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Relicensing Fees as a Secondary Market Strategy

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Relicensing Fees as a Secondary Market Strategy

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Abstract

Secondary markets in the IT industry where used, refurbished or remanufactured equipment is traded have been growing steadily. For Original Equipment Manufacturers (OEMs) in this industry, the importance of secondary markets has grown in parallel, not only as a source of revenue, but also because of their impact on these firms' competitive advantage and overall strategy. Recent articles in the press have severely criticized some OEMs who are perceived to be actively trying to eliminate the secondary market for their products. From a research perspective, the critical tradeoff that needs to be examined is whether the indirect benefit from maintaining an active secondary market (the resale value effect) can outweigh the potentially negative effect of the sales of used products at the expense of new ones (the cannibalization effect). The goal of this paper is to understand how an OEM's incentives and optimal strategies are shaped contingent on her relative competitive advantage, product characteristics and consumer preferences. To that end, we develop a model where the OEM can directly affect the salvage value of her product through a relicensing fee charged to the buyer of the used equipment. Moreover, we introduce a measure of the consumers' willingness to return their used products to account for the fact that the higher the price offered by a third-party entrant, the higher the ratio of returned products at their end-of-use. By doing so, we endogenize both the size and the competitiveness of the secondary market. We analyze the OEM's decision problem in both the monopoly and the duopoly case. We characterize the optimal relicensing fee of the OEM and draw conclusions on the conditions that favor stimulating or deterring the secondary market.

Keywords: Cannibalization, Secondary Market, License Fee, Remanufacturing, Closed-Loop Supply Chain

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1 Introduction

Original Equipment Manufacturers (OEMs) in the Information Technology (IT) industry today often face difficult decisions when forming strategies involving secondary markets for their products. In the years before the dot-com bubble of the late 1990s, there was a limited secondary IT market. Some reasons for this lack of demand for used IT equipment included: 1) IT OEMs focused on their primary sales channels and discouraged customers from considering used equipment; 2) buyers of IT equipment were leery of the quality level of a used product; and 3) there was a lack of independent secondary market firms to refurbish, resell, and support used IT equipment. Shortages of higher-end IT equipment such as servers and routers during the late 1990s however, led to unmet demand that was often satisfied by a new market of third-party IT equipment brokers and refurbishers (Third-party refurbishers do not manufacture their own products, but instead rebuild and reconfigure used OEM products sold to them when a customer upgrades or no longer needs those products). In the years following, the dot-com bust resulted in a large amount of surplus, sometimes barely used IT equipment for sale from companies who failed when the bubble burst. The availability of so much inexpensive used IT equipment led to significant price discounts compared to the price of new equipment and even more brokers and refurbishers entering the secondary market (Berinato 2002).

One of the lasting effects from the dot-com era is that major customers of IT equipment have started accepting used IT equipment as a viable alternative to new equipment and a new body of IT refurbishers has entered the market to meet this demand. According to a 2002 survey of 187 IT executives in CIO magazine, 77 percent said they were purchasing secondary market equipment and 46 percent expect to increase their spending on used equipment next year by an average of 15% (Berinato 2002). In another article, Computer Business Review highlights that "third-party companies have built $100+ million per year businesses in buying used computer equipment, refurbishing it, and selling or leasing it out to someone else" (http://www.cbronline.com/news_archives.asp?show=2005-09). Given the size and growth of the secondary market, the days of ignoring it and only focusing on the sale of new products are over for all major IT OEMs. OEMs may either embrace the secondary market or try to eliminate it, but one thing is now evidently clear, they must form strategies to respond to it.

Some of the major OEMs in the IT industry not only have embraced the existence of a secondary market but also deploy it to obtain a competitive advantage over their rivals. IBM and Hewlett

Packard, for instance, create high resale values for their used equipment by fully supporting the resale process (e.g. offering maintenance, inspection, small relicensing fees) so that the original customers can gain the maximum lifetime benefits from their new product purchases. Such a proactive, and in a sense cooperative, relationship with third-party brokers and refurbishers, however, is not standard policy among IT OEMs. An alternative strategy is to institute policies and fees that attempt to eliminate the secondary market. For example, Sun Microsystems (SUN), one of the leading firms in the IT server business, was "under fire for deliberately attempting to eliminate the secondary market for its machines worldwide through their new pricing and licensing schemes" (Marion 2004). Cisco is another company that requires each new user (buyer of used equipment) to pay high relicensing fees for the proprietary software that makes the equipment run.

The following excerpt, typical of the IT industry, sheds some light on how the licensing mechanism works. "Cisco adopts a policy of non-transferability of its software in order to protect its intellectual property rights. What this means in practice is that owners of Cisco products are only allowed to transfer, re-sell, or re-lease used Cisco hardware and not the embedded software that runs on the hardware." This practice in effect eliminates the secondary market and creates customer dissatisfaction. Cisco's response to this criticism was to institute (significant) relicensing fees: "As Cisco's installed base of equipment has grown to such large numbers over the years, our customers have become more interested in selling and leasing used Cisco equipment on the secondary market. In order to provide our valued customers and partners with this capability, Cisco is now setting up a program where companies who are interested in buying used equipment, may now purchase a new software license to do so" (http://www.cisco.com/warp/public/csc/refurb_equipment/swlicense.html).

Despite such statements that a relicensing fee mechanism allows reselling used equipment on the secondary market, many industry observers argue that some OEMs use unreasonably high licensing fees as a means of limiting the secondary market. In the case of SUN, Marion (2004) highlights the fact that the relicensing fee has been deliberately set so high that the overall cost of used equipment, including hardware and software, reaches that of a new one: "In the end, the potential buyer for the used equipment may have no choice but to return to Sun for a new product." He concludes by stressing another interesting facet of the problem: "End users need to know this and take action to adjust the SUN hardware values reflected on their respective balance sheets to account for the impact that SUN's actions described above will have on resale and residual values." In other words, users should be aware that SUN's practices result in very low resale values of used equipment and this information should be factored in the original purchase decision.

From a research perspective, the discussion above highlights the fundamental question addressed in this paper. Given the OEM's ability to interfere with the secondary market through pricing and licensing schemes, is limiting this market or, conversely, encouraging its existence a more profitable strategy? Our goal is to understand how the OEM's incentives and optimal strategies are shaped contingent on her relative competitive advantage, product characteristics and consumer preferences. Motivated by the industry articles concerning SUN, a company that historically was considered the premium brand in the server market, it is worth examining whether such a quality premium could justify an aggressive secondary market strategy. Addressing this question could shed some light on why different OEMs have chosen so diametrically opposed strategies. If one strategy is dominant over the other, the winner is currently not clear based on anecdotal evidence alone. Although there have been several papers that studied the impact of secondary markets on the OEM's profits, a shortcoming of previous work is the assumption of a monopolistic OEM. Insightful as those models are, they fall short of explaining the significant differences in policy between companies such as SUN and IBM.

We begin our analysis by studying the optimal strategies of a monopolist OEM in the IT industry. Our key finding is that consumers' awareness of the salvage value of the product is a key determinant of the market equilibrium. When consumers act strategically, taking into account the resale value effect in their purchase of new products, it is never optimal for the OEM to completely shut down the secondary market. In contrast, when consumers act myopically, it might be optimal for the OEM to eliminate the secondary market by charging a high enough re-licensing fee. By relaxing the assumption of a perfectly competitive secondary market, we are able to examine how the OEM's strategy changes as the number of the independent resellers increases. We find that both the OEM's profits as well as the size of the secondary market grow in the number of entrants. In a differentiated duopoly setting, our results suggest that the high-end OEM can charge a significantly higher relicensing fee for her used products. Interestingly, however, there is a non-monotonic relationship between the quality premium of the high-end firm and its relicensing fee. In fact, there is a range of values for which the cannibalization effect between its new and used products drives the high-end firm to increase its relicensing fee and adopt a more aggressive strategy towards its secondary market. As competition in the primary market becomes fiercer, however, the high-end firm has to rely on the "resale value effect" and thus, supports the secondary market. Moreover, we identify the conditions under which the low-end firm will have a greater share in the secondary market.

