007-0025

The existence of the core in a three-echelon supply chain

Takamichi Hosoda

Logistics Systems Dynamics Group, Cardiff Business School, Cardiff University, Aberconway Building, Colum Drive, Cardiff CF10 3EU, Wales, UK

hosodat@cardiff.ac.uk

POMS 18th Annual Conference
Dallas, Texas, U.S.A.
May 4 to May 7, 2007

Assuming a stochastic external market demand, this paper studies the existence of the core in a serially linked three-echelon supply chain where payoffs depend on the entire coalition structure. Each player's cost is represented by the infinite horizon standard deviation of the net stock levels. To represent the activity of a player in a supply chain, the generalized order-up-to policy proposed by Hosoda and Disney [7] is exploited. It is shown that even though the grand coalition can produce a large cost reduction to the overall supply chain (26%), the existence of the non-empty core depends on the characteristics of the external market demand and replenishment lead-times. A stable implementable coalition structure is also presented along with its benefits.

---

1 Introduction

In this paper, we examine the issue of stability in the case of the serially linked three-echelon supply chain under the existence of positive externalities. It has been advocated that to improve the multi-echelon supply chain performance, cooperation between players in a supply chain is essential. From a system optimization perspective, it might be quite reasonable to assume that a sequence of optimum policies does not bring a globally optimum solution. This means that at least one player in a multi-echelon supply chain should display altruistic behavior, employing a different policy which is not optimal for itself, but optimal for global performance. The benefit depending on this altruistic behavior should be then redistributed among the participants in the proper manner. Exploiting discrete control theory, Hosoda and Disney [7] have shown that in a two-echelon serial supply chain, the altruistic behavior at the first echelon enables the second echelon player to develop enough benefit (i.e. reduction in inventory related costs) to compensate for the loss at the first

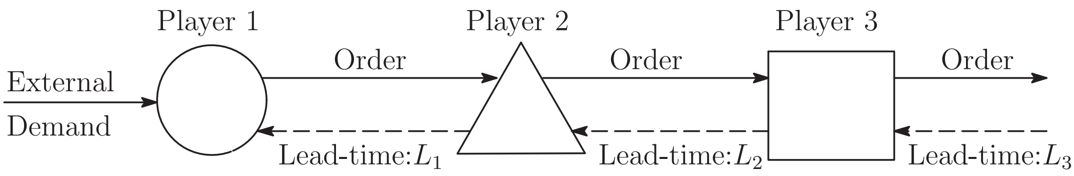

Figure 1: Three-echelon serial supply chain model

echelon. They use the Generalized Order-Up-To (GOUT) policy, that may be described as a Traditional Order-Up-To (TOUT) policy (e.g. [8, 14]) with a single proportional controller, F, added to the inventory and the Work In Progress (WIP) feedback loop.

As stated by Hosoda and Disney [7], the success of the altruistic behavior is completely dependent on the redistribution of the cost savings. In the case of a two-echelon supply chain, if the second echelon player becomes a free-rider (i.e. refusing to redistribute its cost savings), the first echelon player has no incentive to be altruistic so that the first echelon player will change its ordering policy to the TOUT policy, which guarantees local cost minimization, as shown in [8] and [14]. As a result of this policy change by the first echelon player, the cost for the second echelon player will be back to the original level. Therefore, there are no incentives for the second echelon player to be a free-rider. In this sense, the two-echelon supply chain with the GOUT policy has a "self-forcing" structure.

In general, a natural requirement for the stability, which is common to any type of cooperation schemes seems to be that no one can achieve higher benefits by not being involved in the cooperation scheme. Under such conditions neither player therefore has a unilateral incentive to change his strategy. If the grand coalition has such an equilibrium, or in other words, such a self-forcing structure, this benefit-based relationship is stable without any additional enforcement scheme. Otherwise, even though the grand coalition provides the overall supply chain with the greatest benefit, that coalition may be dissolved and will shift to a new subcoalition where the overall benefit might be less than that of the grand coalition, but at least one player enjoys a better payoff. In this paper, assuming that the inventory related cost is the only concern for each player, we study the issue of stability in the grand coalition of a serially linked three-echelon supply chain. To address this issue, we exploit the established cooperative game solution concept, i.e. the core.

The GOUT policy is used to represent each player's activity. We assume that the sole interest of each player is to minimize its own inventory related cost, represented by the stable standard deviation of the net stock levels. Direct neighbors are linked together by nature (Figure 1) and this linkage does not change under any coalition structures. While maintaining this linkage constrain, each player may form a new subcoalition. Therefore, both the cost to each player and the stability concepts depend on the entire coalition structure in a supply chain; externalities exist in our model.

Each player will only deviate from the grand coalition and cooperate with other player(s) if its own cost will decrease as a result of this deviation. Further bargaining occurs among the participants of each subcoalition on how to redistribute what they obtained together. Our model, therefore, can be regarded as a two-stage game in coalitional form with a transferable utility, where in the first stage players form smaller subcoalitions and then bargaining occurs among the players, given the coalition structure determined in the first stage. This framework is quite common when externalities are considered (for example, see [2, 13, 17]).

We allow the forming of a subcoalition by two players who are not directly linked together; the coalition by the first and the third players. Under this "indirect cooperation" scheme, since the second player is still located between them, the first player should place its order to the second player and the third player's demand will come from the second player. This structure can be observed in a real supply chain. For example, in a retail supply chain, it is not unusual that the retailer (the first player) and the manufacturer (the third player) form a coalition to cooperate with each other while the distributor (the second player) focuses on only its own interests.

The altruistic behavior achieved by the GOUT policy mitigates the bullwhip effect [7]. Usually, a lower bullwhip brings better benefits to the upper echelon players (for example, see [9, 10]). Thus, the externalities in our model are positive. Yi [17] points out that if a coalition creates positive externalities on non-members, there might be free-riding problems. Therefore, we may reasonably conclude that the second or third echelon player[1], who is a non-member of another subcoalition (i.e. $\{1, 3\}$ or $\{1, 2\}$, respectively) may have reasonable incentives to free-ride on the positive externalities and may achieve a lower cost without any cooperation with others. Furthermore, this free-riding might be the lowest cost strategy for the second or third player.

