i</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>j</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td></td>
<td>0.4</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>0.6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
forces the locations to satisfy their own demand \( W_{i,i} = 1 \). The cost of safety stock and inbound transportation are small in comparison with the cost of transportation.

**Case 6:**
The objective for this case is to see how the solution might change if the inbound transportation cost due to reduction of number of locations decreases significantly. We changed the inbound transportation cost change factor associated with having an additional location, from 5% to 25% for all the location while holding the other cost parameters constant. The selected location and the allocation have changed. Now location 1, 2, and 3 are selected. The cost information is provided in table 3. We can see that the change in inbound transportation cost has significantly increased the cost saving due to consolidation.

**Case 7:**
Finally, we consider how the solution might change if the fixed cost of placing an order decreased from $50 per order to $500 per order. The result is summarized in Table 2.

<table>
<thead>
<tr>
<th>Cases</th>
<th>Locations Selected</th>
<th>$ Before consolidation</th>
<th>$ After consolidation</th>
<th>% Cost reduction</th>
<th>$ Safety stock</th>
<th>$ Cycle stock</th>
<th>$ Outbound Transportation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>1,2,7</td>
<td>27475380</td>
<td>1417525</td>
<td>48.4 %</td>
<td>340746</td>
<td>6167708</td>
<td>6753600</td>
</tr>
<tr>
<td>4</td>
<td>1,2</td>
<td>25704920</td>
<td>11660240</td>
<td>54.6 %</td>
<td>3191522</td>
<td>6359075</td>
<td>6782400</td>
</tr>
<tr>
<td>5</td>
<td>1,2,3,5,7</td>
<td>27475380</td>
<td>1764553</td>
<td>35.7 %</td>
<td>347025</td>
<td>6200504</td>
<td>9576000</td>
</tr>
<tr>
<td>6</td>
<td>1,2,3</td>
<td>86745780</td>
<td>17811310</td>
<td>79 %</td>
<td>325143</td>
<td>6167708</td>
<td>6782400</td>
</tr>
<tr>
<td>7</td>
<td>1,2,7</td>
<td>27764780</td>
<td>14380590</td>
<td>48.2 %</td>
<td>365953</td>
<td>6347834</td>
<td>6753600</td>
</tr>
</tbody>
</table>

Table 2: The result of experiment with Mode2

To see the impact of changes in cost parameters in the allocation decision we changed that parameter for one location significantly keeping the rest of location as in the base case. The model reacts by allocating the inventory as much as possible to the locations with the lower cost and if the change is significant enough it removes the location from the selection.
V. Conclusion and future directions

We presented an optimization models (Model 1) for analyzing the impact of physical consolidation in different practical cases. Model 1 deals with the cases where the existing managerial insight has led one to designated locations. In these cases the purpose is to realize the service level with the least overall safety stock. This is done by serving the right amount at the right location while the physical constraints such as limited capacity extensions are not violated. This model can be used as a decision support tool. We solved small instances of the problem to verify the impact of lead time variability, and correlation between demands and concluded that the two factors impact the improvement in the opposite directions. We also concluded that the increased correlation between demand and lead time decreases the improvement only when the lead time variation there is significant.

We then proposed a more comprehensive cost optimization model (Model 2) for the selection and physical consolidation of inventory. This model incorporates safety and cycle stock inventory costs at the stocking locations, the economies of scale in the shipping cost charged by the suppliers as well as the impact on the outbound transportation costs. We solved small instances of the optimization problem to verify the impact of change in inventory cost, ordering cost, in bound transportation cost, and out bound transportation cost. These problems each were solved in a fraction of second through the use of non linear solver DICOPT. As the selection and allocation decisions are influenced by a combination of factors and cost components reporting the sensitivity of the model to all the combination of parameter changes is not possible. The increase in transportation cost increased the number of locations from three in the base case to five. High inventory cost caused the selection of two locations instead of three in the base case. The combination of high inventory cost and low outbound transportation cost will lead to smaller number of locations and more overall cost savings. Change in these cost parameters impact the decisions in opposite directions. To see the impact of changes in cost parameters in the allocation decision we changed that parameter for one location significantly keeping the rest
of location as in the base case.

To consider the realistic conditions, it is important to test the solution with the actual data to confirm that the model is accurately capturing the relevant costs especially the capture of inbound and out bound transportation cost changes as a function of number of locations. We would also like to report on the run time efficiency by solving larger size problems. It would be nice to get a sense for the overall systems fill rate after consolidation in cases where different locations require different safety factors.

References