2 Literature Review

The interaction between new and used products has been extensively studied in the economics literature. In a series of papers, Swan (1970,1971) addresses the issue of "optimal durability" faced by a monopolist that sells durable products over time. In another classic contribution, Coase (1972) identifies the "time-inconsistency" problem faced by a monopoly when consumers anticipate lower prices in the future periods. These seminal papers were the first to identify that a monopolist may have the incentive to take actions to lower the value of units previously sold (lower durability, leasing, new product introduction) in order to avoid cannibalization in later periods. In the following paragraphs we summarize the contributions related to the impact of secondary markets, a thorough literature review on durable goods theory can be found in Waldman (2003).

Until the early 1970s, the main conclusion regarding the impact of secondary markets on a monopolist's profitability was due to the cannibalization effect between new and used products. In the words of Gaskins (1974), "conventional economic wisdom... contends that the existence of a competitive secondhand market constitutes a major long-run restraint on monopoly in a primary market." Motivated by the market for diamonds, however, Miller (1974) argues that "the buyer of a newly produced diamond pays a price consistent with what the diamond can be sold for to others including members of later generations" and thus "the initial price captures the present value of all subsequent transactions." In essence, he points out the "resale value effect" arguing that a secondary market might increase the value derived by the consumer, and in turn, the price that the monopolist can charge for it. The latter argument is also stressed by Benjamin and Kormendi (1974), Liebowitz (1982), Rust (1986), and Levinthal and Purohit (1989), who all argue that whether or not a monopolist has the incentive to eliminate the secondary market is not clear-cut. A limitation of these papers is the assumption that the demand side is modeled by a representative consumer (homogeneous consumer preferences). Anderson and Ginsburgh(1994) argue that in those models, the size of the second-hand market is indeterminate since this representative consumer buys both new goods and used goods each period and essentially sells the used good to herself. By introducing a model in which consumers have heterogeneous tastes, they show that the existence of a secondary market enables the monopolist to achieve price discrimination between high and low valuation consumers who buy new and used products, respectively.

These ideas are refined in further studies by Waldman (1996a, 1997), Fudenberg and Tirole(1998), and Hendel and Lizzeri (1999). Waldman (1996a) employs the seminal Mussa and

Rosen (1978) analysis of market segmentation and product-line pricing to allow customers to vary in their valuations of quality. His main result is that because of the substitutability effect between new and used products, the price at which old units trade on the secondary market constrains the price that the monopolist can charge for the new units. Therefore, he demonstrates that the monopolist may have an incentive to "shut down" the market by reducing durability to "sufficiently low" values. In a follow-up paper, Waldman (1997) demonstrates that leasing can successfully serve that purpose and argues that eliminating the secondary market might have been the primary reason for many prominent industry leasing cases (United Shoe, IBM, Xerox). Fudenberg and Tirole (1998) show how the secondary market can limit the monopolist's future sales of a high quality product and derive optimal pricing schemes for upgrades and buybacks contingent on the information about the customer base. Finally, Hendel and Lizzeri (1999) study leasing and selling strategies under secondary markets when durability is endogenous. They show that, due to the existence of the secondary market, the monopolist does have an incentive to provide less durable goods. They also derive conditions under which a monopolist could benefit from a "well-functioning" secondary market.

We extend the above literature on the economics of the secondary markets in three directions. First, we relax the assumption of a monopolist OEM by allowing vertically differentiated products to compete in the primary market. Through our numerical analysis, we demonstrate that the relicensing fees exhibit a non-monotonic relationship with respect to the degree of differentiation. Second, we relax the assumption of perfect competition in the secondary market and allow for a profit maximizing entrant to collect the used products. The value offered by the entrant is determined as his optimal response to the OEM's decisions and therefore, prices for new and used products arise as the Nash Equilibrium of the game between the OEM and entrant. This allows us to examine the impact of an increasing number of resellers on the size of the secondary market as well as on the profits of the OEM. Third, by explicitly incorporating the relicensing fee component in our decision framework, we are able to capture not only the economic but also the strategic implications of this widespread mechanism. For many IT OEMs, the existence of a relicensing fee constitutes the most common way of protecting their intellectual property rights and therefore a significant source of revenues.

A rapidly growing stream of literature on remanufacturing has focused on the interaction between new and used products. Debo et al. (2005) focus on a monopolist who considers selling both new and remanufactured products to a customer base that has a lower willingness-to-pay for the remanufactured product. They determine the monopolist's optimal market segmentation and remanufacturability level decisions in an infinite-horizon setting. In the presence of external competition, Majumder and Groenevelt (2001) derive the Nash Equilibrium between and OEM and an entrant contingent on the availability of used products. Ferrer and Swaminathan (2006) study the optimal pricing schemes for the OEM as well as for the entrant in a multiperiod setting where consumers show a higher preference for the OEM's product over the entrant's product. They show that an OEM might forgo some of the first period profits by making additional units to increase the number of cores available for remanufacturing in subsequent periods. A limitation of the above studies is that the proportion of used products available to each company is exogenously given while in our model, the proportion is determined by the price the third-party entrant is willing to pay. Ferguson and Toktay (2006) analyze two common entry-deterrent strategies: remanufacturing and preemptive collection. They find that a firm may choose to remanufacture or preemptively collect its used products to deter entry, even when the firm would not have chosen to do so under a pure monopoly environment. Finally, Atasu et al. (2007) identify the conditions under which remanufacturing can be used as a strategic marketing tool by developing a model that explicitly incorporates specific demand-related issues, such as the existence of a green segment, OEM competition and product life cycle effects. Although the above models provide a theoretical framework for analyzing the competition between the OEM and potential entrants in the secondary market, they do not incorporate the effect of the resale value on the consumers' willingness to pay for new products and, subsequently, on the OEM's profits. We contribute to the existing literature on remanufacturing by endogenizing the salvage value, and hence the size of the secondary market. Instead of assuming a fixed return rate for the used products, we introduce a measure of the consumers' willingness to return their used products to account for the fact that the higher the price offered by a third-party entrant, the higher the ratio of returned products at their end-of-use. Our paper is the first to explicitly link the resale value of a product on a secondary market to a consumer's willingness-to-pay for the new product in the first period under a competition setting from a secondary market of remanufactured goods.

3 Key Assumptions and Notation

In this section, we describe the sequence of events and the market structure. We then discuss the key assumptions underlying our model.

Market Dynamics

Our analysis focuses on an industry in which the OEM holds a monopoly in the new product market, but potential third-party entrants may create a secondary market by refurbishing and reselling used products collected from the OEM's customers. Our goal is to examine the OEM's licensing fee strategies in the face of future remanufacturing competition. The link between current and future sales is captured by developing two-period model with a one-period useful product life; refurbishing extends the product's life to two periods. Other papers that use a two-period model in a remanufacturing competition setting include Majumder and Groenevelt (2001), Ray et al. (2003), Ferrer and Swaminathan (2006), Ferguson and Toktay (2006), and Atasu et al. (2007).