It might be useful to now address the reason why we consider a three-echelon supply chain, instead of a one- or two-echelon supply chain. It has been recognized that in a

[1] The first echelon player may not have any incentives to form a subcoalition $\{1\}$ by deviating from the grand coalition since even he does so, he still faces the same external market demand and there will be no additional benefits for him.

transferable utility game, if the number of players is greater than two, an equilibrium might be provided by a different structural form from that of the one- or two-echelon supply chain. Using the strategic form on an income redistribution game, Nakayama [11] has shown that if the number of players is greater than two, the equilibrium point is described as a Nash equilibrium, however, it will never be a Pareto equilibrium since at least one player will be a "free-rider". This result might imply that the grand coalition does not have an equilibrium in a game of more than 2 players.

The structure of this paper is as follows: after a literature review in the next section, the model we have exploited will be described in Section 3. Section 4 describes numerical investigations under specific conditions. In Section 5, we will discuss the stability of a three-echelon serial supply chain.

2 Literature Review

There are several research studies that have examined supply chain stability using a game theory approach. For example, Wang and Parlar [16] and Hartman *et al.* [5] have studied a one-echelon case. Cachon and Zipkin [4], Cachon [3] and Wang *et al.* [15] have examined a two-echelon case. Zijm and Timmer [18] might be the only research dealing with a three-echelon case so far. To the best of our knowledge, our research might be the first attempt to analyze the stability of a three-echelon serial supply chain with the question of redistribution of payoffs among all the participants of a coalition in the context of positive externalities.

Using a horizontally located three retailer supply chain model,Wang and Parlar [16] investigate the optimal ordering policies for substitutable products in both non-cooperative and cooperative situations. In the cooperative game, it is assumed that retailers cooperate by switching excess inventory among them. An incentive for this cooperation is the savings of the backlog penalty cost, which results in lower inventory related costs for each player. They consider the stability of a grand coalition case and show conditions for the non-empty core when side payments are and are not allowed. Expanding the Newsboy Problem, Hartman *et al.* [5] consider nonempty core conditions for a centralized nonidentical N retailer supply chain model. It is shown that for any combination of normally distributed individual demands, the cost allocation is in the core of a cooperative game.

Some researchers have exploited a strategic form to investigate the stability of a supply chain. Cachon and Zipkin [4] investigate a two-echelon serially linked base stock policy supply chain and show that the games nearly always have a unique Nash equilibrium, which differs from the optimal solution. They assume that each player's concern is to minimize its own inventory related costs. Using a parameter (called α in their paper) which determines the distribution ratio of a consumer backlog penalty cost among these two players, the cost difference between a Nash equilibrium and the system optimal solution is shown. When $\alpha = 0.5$, which means that the backlog cost will be equally split, the Nash equilibrium is quite close to the system optimal solution. Cachon [3] examines a two-echelon supply chain with one supplier and N retailers case. All retailers incur local inventory holding costs and backlog costs. The supplier's concern is to minimize its holding cost and its backlog penalty cost charged for the backlog at the retailers. The objective of this system is to minimize the total inventory holding plus backlog penalty costs. It is assumed that all players in this system exploit the (R, nQ) policy. It is shown that the supply chain optimal reorder points are frequently not a Nash equilibrium, and the benefit of the supply chain cooperation is context specific, depending on the degree of preference toward consumer backlogs. Wang *et al.* [15] study a one supplier and N retailers supply chain model, which is similar to the model of [3]. The uniqueness is that Wang *et al.* [15] assume that each retailer has its own lead-time and holding cost. Furthermore, they consider the case when the supplier has an insufficient supply. Using a numerical study, several contracts, which produce a Nash equilibrium, are proposed.

Zijm and Timmer [18] have investigated the stability of a three-echelon supply chain model using a concept of the Nash equilibrium. In their model, each player's activity is described by the choice of the base stock level, which is assumed to be time invariant. All players are selfish; the minimization of its own inventory cost is the single concern for each player. A lower cost for a coordination consisting of two directly linked players can be achieved by increasing the base stock level at the upstream player. This higher cost of the upstream player then will be fully compensated by the downstream player with additional side payments. In their research, the three-echelon model is considered as a sequence of two two-echelon models. They conclude that the global optimum will be achieved by a Nash equilibrium and all players will be better off.

3 The Model

A periodic review system, backordering, and constant lead-times are assumed. The sequence of events at each echelon is as follows: at the beginning of a period, the replenishment orders placed earlier are received, the demand is fulfilled, the inventory levels and the WIP are reviewed and an ordering decision is made at the end of the period. All model parameters including these for the external market demand and policies used by the players are common knowledge. It is assumed that when players decide to form a new subcoalition, they will take into account the reaction of the non-members to the formation of the subcoalition. Thus, for example, before one player is going to deviate from the grand coalition, this deviating player considers that two residual players will form a new subcoalition since this is the only alternative for these residual players to minimize their costs.

To explain the GOUT policy, let us begin by reviewing the TOUT policy.

$$\begin{aligned} O_t &= \hat{D}_t^L - (WIP_t + NS_t) \\ &= \acute{D}_t^L + \hat{D}_t^{L-1} - (WIP_t + NS_t) \\ &= \acute{D}_t^L + (\hat{D}_t^{L-1} - (WIP_t + NS_t)) \\ &= \acute{D}_t^L + (DIP_t - (WIP_t + NS_t)), \end{aligned}$$

where $\acute{D}_t^L$ is the conditional Minimum Mean Square Error (MMSE) estimate of the demand in time period $t + L$ made at time period t. Therefore, $\hat{D}_t^L = \acute{D}_t^L + \hat{D}_t^{L-1}$, where $\hat{D}_t^L$ is the conditional MMSE estimate of the demand over the lead-time L. DIP_t is the Desired Inventory Position at time period t, that can be described as $\hat{D}_t^{L-1}$. Note that $DIP_t = 0$, if $L = 1$; $DIP_t = \hat{D}_t^{L-1}$, if $L > 1$. The WIP at time t, WIP_t, can be expressed as;

$$WIP_t = \begin{cases} 0 & \text{if } L = 1, \\ \sum_{i=1}^{L-1} O_{t-i} & \text{otherwise.} \end{cases}$$

Incorporating a proportional controller, F, into (1) yields a GOUT policy

$$O_t = \acute{D}_t^L + F(DIP_t - (WIP_t + NS_t)),$$

where $0 < F < 2$ as shown in [6]. Obviously, if $F = 1$, the GOUT policy is identical to the TOUT policy. By manipulating F, the GOUT policy can describe each player's activity policy. A non-cooperative activity is considered by setting $F = 1$, since it minimizes the local inventory related cost.