The sequence of events is as follows: In the first period, the OEM launches the new product in the market as a monopolist. At the end of the first period, an entrant can potentially pay consumers to collect used products, and enter the market in the second period by refurbishing and reselling these products on the same market. Consumers purchasing the used product need to pay a relicensing fee to the OEM to operate it. Thus, in the second period, the new product launched by the OEM faces competition from the used products procured and offered for sale by the entrant. At the same time, the OEM generates license revenues from the used products through a relicensing of the software required to run it.

In this competitive setting, the OEM has a significant advantage over the entrant: she controls the value of the relicensing fee that consumers of used products need to pay on top of the purchase price charged by the entrant. As the relicensing fee increases, the cost to consumers of the used product increases, which in turn reduces the demand for the used product and shifts consumers to new products. At first sight, a high value for the relicensing fee might seem an optimal strategy for the OEM, since it eliminates the competition from the used products. Eliminating the secondary market, however, has an important impact on first period profits as consumers can no longer sell their used products to an entrant. As a result, the consumer willingness to pay for the new product decreases, and consequently the price charged by the monopolist OEM, along with the first period profits, is lower than it would have been had the consumers foreseen a positive salvage value for their used products. Hence, the OEM needs to balance the impact of two opposite forces: a lower relicensing fee leads to competition in the second period but allows the OEM to charge a premium in the first period that reflects the consumer's ability to resell the product in the second period. We now discuss our key assumptions regarding the consumers' characteristics.

Assumption 1. *Consumer willingness-to-pay is heterogeneous and uniformly distributed in the* *interval [0, 1].*

We assume that consumers' types are distributed uniformly in the interval $[0, 1]$ and that in any period, each consumer uses at most one unit. The market size is normalized to 1. In this vertical differentiation model (Tirole 1988), a consumer of type $\theta \in [0, 1]$ has a valuation of θ for a new product. In the first period when only the new product is available, if no secondary market existed for the product (zero salvage value) then the consumer utility function would be $U_1 = \theta - p_1$ where U_1 represents the consumer's utility in the first period and p_1 is the price paid for the new product. This assumption regarding the distribution of consumer utilities leads to the familiar inverse demand function $p_1 = 1 - q_1$, where q_1 is the quantity of new product sold in the first period.

Assumption 2. *Each consumer's willingness-to-pay for a used product is a fraction δ of their willingness-to pay for the new product.*

Under this assumption, a consumer who derives utility θ from the new product will derive utility $\delta\theta$ from the used one. This assumption states that the nature of competition between new and used products is that of vertical differentiation. That is, holding all else fixed, consumers would prefer a new product to a used one. This is mainly driven by the evidence that consumers are concerned about the quality of a used product and this is reflected in their willingness to pay for it. This perspective is reflected in a number of articles both from academics and practitioners (Kandra 2002, Lund and Skeels 1983, Debo et al. 2005, Ferguson and Toktay, 2006). Note that if $\delta = 0$, consumers are not willing to pay anything for the used product; this eliminates the option of maintaining a secondary market. If $\delta = 1$, consumers view the new and used units as being identical and are willing to pay the same amount for either product. Most products fall between the two extremes; we assume $0 < \delta < 1$.

Assumption 3. *The OEM charges a relicensing fee h in the second period to any new user of her product sold in the first period.*

The establishment of a relicensing fee, typically called a Digital License Agreement (DLA), has been widely employed by OEMs as a means of protecting their intellectual property rights. Typically, the price of a new product implicitly includes both the hardware and software cost. Most OEMs, however, will usually publish a separate list where their relicensing policies is explicitly laid out. The relicensing fee constitutes an important element of our model, since it acts as a signal of the potential salvage value. Strategic consumers of new products can infer, through the relicensing fee, the magnitude of the secondary market, which in turn determines the value they will be able to resell their product for.

The utility that each consumer derives from purchasing a product is given by the difference of their valuation and the price. In the case of used products, in addition to the purchase price, the consumer also incurs the extra cost of the relicensing fee, which also reduces his overall utility. Let p_2 and p_u denote the second period prices of new and used products, respectively. The corresponding consumer utilities obtained by purchasing each type product in the second-period are $U_2 = \theta - p_2$ for the new product and $U_u = \delta\theta - p_u - h$ for the used product. From these utility functions, and letting q_2 and q_u represent the second period quantities of new and used product respectively, the inverse demand functions are:

$$p_2 = 1 - q_2 - q_u\delta$$
$$p_u = \delta - q_u\delta - q_2\delta \ - h$$

Assumption 4. *Consumer reservation prices to return their product are a fraction of their original valuation of the new product.*

We assume that a consumer with a valuation θ for a new product will have a reservation price of $\gamma\theta$ (where $0 < \gamma < \delta$) in order to sell his used product to the entrant: he will sell it only if the purchase price s offered to him is more than $\gamma\theta$. According to this assumption, consumers who had a higher valuation for the product at the moment of purchase are expected to have higher reservation prices in order to return the product. This assumption is appropriate when consumers who can afford to pay higher prices for a new product are less inclined, for a given price, to spend the effort needed to sell a used product to the entrant than a consumer who could only afford a lower price. This behavioral characteristic is the basis behind the common use of product rebates that allow price discrimination between consumers who will take the time to send in the rebate and those who will not. Obviously, the higher the rebate, the higher the percentage of customers that claim it, just as in our setting where the higher the price offered by the entrant, the higher the percentage of customers who will sell him the used product. Similar assumptions regarding consumers' willingness to return their product have been used by Ghose et al. (2005) and Shulman and Coughlan (2006).

Reservation prices are an important element of our model since they account for the fact that only those consumers with reservation prices lower than the salvage value, offered by the entrant, will be willing to return their product while the rest will retain the product. Moreover, in line with previous research on reverse logistics and remanufacturing, this assumption ensures that the average cost of collection increases in the quantity of the products collected (Guide 2001, Guide and Van Wassenhove 2001, Ferguson and Toktay 2006).

Assumption 5. *Customers are strategic.*

We assume that customers take into account the future residual value of the product in making their purchase decisions. A strategic consumer of type θ will derive an overall first period utility of: $U_1 = \theta - p_1 + \max\{s, \gamma\theta\}$.At the end of period 1, the consumer chooses between reselling the used product to the third-party remanufacturer for a value s, or retaining the product, which generates a reservation utility of $\gamma\theta$. Note that we assume the consumers cannot resell a used product directly to each other. This assumption reflects the current practice in the used IT market where most used equipment, before it can be resold, requires some costly remanufacturing or refurbishing effort that the original consumers do not have the technical capability to perform. We also assume a product that is not remanufactured or refurbished only has a useful life of one period. A consumer who chooses not to resell her product to the third-party remanufacturer cannot continue to use the product in the second period, and thus re-enters the market for either new or used equipment in the second period.

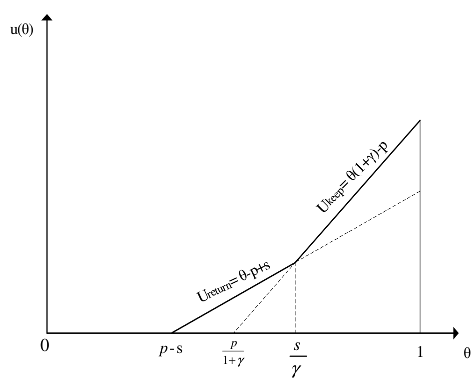

Figure 1: Consumer State Space

As shown in Figure 1, contingent on their type, consumers at the end of the first period will either resell or retain the product. Mathematically,

$if \;\; \theta \leq p - s,$	consumers do not purchase the new product
$if \;\; p - s < \theta \leq \frac{s}{\gamma}$	consumers purchase the new product and subsequently resell it
$if \;\; \frac{s}{\gamma} < \theta \leq 1$	consumers purchase the new product, and do not resell it

4 Analysis

The analysis in this section aims to answer the question of how OEMs should use relicensing fees to impact the secondary market for their products. Our baseline scenario is of a single OEM who sells the new product in the first period and charges a relicensing fee for used products that are resold by a single entrant in the second period. Under this scenario, we find that it is never optimal for the OEM to charge a relicensing fee that is so high that it effectively eliminates the secondary market. We then check to see if our result holds when consumers are non-strategic and find that there are cases with non-strategic customers where the OEM will eliminate the secondary market through a high relicensing fee. We also model competition at the entrant and OEM levels. When we allow more than one third-party reseller, the OEM has to lower the value of the relicensing fee to accommodate the decreasing marginal profits of the resellers. Yet, due to the resale value effects, the OEM's profits are concave increasing in the number of resellers. When there is competition at the OEM level between an high-end product OEM and a low-end product OEM, we show that it is never optimal for either OEM to eliminate the secondary market.

4.1 Monopolist OEM, Single Entrant

We begin by characterizing the optimal pricing and licensing schemes in the monopolist-OEM, single-entrant case. This scenario will serve as a baseline for the model extensions we present. In the first period, the OEM is the only firm in the market and only sells the new product. In the second period, the third-party entrant buys the used products from a portion of the customers who purchased the new product in the first period, and remanufactures and resells the used product in the second period to the lower willingness-to-pay consumers. The OEM sells new products again in this period. We solve the problem in two stages, starting with the second period.

Second-period analysis.

Let Π_2 and Π_u denote the OEM's and the entrant's second-period profit, respectively. The OEM's second-period objective given the entrant's choice of q_u is

$$Max_{q_2} \qquad \Pi_2(q_2|q_u) = (p_2 - c)q_2 + hq_u = (1 - q_2 - q_u\delta - c)\, q_2 + hq_u \quad s.t \quad q_2 \geq 0$$

The first part of the equation above captures the profits obtained from selling q_2 units of new products while the second part represents the profits from the relicensing fees (h) obtained by the q_u units of used products that the entrant launched in the market. The quantity of new products to sell is the only decision variable for the OEM in the second period as the relicensing fee is set in the first period.

The entrant's corresponding objective given the OEM's choice of q_2 is

$$Max_{q_u,s}(q_u, s|q_2) \qquad \Pi_u = (p_u - s)q_u \tag{1}$$
$$s.t \quad q_u \leq s(\frac{1}{\gamma} + 1) - p_1$$
$$q_u \leq q_1, \quad q_u \geq 0$$

For simplicity, we normalize the refurbishment cost and collection-related costs other than the price s to 0. With these assumptions, the entrant's objective function is $(p_u - s)q_u$. The entrant has to determine both the price s that she will offer to the consumers to obtain used products, and the quantity of used products that she will make available in the market, q_u. The first constraint ensures that the latter quantity can not be greater than the number of units collected from the consumers, given by $s(\frac{1}{\gamma} + 1) - p_1$. The second constraint ensures the quantity of used products cannot be greater than the quantity of new products manufactured in period 1.

Lemma 1 *At optimality, the entrant has no incentive to collect more products than the ones she intends to launch in the market. That is, the first constraint in (1) is satisfied as an equality and the optimal salvage value offered by the entrant is*

$$s^* = \frac{\gamma(q_u^* + p_1)}{1 + \gamma} \tag{2}$$

Proof All proofs can be found in Oraiopoulos et al. (2007) ■

Based on Lemma 1 and the joint concavity with respect to s and q_u we can rewrite the entrant's problem as an optimization problem in q_u

$$Max_{q_u} \qquad \Pi_u = (p_u - \tfrac{\gamma(q_u+p_1)}{1+\gamma})q_u \qquad s.t \;\; 0 \leq q_u \leq q_1$$

Solving simultaneously for the quantities of the two competitors, we obtain the following Nash Equilibria (N.E):

$$q_2^* = \frac{\delta\, h^*(1+\gamma) + \delta\,\gamma p_1 + 2\,(1-c)(\delta + \gamma(\delta+1)) - \delta^2(1+\gamma)}{\delta(1+\gamma)(4-\delta) + 4\gamma} \tag{3}$$

$$q_u^* = \frac{\delta\,\gamma - 2\,h(1+\gamma) + \delta + \delta\,c - 2\,\gamma p_1 + \delta\,\gamma c}{\delta(1+\gamma)(4-\delta) + 4\gamma} \tag{4}$$

As we can readily see from the latter expression, the quantity of used product, q_u^*, is decreasing in the relicensing fee h^* as well as in the price of new products p_1^*. Substituting q_u^* from equation (4) into (2) gives:

$$s^* = \frac{\gamma[(2\,\gamma + 4\,\delta\,(1+\gamma) - \delta^2(\gamma+1))]p_1 + \gamma(1+\gamma)[\delta(1+\,c) - 2h]}{[\delta(1+\gamma)(4-\delta) + 4\gamma]\,(1+\gamma)}. \tag{5}$$

As can be seen in Figure 1, the quantity of new units sold in the first period by the OEM is given by $q_1 = 1 - p_1 + s$. Substituting $p_1 = 1 - q_1 + s$ into (5) we get the optimal price s^* that the entrant is willing to pay first-period consumers in order to collect used products as a function of q_1:

$$s^* = \gamma\frac{(-4\delta - 2\gamma - 4\delta\gamma + \delta^2\gamma + \delta^2)q_1 - 2h(1+\gamma) + \delta(1+\gamma)(5+c-\delta)}{2\,\gamma(\gamma+2) + \delta(1+\gamma)(4-\delta)}.$$

The above expression reveals two interesting properties of the salvage value. First, s^* decreases in the quantity of new products launched in the first period. This observation is consistent with the residual values we observe in practice: whenever a large supply of a specific used model becomes available, its residual value drops dramatically. Second, s^* decreases in the relicensing fee h. This captures an essential tradeoff in our model: first period consumers who observe the relicensing fee can infer the potential value they will receive for their returned product. A low value of h signals a competitive secondary market, and thus, a higher salvage value which increases the consumers' willingness-to-pay for the new product in the first period. After characterizing the optimal second period decisions given p_1 and h we now solve for the optimal first period decisions p_1^* and h^*.