Table 1: Cost function notations

Whole coalition structure	Cost function notation	Corresponding coalition
$\{\{1,2,3\}\}$	$v(123)$	$\{1,2,3\}$
$\{\{1\},\{2,3\}\}$	$v(1)$	$\{1\}$
	$v(23)$	$\{2,3\}$
$\{\{2\},\{1,3\}\}$	$v(2)$	$\{2\}$
	$v(13)$	$\{1,3\}$
$\{\{3\},\{1,2\}\}$	$v(3)$	$\{3\}$
	$v(12)$	$\{1,2\}$
$\{\{1\},\{2\},\{3\}\}$	$v1$	$\{1\}$
	$v2$	$\{2\}$
	$v3$	$\{3\}$

Let $N = \{1, 2, 3\}$ be the set of players. An arbitrary player $i \in N$ who is the player in echelon i, has strategy space S_i, and let s_i denote an arbitrary member of this strategy space. Each player's strategy space, therefore, can be represented as $0 < S_i < 2$, in which case a certain strategy s_i is a quantity choice F_i. A coalition structure $\mathcal{P}$ is a partition of N, and each element of $\mathcal{P}$ is called a coalition. We will use $\mathcal{R}(\mathcal{P})$ to denote all coalitional structures that are refinements of $\mathcal{P}$. In a three-echelon case, $\mathcal{R}(\{\{1, 2, 3\}\})$, the set of refinements of the grand coalition, has the following four coalition structures:

$$\{\{1\},\{2,3\}\},\ \{\{2\},\{1,3\}\},\ \{\{3\},\{1,2\}\},\ \{\{1\},\{2\},\{3\}\}.$$

It might be useful to mention that the case of $\{\{1\}, \{2\}, \{3\}\}$ has been studied in [8].

3.1 The Grand Coalition and its Equilibrium

Since our central feature is whether the grand coalition has an equilibrium, we will only consider the case where a new subcoalition is created by the dissolving of the grand coalition. The cases considered here, therefore, are: Case 1: a breakaway by a single player with two residual players, Case 2: a joint breakaway by two players with a single residual player, and Case 3: an individual breakaway by three players for independent optimization. Table 1 shows the cost function notations used herein to represent the minimized inventory related costs for each coalition.

Theorem 1 *An equilibrium, the core, is given, when the grand coalition meets the following* *criteria:*

$$\begin{aligned} v(1)+v(2)+v(3) &\geq v(123), \quad (1)\\ \frac{1}{2}\left(v(12)+v(23)+v(13)\right) &\geq v(123), \quad (2)\\ v1+v2+v3 &\geq v(123). \quad (3) \end{aligned}$$

The proof is shown in the Appendix.

3.2 External Market Demand Pattern

For the external market demand pattern, we assume an AR(1) pattern. This assumption is common when an autocorrelation exists among the demand process. The AR(1) process can be expressed as,

$$X_t = d + \rho(X_{t-1} - d) + \varepsilon_t,$$

where X_t is the external market demand at time period t, d is the mean demand, ρ is the autoregressive coefficient, $|\rho| < 1$, and ε_t is an i.i.d. white noise process with a mean of zero and a variance of σ_ε^2. Note that this white noise process can be drawn from any continuous distribution, e.g. normal, gamma, uniform, etc. In this paper, we set $d = 0$ without any loss of generality since we are studying a linear system. Setting $\rho = 0$, therefore, we can have an i.i.d. white noise external demand. It should be noted that we do not assume any specific types of probabilistic distributions on ε_t in this research study.

3.3 Cost Functions and the Values of F

Using the *Principle of Optimality* [1], Hosoda and Disney [7] show that to minimize the objective function of a multi-echelon supply chain (i.e. the sum of the net stock level standard deviations), the highest echelon player i should set its value of F_i to unity. Therefore, in a supply chain, both the player at the highest echelon in a coalition and the player who does not cooperate with anybody have a unity of F to minimize the objective function for the coalition and its own local cost, respectively. From this, in the case of a three-echelon supply chain, we can conclude that F_3 should be always unity. Thus, we have the following lemma.

Lemma 1 *F_3 is always unity under any coalitional strategies and F_i is always unity if the player i is not cooperative with any other players.*

In the rest of this paper, we will set $F_3 = 1$.

Let us use $C_i(\cdot)$ to represent the inventory related cost at echelon level i in the supply chain. Since we assume that $C_i(\cdot)$ is the stable standard deviations of the net stock levels at echelon level i, $\sqrt{Var[NS_i]}$, we will have the following:

$$\sqrt{Var[NS_1]} = C_1(F_1|\rho, L_1, \sigma_\varepsilon^2) = \sqrt{\left(\begin{array}{l} \frac{\left(L_1(1-\rho^2) + \rho(1-\rho^{L1})(\rho^{L_1+1} - \rho - 2)\right)}{(1-\rho)^2(1-\rho^2)}\sigma_\varepsilon^2 + \\ \frac{(\rho^{L_1}-1)^2(F_1-1)^2}{(\rho-1)^2(2-F_1)F_1}\sigma_\varepsilon^2 \end{array} \right)}, \tag{4}$$

$$\sqrt{Var[NS_2]} = C_2(F_2|F_1, \rho, L_1, L_2, \sigma_\varepsilon^2) = \sqrt{\left(\begin{array}{l} \frac{\sigma_\varepsilon^2}{(\rho-1)^2}\left(L_2 + \frac{\left((1-F_1)^{2L_2}-1\right)(\rho^{L_1}-1)^2(1-F_1)^2}{(1-F_1)^2-1} + \right. \\ \frac{\rho^{L_1+1}\left(\rho^{L_2}-1\right)\left(\rho^{L_1+L_2+1}+\rho^{L_1+1}-2\rho-2\right)}{\rho^2-1} + \\ \left. 2(\rho^{L_1}-1)(1-F_1)\left(\frac{1-(1-F_1)^{L_2}}{F_1} - \frac{\rho^{L_1+1}\left((\rho(1-F_1))^{L_2}-1\right)}{\rho(1-F_1)-1}\right)\right) + \\ \frac{\left(\left(1-(1-F_1)^{L_2}\right) - \rho^{L_1}\left(\rho^{L_2}-(1-F_1)^{L_2}\right)\right)^2(F_2-1)^2}{(\rho-1)^2(2-F_2)F_2}\sigma_\varepsilon^2, \end{array} \right)}, \tag{5}$$

$$\sqrt{Var[NS_3]} = C_3(1|F_1, F_2, \rho, L_1, L_2, L_3, \sigma_\varepsilon^2) = \sqrt{\left(\begin{array}{l} \xi^2 \cdot \sigma_\varepsilon^2 + \\ \sum_{r=2}^{L_3}\left(\left(\sum_{i=2}^{r}\left(\frac{-1}{(F_1-1)(\rho-1)\rho}\right.\right.\right. \\ \left((1-F_2)^{i-1}\left((F_1-1)(F_2-1)(\rho-1)\rho^{L_1+L_2}\left(\frac{\rho}{1-F_2}\right)^i - \right.\right. \\ \rho \cdot F_1(1-F_1)^{L_2}\left(\frac{F_1-1}{F_2-1}\right)^i(F_2-1)\left(\rho^{L_1}-1\right) + \\ \left.\left.\left.\left.\left.\rho \cdot F_2(F_1-1)\left(1-(1-F_1)^{L_2}+\rho^{L_1}((1-F_1)^{L_2}-\rho^{L_2})\right)\right)\right)\right) + \xi\right)^2\right)\sigma_\varepsilon^2 \end{array} \right)}, \tag{6}$$

where

$$\xi = \frac{(1-F_1)^{L_2}(F_2-F_1)(1-\rho^{L_1}) + \rho^{L_1+L_2}(\rho+F_2-1) - F_2}{\rho - 1}.$$

Details of the process to reach (4), (5), and (6) are shown in [6]. It should be noted that (6) cannot be used when $F_1 = 1$, $F_2 = 1$, and/or when $\rho = 0$ because of a singularity in the denominator. However, solutions do exist at the singularity, and they are also shown in [6].

Here, using expressions $C_1(\cdot)$, $C_2(\cdot)$, and $C_3(\cdot)$ shown above, we will describe the cost function for each coalition, given the whole coalition structure.

3.3.1 The case of $\{\{1,2,3\}\}$

The cost function of the grand coalition is expressed as

$$\begin{aligned} v(123) &= C_1(F^*_{1,v(123)}|\rho, L_1, \sigma^2_\varepsilon) + \\ &\quad C_2(F^*_{2,v(123)}|F^*_{1,v(123)}, \rho, L_1, L_2, \sigma^2_\varepsilon) + \\ &\quad C_3(1|F^*_{1,v(123)}, F^*_{2,v(123)}, \rho, L_1, L_2, L_3, \sigma^2_\varepsilon), \end{aligned}$$

where $F^*_{1,v(123)}$ and $F^*_{2,v(123)}$ are the solutions of

$$\min_{F_1,F_2} \left[C_1(F_1|\rho, L_1, \sigma^2_\varepsilon) + C_2(F_2|F_1, \rho, L_1, L_2, \sigma^2_\varepsilon) + C_3(1|F_1, F_2, \rho, L_1, L_2, L_3, \sigma^2_\varepsilon)\right].$$

3.3.2 The case of $\{\{1\},\{2,3\}\}$

The first echelon player can achieve its objective by setting $F_1 = 1$, since it is selfish player. Thus, we have $v(1) = C_1(1|\rho, L_1, \sigma^2_\varepsilon)$. The coalition consisting of the second and the third players will try to achieve the following condition;

$$\min_{F_2} \left[C_2(F_2|1, \rho, L_1, L_2, \sigma^2_\varepsilon) + C_3(1|1, F_2, \rho, L_1, L_2, L_3, \sigma^2_\varepsilon)\right].$$

Using the solution of the objective function shown above, $F^*_{2,v(23)}$, we can describe $v(23)$, which is

$$v(23) = C_2(F^*_{2,v(23)}|1, \rho, L_1, L_2, \sigma^2_\varepsilon) + C_3(1|F^*_{2,v(23)}, \rho, L_1, L_2, L_3, \sigma^2_\varepsilon).$$

3.3.3 The case of $\{\{2\},\{1,3\}\}$

Under this whole structure, we set $F_2 = 1$ as the second echelon player is a selfish player. $v(2)$, therefore, can be represented as

$$v(2) = C_2(1|F^*_{1,v(13)}, \rho, L_1, L_2, \sigma^2_\varepsilon),$$

where $F^*_{1,v(13)}$ is the solution of

$$\min_{F_1} \left[C_1(F_1|\rho, L_1, \sigma^2_\varepsilon) + C_3(1|F_1, 1, \rho, L_1, L_2, L_3, \sigma^2_\varepsilon)\right].$$

Thus, $v(13)$ can be written as

$$v(13) = C_1(F^*_{1,v(13)}|\rho, L_1, \sigma^2_\varepsilon) + C_3(1|F^*_{1,v(13)}, 1, \rho, L_1, L_2, L_3, \sigma^2_\varepsilon).$$

3.3.4 The case of $\{\{3\},\{1,2\}\}$

Since the second echelon player is located at the highest echelon in the coalition $\{1,2\}$, we set $F_2 = 1$. The selfish third echelon player's cost can then be described as

$$v(3) = C_3(1|F^*_{1,v(12)}, 1, \rho, L_1, L_2, L_3\sigma^2_\varepsilon),$$

where $F^*_{1,v(12)}$ is the solution of

$$\min_{F_1}\left[C_1(F_1|\rho, L_1, \sigma^2_\varepsilon) + C_2(1|F_1, \rho, L_1, L_2, \sigma^2_\varepsilon)\right].$$

Hence, $v(12)$ becomes

$$v(12) = C_1(F^*_{1,v(12)}|\rho, L_1, \sigma^2_\varepsilon) + C_2(1|F^*_{1,v(12)}, \rho, L_1, L_2, \sigma^2_\varepsilon).$$

3.3.5 The case of $\{\{1\},\{2\},\{3\}\}$

In this traditional supply chain case, each player only focuses on its own cost minimization without any cooperation with others. All players, therefore, employ the TOUT policy which minimizes the local cost. The TOUT policy is established by setting $F_i = 1$ for all i. The cost function for each player can be written as

$$\begin{aligned} v1 &= C_1(1|\rho, L_1, \sigma^2_\varepsilon), \\ v2 &= C_2(1|1, \rho, L_1, L_2, \sigma^2_\varepsilon), \\ v3 &= C_3(1|1, 1, \rho, L_1, L_2, L_3, \sigma^2_\varepsilon). \end{aligned}$$

Further analytical results are difficult to present due to the rather unwieldy expressions of the cost functions so that we will exploit the numerical investigations in the next section.