First period analysis

In the first period, the OEM acts as a monopolist since there are only new products. The OEM's decisions include the quantity of new units to sell as well as the relicensing fee to charge consumers who purchase the used product in the second period. More specifically, the OEM's problem is the following:

$$Max_{q_1,h}\ \Pi(q_1,h) = \Pi_1 + \Pi_2^*(q_1,h) = (p_1 - c)q_1 + \Pi_2^*(q_1,h),$$

where the first part reflects the profits from the sale of new units while the * on the second part denotes the optimal second-period profits for given q_1 and h . For the problem to be well-defined, we need to incorporate the associated second-period non-negativity constraints on the first-period problem. That is,

$$\begin{array}{c} Max_{q_1,h} \qquad \Pi(q_1,h) \\ s.t \quad q_2^* \geq 0,\ q_u^* \geq 0,\ q_1 \geq 0,\ \text{and}\ h \geq 0 \end{array}$$

Proposition 1 *It is never optimal for the OEM to eliminate the secondary market: $q_u^* > 0$. The relicensing fee depends on the unit production cost. If $c \geq \widetilde{c}(\delta,\gamma) \doteq \frac{\delta^2(3\gamma^2-3\delta\gamma+11\gamma-3\delta+8)-8\gamma^2(1+\delta)}{\delta^2(\delta+\delta\gamma-\gamma)}$, then $h^* = 0$. For $c < \widetilde{c}(\delta,\gamma)$, $h^* > 0$.*

Proposition 1 illustrates the importance of the secondary market in the total OEM's profit. Despite the competition faced in the second period from the entrant, the OEM is always better off by maintaining a secondary market. Through this secondary market, consumers in the first period obtain higher utility from purchasing the new product by factoring in the salvage value (s) they can resell the product for at the end of the first period. This increased utility, in turn, allows the OEM to charge a higher price (premium) for his product in the first period and thus leads to higher profits. According to Proposition 1, this increase in the first period profit always outweighs the reduced profits in the second period due to the competition effect.

The optimal policy depends on the unit production cost. If the production cost is above a certain threshold ($c \geq \widetilde{c}(\delta,\gamma)$), the OEM is better off by increasing the premium charged in the first period through fully promoting the secondary market ($h^* = 0$). It might even be optimal (when $c \geq \overline{c}(\delta,\gamma) > \widetilde{c}(\delta,\gamma)$), to completely abstain from the second period ($q_2^* = 0$). In contrast, for low values of production cost ($c \leq \widetilde{c}(\delta,\gamma)$), the OEM maximizes his profits by a more aggressive competition strategy in the second period. By charging a positive relicensing fee, the OEM reduces the utility offered by used products, which leads to higher demand for the new products offered by the OEM in the second period.

Remark 1 *When $h^* > 0$ ($c \leq \widetilde{c}(\delta,\gamma)$), the following properties hold: The optimal relicensing fee increases in δ and decreases in γ. If the OEM does not launch a new product in the second period, the optimal relicensing fee increases in c, otherwise it decreases.*

The relationship between δ and h^* is to be expected: All other things being equal, when consumers value the used product more, the OEM can charge a higher fee for it. Similarly, the more willing they are to return their products (low γ), the lower the procurement cost for the entrant, and therefore the OEM can charge more for the relicensing fee without affecting the positive impact of the secondary market on her profits. Finally, when it is optimal for the OEM not to launch new products in the second period, as the production cost increases, the OEM prefers to increase the relicensing fee despite the negative impact on the first period sales. Intuitively, when the marginal profits from new products decrease, the OEM exploits the revenues from the relicensing fees. Conversely, when the OEM optimally launches a new product in the second period, and there is competition between new and used products, a higher production cost leads to a lower relicensing fee in order for the OEM to further enhance the secondary market. These relationships are illustrated in Figure 2.

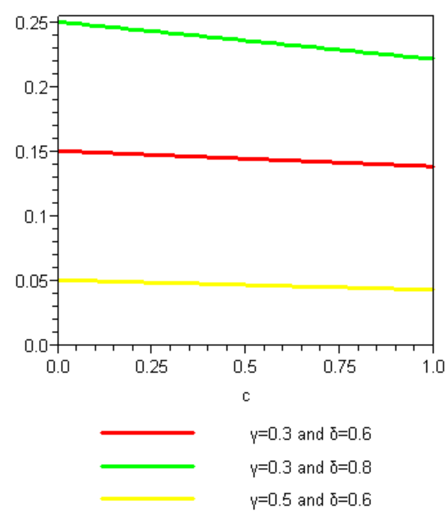

Figure 2: Optimal Relicensing Fees

4.1.1 What if consumers are non-strategic?

In this subsection, we examine the case where first period consumers act non-strategically. That is, in their purchase decision they do not take into account the potential salvage value of the product. Therefore, the OEM can no longer charge a premium to first-period consumers even though a secondary market may form in the future. Under those circumstances, it is interesting to see whether the optimality of never shutting down the secondary market – our key finding in the above analysis – still holds.

In terms of the utility of first period consumers, acting myopically means that the derived utility is given by $U_1 = \theta - p_1$ and consumers with $\theta \geq p_1$ will buy the product. At the end of the first period, consumers will resell their products as long as $s \geq \theta\gamma \Rightarrow \theta \leq \frac{s}{\gamma}$. Therefore, the total supply of used products is $q_s = \frac{s}{\gamma} - p_1 < q_{s,strategic}$. The rest of the analysis remains similar to our benchmark case.

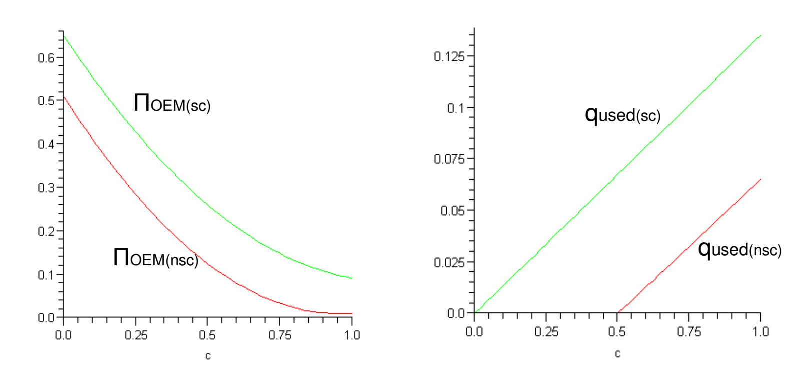

Figure 3 : Comparison of OEM's profts (left) and size of secondary market(right) under strategic (sc) versus non-strategic (nsc) customers for δ=0.7 and γ=0.3

As we can see in Figure 3, the existence of non-strategic consumers has a significant impact on the OEM's total profits (left). In particular, even though the OEM charges a higher relicensing fee when consumers are non-strategic, this additional revenue does not make up for the loss of sales in the second period due to the competition from the third-party entrant. Thus, the OEM's benefit from supporting a secondary market is significantly weakened, and shutting down the secondary market may become an optimal strategy. Therefore, Proposition 1 does not hold when consumers are non-strategic.

4.2 Competition in the secondary market (N entrants)

In this section, we study the impact of competition within the secondary market. Realizing the significant profit opportunities by reselling used equipment, a number of firms have been founded with the sole purpose of buying and reselling used IT equipment (http://www.cbronline.com/news_archives.asp?show=2005-09). In this setting, while the OEM maintains her monopolistic position in the market for new products, N independent entrants compete on both acquiring the used products from the customers as well as reselling them in the secondary market. This raises two interesting questions: First, does an increasing number of resellers correspond to larger secondary market or does the internal competition reduce marginal profits, and thus, lead to a decrease in overall quantity? Second, how is the OEM's profitability and relicensing fee strategy affected by the number of entrants N?