4 Numerical Investigations

In this section, the model will be numerically investigated with two lead-time settings $L_1 = 2, L_2 = 2, L_3 = 3$ and $L_1 = 1, L_2 = 3, L_3 = 3$. $\sigma^2_\varepsilon = 1$ is assumed. The values of the objective function for the grand coalition

$$C_1(F_1|\rho, L_1, \sigma^2_\varepsilon) + C_2(F_2|F_1, \rho, L_1, L_2, \sigma^2_\varepsilon) + C_3(1|F_1, F_2, \rho, L_1, L_2, L_3, \sigma^2_\varepsilon) \quad (7)$$

have been plotted in Figure 2 with the restriction that $0 < F_i < 2$ when $\rho = -0.7, 0.0$ and 0.7 for both lead-time settings. From these figures, it can be seen that the shape is not purely convex, but there is a unique minimum value for the given values of ρ, L_1, L_2 and L_3.

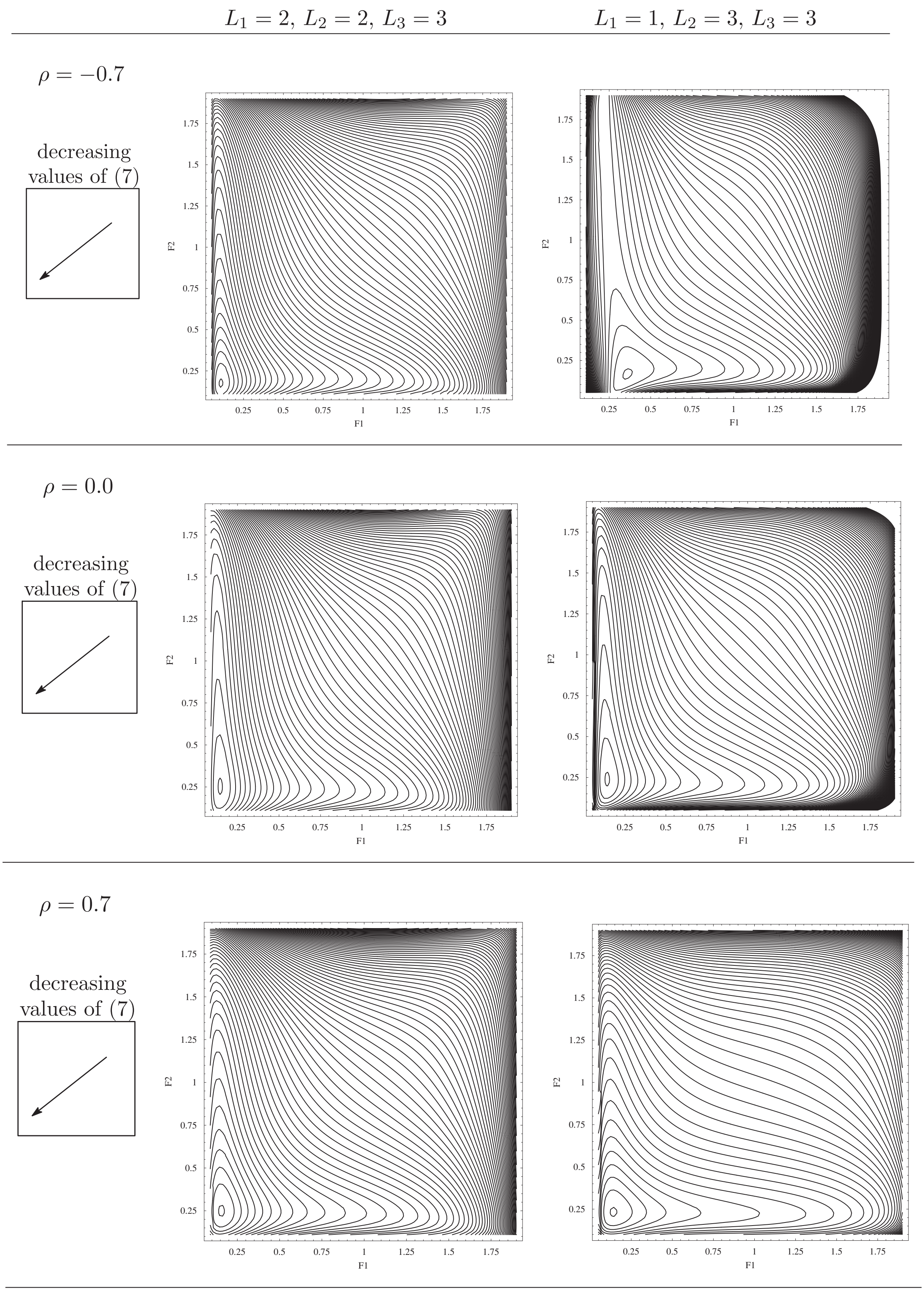

Figure 2: The shape of the objective function for $\{\{1, 2, 3\}\}$.