To answer these questions, we assume N symmetric entrants who are in Cournot competition (similar to Debo et al. 2005) with each other, and at the same time face competition from new products under the vertical differentiation model outlined in the previous section. Thus, if we let q_u^i denote the quantity of used products offered by the entrant i, the total quantity of used products in the second period will be $\sum_{i=1}^{N} q_u^i$. Paralleling our analysis in section 4.1, the inverse demand function is given by:

$$p_2 = 1 - q_2 - \delta \sum_{i=1}^{N} q_u^i$$
$$p_u = \delta - q_2\delta \ - h - \delta \sum_{i=1}^{N} q_u^i$$

and the OEM's second-period objective is given by

$$Max_{q_2} \qquad \Pi_2 = \left(-q_2 + 1 - \delta \sum_{i=1}^{N} q_u^i \ - c \right) q_2 + h \sum_{i=1}^{N} q_u^i$$
$$s.t \qquad q_2 \geq 0$$

while the $i's$ entrant problem can be described as follows:

$$Max_{q_u^i} \qquad \Pi_u = (p_u - s) q_u^i$$
$$s.t \qquad \sum_{i=1}^{N} q_u^i \leq s(\tfrac{1}{\gamma} + 1) - p_1$$
$$\sum_{i=1}^{N} q_u^i \leq q_1$$
$$q_u \geq 0$$

As the entrants are symmetric, at equilibrium we have that $Q_u = \sum_{i=1}^{N} q_u^{i\,*} = N q_u^{1\,*}$. In addition, the market clearing price s for used products should satisfy $N q_u^{1\,*} = s(\frac{1}{\gamma} + 1) - p_1$. Using a parallel development as in Section 4.1 (presented in the appendix) we obtain the following result:

Proposition 2 *The overall quantity of used products in the market as well as the total profits of the OEM are concave increasing in the number of resellers N. The quantity per reseller, however, is convex decreasing in N. Moreover, similar to Proposition 1, there is a threshold value for the production cost ($\widetilde{c}(\delta, \gamma, N)$) above which the optmal relicensing fee is set to zero and below which it is convex decreasing in N.*

Proposition 2 reveals some interesting insights about the internal competition among the resellers and how this competition affects consumers purchase behavior along with OEM's profits. First, as we would expect, as the number of resellers increases, the marginal profit per reseller is decreasing and thus the optimal quantity (per reseller) is decreasing. That is, internal competition drives the prices of used products to lower levels. As a result, the secondary market attracts more consumers and the overall quantity of used products increases. An indirect effect of this expansion is that consumers of new products will enjoy a higher resale (salvage) value at the end of use. This value, in turn, can be captured by the OEM through the price that she charges for the new products. Note, however, that the OEM has to lower the value of the relicensing fee to accommodate the decreasing marginal profits of the resellers. Yet, due to the resale value effects, the OEM's profits are concave increasing in the number of resellers. Figure 4 illustrates the above results.

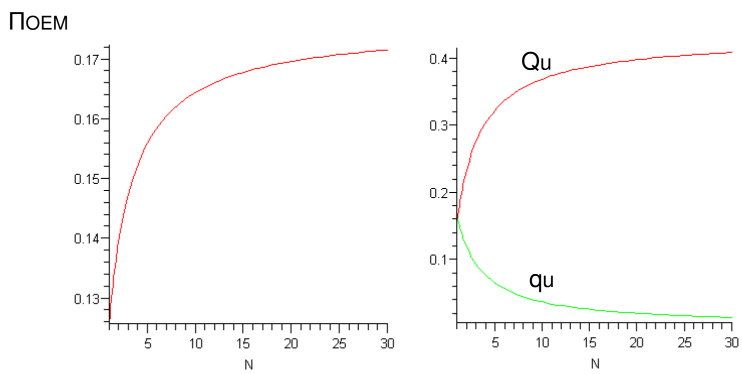

Figure 4: OEM's profits (left) and Quantity of Used Products (right) per reseller (qu) and overall (Qu) for γ=0.1,δ=0.7,and c=0.6

4.3 Competition in both Primary and Secondary Market

The motivation for this paper is to understand potential drivers of the disparate relicensing policies observed in practice. In our analysis so far, we have assumed a monopolist setting in the primary market with the competition being restricted to the secondary market. We found that unless customers are non-strategic, relicensing fees should not be so high as to eliminate the secondary market. Since firms in the same industry typically face similar customer profiles, this explanation, while interesting in its own right, falls short of explaining the significant differences in policy between companies such as SUN and IBM. Motivated by the historical edge that SUN commanded in the high-end server market (http://www.sun.com/servers/highend/sunfire15k/performance.html), we hypothesize that brand premium differences between companies may explain the differences in secondary market strategies.

To explore this direction, we relax the monopoly assumption and develop a vertical differentiation model in which consumers place a higher value on firm A's product than on firm B's product.

This assumption allows us to address two critical questions regarding the markets for used products. First, how does the quality differentiation influence the OEMs' incentives to interfere with their corresponding secondary markets? In particular, we examine whether a quality premium corresponds to a higher or lower willingness to support the secondary market. Second, how does competition in the primary market affect the overall size of the secondary market?

We capture the difference in the perceived quality between firms by the parameter $a:$ a consumer who derives utility θ from a new product by firm A will derive utility $a\theta$ from a new product by firm B. Without loss of generality, we assume that $a < 1$ so that firm B represents the low-end firm. We also assume an equal rate of perceived quality depreciation for both firms. That is, a consumer derives utility $\delta\theta$ from firm A's used product, while he derives utility $\delta\alpha\theta$ from a used product by firm B. This assumption allows us to isolate the effect of the quality premium (α) on the OEMs' secondary market strategies without making any additional assumptions about the nature of the products. In addition, we assume that $\delta < \alpha$ so that consumers evaluate firm B's new product strictly more than firm A's used product. This is a reasonable assumption in a competitive industry that eliminates the trivial case where one firm dominates both the primary and secondary markets. Lastly, we assume perfectly competitive secondary markets. Given the large number of IT resellers, this is a reasonable assumption and allows us to maintain tractability since allowing a third profit-maximizing firm would further complicate the equilibrium conditions without offering additional managerial insights (our analysis in the previous section provides the main intuition regarding an oligopolistic secondary market). Similarly to our benchmark case, we formulate the problem in two stages, starting with the second period and solving backwards. Unlike our previous analysis, however, ensuring the uniqueness of the N.E for any arbitrary set of parameters is not a trivial task (extremely long expressions). Instead, in our extensive numerical analysis, we derive the unique equilibrium quantities for a given set of values and subsequently we ensure that all constraints are being satisfied. We, therefore outline the critical components of our model formulation without providing the exact expressions for the equilibrium outcomes.

Second Period Analysis

The following equations provide the utilities for firm's A new products, firm's B new products, firm's A used products, and firm's B used products, respectively.

$$U_{2A} = \theta - p_{2A}$$
$$U_{2B} = a\theta - p_{2B}$$
$$U_{2AU} = \delta\theta - p_{2AU} - h_A$$
$$U_{2BU} = a\delta\theta - p_{2BU} - h_B$$

Solving for the marginal consumer, we get :

$\theta_1 = \frac{p_{2A}-p_{2B}}{1-a}, \theta_2 = \frac{p_{2B}-p_{2AU}-h_A}{a-\delta}, \theta_3 = \frac{p_{2AU}-p_{2BU}+h_A-h_B}{(1-a)\delta}, \theta_4 = \frac{p_{2BU}+h_B}{a\delta}$

In our analysis hereafter we assume that $0 \leq \theta_4 \leq \theta_3 \leq \theta_2 \leq \theta_1 \leq 1$ so that there is no overlapping among the regions. We believe that our assumption is consistent with what we observe in practice where new and used products from each OEM are available in the market. Moreover, our numerical analysis allows us to identify the conditions under which the competition intensity would force one of the segments to disappear.

Under this assumption, the demand for each product can be expressed as follows: $q_{2A} = 1 - \theta_1$, $q_{2B} = \theta_1 - \theta_2$, $q_{UA} = \theta_2 - \theta_3$, $q_{UB} = \theta_3 - \theta_4$. Figure (5) illustrates the four market segments.

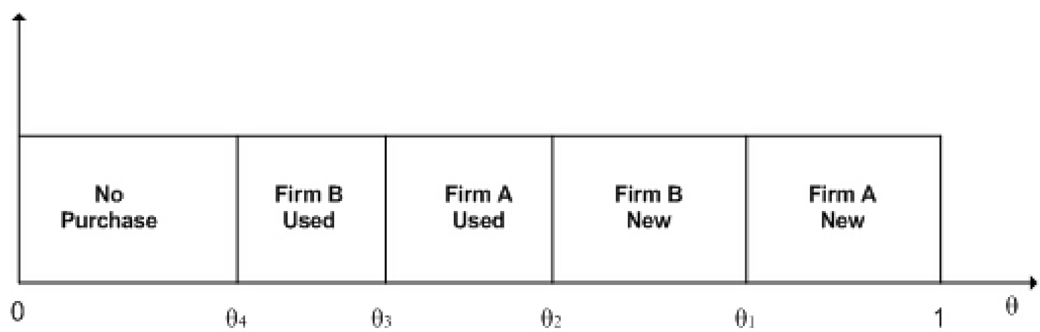

Figure 5: Consumer State Space in the second period

Under perfect competition in the secondary markets, the used products will be traded at a price equal to their procurement cost : $p_{2AU} = s_A$ and $p_{2BU} = s_B$ with corresponding inverse demand functions:

$$p_{2A} = (\delta - a)q_{2B} + h_A - (1-\delta)q_{2A} + 1 - \delta + s_A$$
$$p_{2B} = (\delta - a)q_{2A} + h_A - (a-\delta)q_{2B} + a - \delta + s_A$$

Finally, the second-stage optimization problems for firms A and B are

$$Max_{q_{2A}} \quad \Pi_{2A}(q_{2A}|q_{2B}) = (p_{2A} - c)q_{2A} + h_A q_{UA} \quad s.t \;\; q_{2A} \geq 0$$
$$Max_{q_{2B}} \quad \Pi_{2B}(q_{2B}|q_{2A}) = (p_{2B} - c)q_{2B} + h_B q_{UB} \quad s.t \;\; q_{2B} \geq 0$$

Note that once we substitute the price equations for each product, the only second-period decision variables are the quantities q_{2A} and q_{2B}. By solving the first-order conditions simultaneously, we can derive the Nash Equilibrium of this game, $q^*_{2A}(h_A, h_B, s_A, s_B)$ and $q^*_{2B}(h_A, h_B, s_A, s_B)$, and subsequently the quantities $q^*_{UA}(h_A, h_B, s_A, s_B)$ and $q^*_{UB}(h_A, h_B, s_A, s_B)$.

First period analysis

Similarly to our analysis for the monopolistic OEM, if s_i denotes the salvage value of firm i's new product at the end of period 1, then customers of firms A and B will derive the corresponding utilities:

$$U_{1A} = \theta - p_{1A} + \max\{s_A, \gamma\theta\}$$
$$U_{1B} = a\theta - p_{1B} + \max\{s_B, \gamma a\theta\}$$

Figure (6) illustrates the total demand in the first period as well as the fraction of consumers who decide to return their product for each firm.

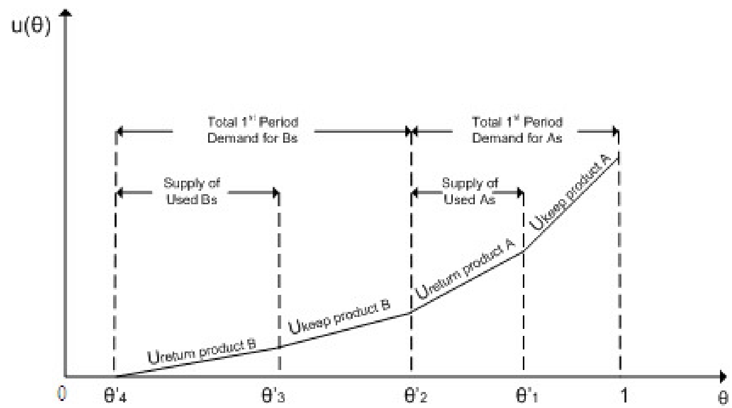

Figure 6: Consumer state space in the first period

The type of the marginal consumers are $\theta'_1 = \frac{s_A}{\gamma}$, $\theta'_2 = \frac{p_{1A}-p_{1B}-s_A}{1-a-\alpha\gamma}$, $\theta'_3 = \frac{s_B}{a\gamma}$ and $\theta'_4 = \frac{p_{1B}-s_B}{a}$, with respective demands for new products of $q_{1A} = 1 - \theta'_2 = 1 - \frac{p_{1A}-p_{1B}-s_A}{1-a-\alpha\gamma}$, and $q_{1B} = \theta'_2 - \theta'_4 = \frac{p_{1A}-p_{1B}-s_A}{1-a-\alpha\gamma} - \frac{p_{1B}-s_B}{a}$, and respective supplies of used products or $q_{UA} = \theta'_1 - \theta'_2 = \frac{s_A}{\gamma} - \frac{p_{1A}-p_{1B}-s_A}{1-a-\alpha\gamma}$, and $q_{UB} = \theta'_3 - \theta'_4 = \frac{s_B}{a\gamma} - \frac{p_{1B}-s_B}{a}$.

By setting those quantities equal to the equilibrium secondary market sizes of the second period $q^*_{UA}(h_A, h_B, s_A, s_B)$ and $q^*_{UB}(h_A, h_B, s_A, s_B)$, we can express the salvage values in terms of the prices of new products and the relicensing fees: $s_A(h_A, h_B, p_{1A}, p_{1B})$ and $s_B(h_A, h_B, p_{1A}, p_{1B})$.

First period profits are given by

$$\Pi_{1A}(q_{1A}|q_{1B}) = (p_{1A} - c)q_{1A}$$
$$\Pi_{1B}(q_{1B}|q_{1A}) = (p_{1B} - c)q_{1B}$$

while the total profits over the two period horizon are:

$$Max_{q_{1A},h_A} \quad \Pi_A(q_{1A},h_A|q_{1B},h_B) = (p_{1A} - c)q_{1A} + \Pi^*_{2A}(q_{1A},h_A|q_{1B},h_B)$$
$$Max_{q_{1B},h_B} \quad \Pi_B(q_{1B},h_B|q_{1A},h_A) = (p_{1B} - c)q_{1B} + \Pi^*_{2B}(q_{1B},h_B|q_{1A},h_A)$$

Once we verify that the conditions for a unique N.E are met (convex strategy set, strictly concave profit functions) we can solve the first-order conditions simultaneously for all the decision variables and derive the values $q^*_{1A,}h^*_A, q^*_{1B,}h^*_B$.