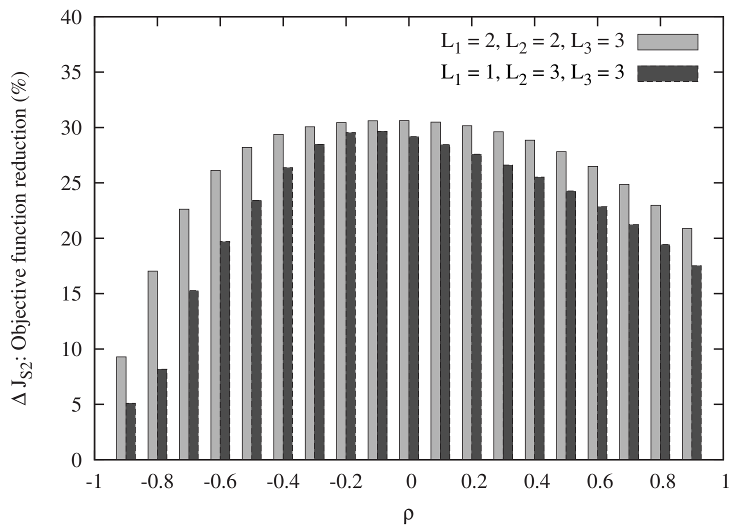

Figure 3: $\Delta v(123)$: Cost reduction by the grand coalition (%)

Before moving onto the issue of stability, let us show the benefit of the grand coalition. The measure of the cost reduction by the grand coalition, $\Delta v(123)$, where

$$\Delta v(123) = \frac{v1 + v2 + v3 - v(123)}{v1 + v2 + v3},$$

has been plotted in Figure 3. The average values of the $\Delta v(123)$ are 26.1% and 22.6% for the lead-time settings $L_1 = 2$, $L_2 = 2$, $L_3 = 3$ and $L_1 = 1$, $L_2 = 3$, $L_3 = 3$, respectively.

Now, the question is whether an equilibrium exists in the grand coalition. To have a feasible set of cost vectors in a supply chain, the conditions given by (1)-(3) must be met. Table 2 and Table 3 show the results. In the case of the lead-time setting $L_1 = 2$, $L_2 = 2$, $L_3 = 3$ (Table 2), it has been shown that when $-0.3 \leq \rho \leq 0.4$, the equilibrium does not exist even though the grand coalition has achieved a relatively higher cost savings (see, Figure 3). Let's have a look at the case $\rho = 0$ where $\Delta v(123)$ has achieved its maximum value (30.6%). The values of $v(1)$, $v(2)$, and $v(3)$ are 1.414, 0.380 and 1.262, respectively, which yields $v(2) < v(3) < v(1)$. This value of $v(2)$ is the acceptable maximum cost for the second player. We may conclude, therefore, that this very low value of $v(2)$ makes it difficult to set imputation vectors for the grand coalition. The best strategy for the second player, in this case, might be to deviate from the grand coalition and incur the cost of $v(2)$.

The reason for the lower values of $v(2)$ is the positive externalities created by the sub-coalition $\{1, 3\}$. The first player will mitigate the bullwhip to generate benefits at the third echelon. However, there is the second echelon player who is not a member of $\{1, 3\}$ in the middle of them, and basically, this middle player will increase the bullwhip, since s/he uses the TOUT policy for its own benefits. Knowing this increase, the first echelon player tries to mitigate the bullwhip as much as possible to compensate the loss at the second echelon. As a result, the demand faced by the second echelon player is smoothed well enough to enjoy such a very low value of $v(2)$.

However, in the case of lead-time setting $L_1 = 1$, $L_2 = 3$, $L_3 = 3$, the situation is different (see Table 3). The core exits at any values of ρ. Interestingly, under this setting, the value of $F^*_{1,v(13)}$ is unity. This means that even though the first echelon player does altruistic behavior, the benefit created at the third echelon player is not sufficient to compensate for the loss at the first echelon so that the best strategy for the subcoalition $\{1,3\}$ is to stop the cooperation and to act selfishly (i.e. use the TOUT policy), which can be yield by setting $F^*_{1,v(13)} = 1$.

When $F^*_{1,v(13)} = 1$, the GOUT policy employed by the first echelon player becomes the TOUT policy. Since the TOUT does not create any positive externalities, there are no incentives for the second echelon player to deviate from the grand coalition. Therefore, under the given setting, the unity value of $F^*_{1,v(13)}$ not only minimizes the total cost of the subcoalition $\{1,3\}$, but also dissuades the second echelon player from deviation, which brings stability to the grand coalition.

Table 2: The values of $v(\cdot)$ and F when $L_1 = 2, L_2 = 2, L_3 = 3$

Whole Structure		ρ −0.9	−0.8	−0.7	−0.6	−0.5	−0.4	−0.3	−0.2	−0.1	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
{{1, 2, 3}}	$F^*_{1,v(123)}$	0.073	0.096	0.108	0.117	0.124	0.130	0.135	0.139	0.143	0.146	0.148	0.150	0.151	0.153	0.154	0.155	0.156	0.156	0.157
	$F^*_{2,v(123)}$	0.105	0.147	0.174	0.195	0.213	0.228	0.238	0.245	0.249	0.250	0.249	0.248	0.247	0.246	0.246	0.247	0.248	0.249	0.250
	F_3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$v(123)$	2.814	2.489	2.339	2.302	2.339	2.427	2.554	2.717	2.918	3.164	3.463	3.832	4.291	4.869	5.608	6.562	7.806	9.439	11.587
{{1}, {2, 3}}	F_1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$F^*_{2,v(23)}$	0.137	0.178	0.198	0.209	0.217	0.222	0.225	0.226	0.227	0.227	0.227	0.227	0.228	0.229	0.230	0.232	0.234	0.236	0.238
	F_3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$v(1)$	1.005	1.020	1.044	1.077	1.118	1.166	1.221	1.281	1.345	1.414	1.487	1.562	1.640	1.720	1.803	1.887	1.972	2.059	2.147
	$v(23)$	1.934	1.722	1.660	1.683	1.756	1.860	1.991	2.150	2.342	2.576	2.862	3.214	3.653	4.208	4.921	5.846	7.057	8.654	10.764
{{2}, {1, 3}}	$F^*_{1,v(13)}$	0.070	0.097	0.116	0.132	0.145	0.155	0.164	0.172	0.178	0.183	0.187	0.191	0.194	0.196	0.198	0.200	0.202	0.204	0.206
	F_2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	F_3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$v(2)$	0.822	0.680	0.566	0.479	0.414	0.369	0.342	0.334	0.346	0.380	0.441	0.535	0.666	0.840	1.063	1.338	1.673	2.072	2.543
	$v(13)$	2.087	1.957	1.942	1.994	2.088	2.212	2.361	2.533	2.732	2.960	3.227	3.544	3.929	4.410	5.023	5.819	6.867	8.258	10.112
{{3}, {1, 2}}	$F^*_{1,v(12)}$	0.142	0.182	0.205	0.221	0.234	0.246	0.257	0.267	0.276	0.282	0.288	0.292	0.296	0.299	0.302	0.305	0.307	0.309	0.311
	F_2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	F_3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$v(3)$	1.082	0.925	0.860	0.845	0.860	0.899	0.957	1.036	1.136	1.262	1.422	1.630	1.903	2.270	2.767	3.446	4.375	5.646	7.378
	$v(12)$	1.850	1.762	1.724	1.726	1.762	1.824	1.910	2.019	2.152	2.311	2.501	2.727	2.993	3.304	3.664	4.079	4.554	5.094	5.704
{{1}, {2}, {3}}	F_1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	F_2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	F_3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$v1$	1.005	1.020	1.044	1.077	1.118	1.166	1.221	1.281	1.345	1.414	1.487	1.562	1.640	1.720	1.803	1.887	1.972	2.059	2.147
	$v2$	0.928	0.902	0.908	0.935	0.976	1.031	1.098	1.182	1.286	1.414	1.570	1.759	1.985	2.252	2.565	2.929	3.348	3.830	4.378
	$v3$	1.169	1.079	1.071	1.105	1.164	1.240	1.333	1.444	1.575	1.732	1.924	2.165	2.472	2.871	3.401	4.111	5.069	6.366	8.120
Inequality	(1)	√	√	√	√	√	√	n/s	n/s	n/s	n/s	n/s	n/s	n/s	n/s	√	√	√	√	√
	(2)	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
	(3)	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√

√: inequality is satisfied, n/s: inequality is not satisfied.

Table 3: The values of $v(\cdot)$ and F when $L_1 = 1, L_2 = 3, L_3 = 3$

Whole Structure		ρ: −0.9	−0.8	−0.7	−0.6	−0.5	−0.4	−0.3	−0.2	−0.1	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
{{1, 2, 3}}	$F^*_{1,v(123)}$	0.288	0.249	0.358	0.279	0.230	0.194	0.167	0.149	0.142	0.141	0.140	0.141	0.141	0.141	0.140	0.140	0.140	0.140	0.141
	$F^*_{2,v(123)}$	1.819	1.839	0.164	0.173	0.189	0.210	0.233	0.247	0.245	0.237	0.231	0.228	0.227	0.227	0.229	0.230	0.233	0.235	0.238
	F_3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$v(123)$	2.944	2.757	2.564	2.506	2.497	2.527	2.600	2.727	2.915	3.161	3.464	3.834	4.289	4.858	5.583	6.518	7.738	9.343	11.459
{{1}, {2, 3}}	F_1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$F^*_{2,v(23)}$	0.136	0.176	0.193	0.201	0.206	0.208	0.209	0.208	0.208	0.207	0.208	0.209	0.211	0.213	0.216	0.219	0.222	0.226	0.229
	F_3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$v(1)$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$v(23)$	1.939	1.741	1.700	1.750	1.851	1.987	2.150	2.341	2.563	2.824	3.132	3.502	3.954	4.517	5.232	6.156	7.364	8.955	11.058
{{2}, {1, 3}}	$F^*_{1,v(13)}$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	F_2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	F_3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$v(2)$	0.933	0.924	0.956	1.017	1.097	1.192	1.302	1.428	1.570	1.732	1.917	2.130	2.373	2.652	2.971	3.337	3.755	4.232	4.773
	$v(13)$	2.169	2.079	2.071	2.105	2.164	2.240	2.333	2.444	2.575	2.732	2.924	3.165	3.472	3.871	4.401	5.111	6.069	7.366	9.120
{{3}, {1, 2}}	$F^*_{1,v(12)}$	0.655	0.544	0.466	0.405	0.357	0.318	0.290	0.273	0.265	0.262	0.261	0.261	0.261	0.261	0.261	0.261	0.262	0.263	0.264
	F_2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	F_3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$v(3)$	1.157	1.036	0.984	0.964	0.962	0.975	1.012	1.084	1.195	1.345	1.534	1.773	2.080	2.479	3.010	3.722	4.683	5.985	7.746
	$v(12)$	1.905	1.844	1.812	1.805	1.818	1.854	1.916	2.007	2.127	2.278	2.458	2.671	2.917	3.203	3.532	3.910	4.341	4.832	5.389
{{1}, {2}, {3}}	F_1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	F_2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	F_3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$v1$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$v2$	0.933	0.924	0.956	1.017	1.097	1.192	1.302	1.428	1.570	1.732	1.917	2.130	2.373	2.652	2.971	3.337	3.755	4.232	4.773
	$v3$	1.169	1.079	1.071	1.105	1.164	1.240	1.333	1.444	1.575	1.732	1.924	2.165	2.472	2.871	3.401	4.111	5.069	6.366	8.120
Inequality	(1)	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
	(2)	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
	(3)	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√

√: inequality is satisfied, n/s: inequality is not satisfied.

5 Discussion and Conclusion

We have investigated the issue of stability of a cooperative three-echelon supply chain when positive externalities exist. A cooperative game solution concept, the core, is exploited herein. We have shown that under a specific market demand and lead-time settings, the grand coalition has no imputation vectors which meet the minimum requirement of all three players at the same time, even though the grand coalition achieves the high cost savings.

To be stable under any kind of setting, additional binding scheme, such that *KEIRETSU* might be necessary, irrespective of the market demand characteristics and the lead-time settings. Another way to obtain a stability in a three-echelon supply chain is to limit the cooperation only between direct neighbors, as Zijm and Timmer [18] assumed. When such two players cooperate with each other, the optimum value of F of the lower echelon player never has a unit value [6], which means that there must be a benefit for any values of ρ and the lead-time settings.

In addition, it might be useful to mention that the grand coalition depending on the altruistic behavior has another practical drawback; the standard deviation of the net stock levels at the first echelon will increase due to the altruistic behavior which is the key cooperation activity in the coalition. Since the level of safety stock is directly proportional to the standard deviation of the net stock levels, in the grand coalition, the volume of the net stock levels at the first echelon will increase. This higher volume of the safety stock level might be a very serious problem for retail stores where the stock space is limited and it is difficult to expand shelf and storage capacities.

To overcome this drawback, the coalition structure $\{\{1\}, \{2, 3\}\}$ is attractive. This coalition structure may be a case of a three-echelon supply chain that is governed by two organizations: the first echelon inventory is managed by a retailer, and both the second and third echelon inventories are managed by a supplier, for instance. This VMI type supply chain can be seen in the UK grocery supply chain (e.g. [12]). Under the coalition structure $\{\{1\}, \{2, 3\}\}$, the retailer focuses on minimization of the local inventory related cost at the store level. The supplier, on the other hand, manages its inventories at both the retailer's warehouse (the second echelon) and its own (the third echelon), and his concern is the minimization of the aggregate inventory related cost. To achieve the goal independently, the retailer may use the TOUT policy, which minimizes its local standard deviation of the net stock level, and the supplier exploits the GOUT policy at the second echelon and employs the TOUT policy at the third echelon to minimize its total inventory related costs. If the following expression

$$\Delta v(23) = \frac{v2 + v3 - v(23)}{v2 + v3}$$

is used to measure the benefit for the subcoalition $\{2, 3\}$, this coalition can enjoy 16.6% and 16.8% cost reductions on average for each lead-time setting, respectively. Since it is the supplier who plays the role of the altruistic behavior and also it is the supplier who enjoys the benefit from this behavior, the structure $\{\{1\}, \{2, 3\}\}$ may be more acceptable to a real business world than the grand coalition structure.

A Proof of Theorem 1

Any breakaway can happen if and only if under the new coalition structure, the breakaway player(s) can enjoy lower costs than under the current grand coalition. Let x_i be the cost for player i after a cost redistribution among players in the grand coalition has been done:

$$x_1 + x_2 + x_3 = v(123).$$

Case 1: Breakaway by a single player. To be stable against the breakaway of a single player, the grand coalition should meet the individual rationality condition which is represented by the following set of inequalities:

$$x_1 \leq v(1),\ x_2 \leq v(2),\ x_3 \leq v(3).$$

By summing both sides of the above set of inequations, we have (1).

Case 2: Breakaway by two players. Against a subgroup of two players who is willing to block the grand coalition, the following set of inequalities is necessary.

$$x_1 + x_2 \leq v(12),\ x_2 + x_3 \leq v(23),\ x_1 + x_3 \leq v(13).$$

By summing both sides of the above set of inequations, some algebraic simplification yields (2).

Case 3: Breakaway by all players. To meet the individual rationality against the non-cooperative supply chain scheme, the grand coalition has to meet the following set of inequalities.

$$x_1 \leq v1, \ x_2 \leq v2, \ x_3 \leq v3.$$

Adding both sides yields (3). □

References

[1] R. Bellman, *Dynamic Programming.* Princeton, NJ: Princeton University Press, 1957.

[2] F. Bloch, "Endogenous structures of association in oligopolies," *The RAND Journal of Economics*, vol. 26, no. 3, pp. 537–556, 1995.

[3] G. P. Cachon, "Stock wars: Inventory competition in a two-echelon supply chain with multiple retailers," *Operations Research*, vol. 49, no. 5, pp. 658–674, 2001.

[4] G. P. Cachon and P. H. Zipkin, "Competitive and cooperative inventory policies in a two-stage supply chain," *Management Science*, vol. 45, no. 7, pp. 936–994, 1999.

[5] B. C. Hartman, M. Dror, and M. Shaked, "Cores of inventory centralization games," *Games and Economic Behavior*, vol. 31, pp. 26–49, 2000.

[6] T. Hosoda, "The principles governing the dynamics of supply chains," Ph.D. dissertation, Cardiff University, Wales, August 2005.

[7] T. Hosoda and S. M. Disney, "The governing dynamics of supply chains: The impact of altruistic behaviour," *Automatica*, vol. 42, no. 8, pp. 1301–1309, 2006.

[8] ——, "On variance amplification in a three-echelon supply chain with minimum mean square error forecasting," *OMEGA, The International Journal of Management Science*, vol. 34, no. 4, pp. 344–358, 2006.

[9] H. L. Lee, V. Padmanabhan, and S. Whang, "The bullwhip effect in supply chains," *Sloan Management Review*, vol. 38, no. 3, pp. 93–102, 1997.

[10] R. Metters, "Quantifying the bullwhip effect in supply chains," *Journal of Operations Management*, vol. 15, pp. 89–100, 1997.

[11] M. Nakayama, "Nash equilibria and pareto optimal income redistribution," *Econometrica*, vol. 48, no. 5, pp. 1257–1264, 1980.

[12] A. Potter, C. Lalwani, T. Hosoda, and H. Al-Kaabi, "Vendor management inventory in a grocery supply chain: What are the benefits?" The 10th International Symposium on Logistics, Lisbon, Portugal, 2005.

[13] D. Ray and R. Vohra, "Equilibrium binding agreements," *Journal of Economic Theory*, vol. 73, pp. 30–78, 1997.

[14] H. J. Vassian, "Application of discrete variable servo theory to inventory control," *Journal of the Operations Research Society of America*, vol. 3, no. 3, pp. 272–282, 1955.

[15] H. Wang, M. Guo, and J. Efstathiou, "A game-theoretical cooperative mechanism design for a two-echelon decentralized supply chain," *European Journal of Operational Research*, vol. 157, pp. 372–388, 2004.

[16] Q. Wang and M. Parlar, "A three-person game theory model arising in stochastic inventory control theory," *European Journal of Operational Research*, vol. 76, pp. 83–97, 1994.

[17] S. Yi, "Stable coalition structures with externalities," *Games and Economic Behavior*, vol. 20, pp. 201–237, 1997.

[18] H. Zijm and J. Timmer, "Coordination mechanisms for inventory control in three-echelon serial and distribution systems," Memorandum No. 1738 (ISSN 0169-2690), Department of Applied Mathematics, Faculty of Electrical Engineering, Mathematics and Computer Science, University of Twente, Netherlands, 2004.