Analysis

We now explore the optimal OEM strategies (relicensing fee and quantity decisions) change as a function of their relative difference in quality. This allows us to study how the market size, both in new and used products, evolves for different values of a. Figure 7 plots h^*_A and h^*_B for a wide range of values in which $0 \leq \theta_4 \leq \theta_3 \leq \theta_2 \leq \theta_1 \leq 1$

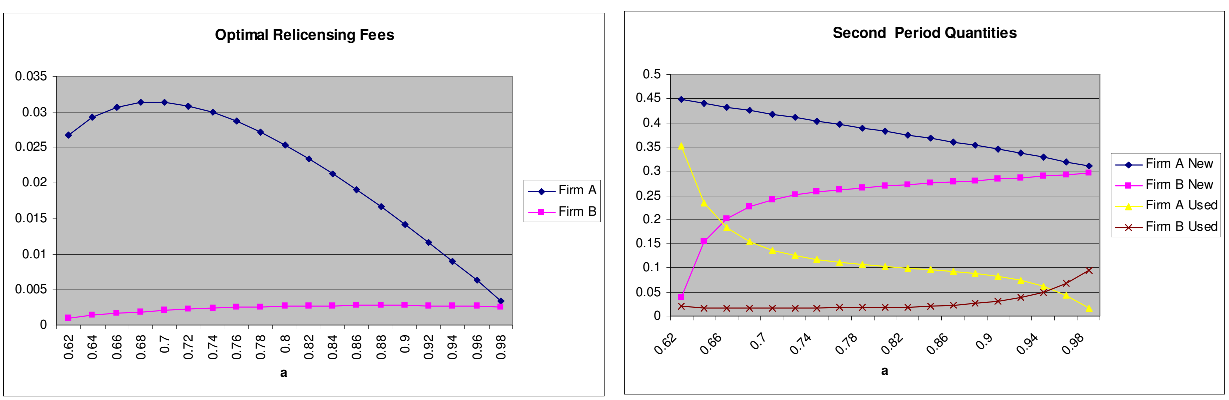

Figure 7: Optimal Relicensing Fees and Second Period Quantities for δ=0.6, γ=0.015, and c=0.05

Observation 1: *The high-end OEM always charges a higher relicensing fee. Yet, there is a non-monotonic relationship between the licensing fee and the quality premium: for very low or very high values of a we observe a low h^*_A, while a higher h^*_A is charged for intermediate values of a.*

As we would expect, the high-end firm A is able to charge a higher relicensing fee. Rather counter to intuition, however, we find that there is a non-monotonic relationship between h^*_A and a: for low values of a, as firm B becomes more competitive (a increases) firm A increases its optimal relicensing fee despite the fact that its products face fiercer competition from products B. This would never be the case in a setting with only primary markets. In the context of secondary markets, however, recall that the optimal relicensing fee balances two opposing forces: the profits generated from first-period customers via the resale value effect versus the cannibalization of new product sales in the second period. For low values of a (close to δ), used products of firm A compete with new products of firm B while the sales of new products of firm A face no significant threat from firm B. As a increases, firm A loses its market share with decreasing margins both in the primary and the secondary market. Instead of decreasing its relicensing fee and strengthening its secondary market, firm A chooses to increase the relicensing fee, further limiting its secondary market. Due to the cannibalization effect, however, this strategy moderates the impact on the sales in the primary market, leading to a smoother drop on firm's A new product margins. In other words, firm A accelerates the losses from the secondary market in order to sustain its competitive advantage in the primary. As a increases further, and firm's A secondary market drops sharply, cannibalization becomes less significant (since now firm B has a significant share in the secondary market) while the increased intensity of competition in the first period (higher a) directs firm A to raise the resale value of its product by charging a lower relicensing fee. Conversely, firm's B increasing quality allows it to charge a higher relicensing fee.

Observation 2: *There is a threshold value for the quality differentiation a above which the low-end OEM might have a larger share on the secondary market. As a increases, the secondary market for the high-end OEM approaches elimination. Therefore, whether a high-end or a low-end quality OEM has a greater incentive to support its secondary market is not clear-cut.*

Recall that higher values of a reflect a smaller gap between the quality performance of the two firms' products. Our numerical experiments suggest that the intersection point above which firm B has a larger secondary market decreases in either one of the following cases: c is decreasing, δ is decreasing, or γ is decreasing. Note that the former condition makes the primary market more profitable, while the latter two reduce the competitiveness of the secondary market. Essentially, firm A exhibits a leader strategy focusing on the market with the higher margins. The above remarks are in line with Ferguson and Toktay (2006), who find that as the unit manufacturing cost increases or as the relative willingness-to-pay δ increases, the relative profitability of the remanufacturing strategy increases. Figure 8 illustrates this case.

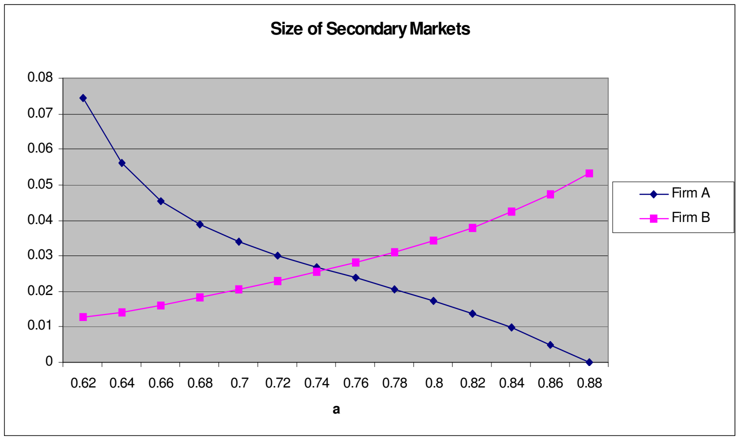

Figure 8: Size of Secondary Market for each firm for for δ=0.6, γ=0.05, and c=0.03

Observation 3: *For low values of willingness-to-pay δ, the low-end OEM is forced to exit the secondary market. On the other hand, for high values of δ, the high-end OEM dominates both the primary and the secondary market.*

Figure 9 plots the second period demand for both new and used products. As we can see, for low values of δ, firm B has no secondary market since the perceived quality of its used products is too low. As δ increases, firm's B secondary market increases as well but its primary market shrinks since firm's A used products become more competitive. For even higher values of δ, firm A dominates both primary and secondary markets since in that case consumers can obtain relatively high quality used products at low prices due to the perfectly competitive secondary market. On the other hand, the relatively high production cost of firm B's new products prohibits any further price reduction.

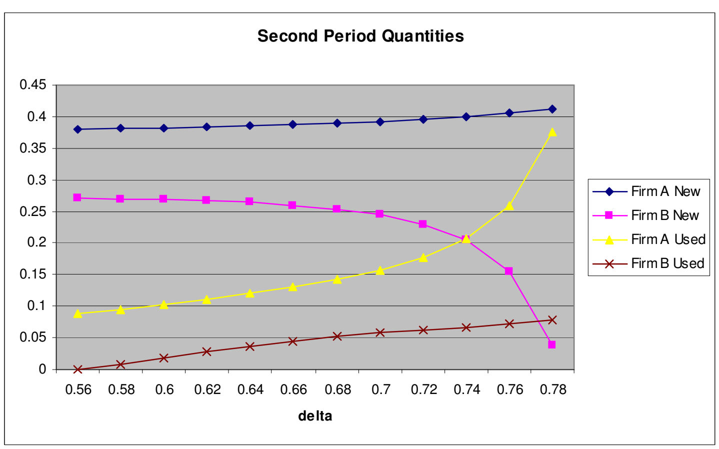

Figure 9: Size of Secondary Market for each firm for a=0.8, γ=0.015, and c=0.05

Note that in our analysis so far we have assumed equal production costs for both firm A and B, even though firm A is perceived to be of higher quality than firm B. This is a reasonable assumption for many IT products since they can be characterized as development-intensive products. The distinguishing feature of this type of products is that their fixed costs of development far outweigh the unit variable costs (Krishnan and Zhu 2006). The assumption of equal production costs, however, is not restrictive since the implication of modifying either firm's cost is apparent. For example, an increasing production cost for firm A would result in lower quantities for its primary market and a corresponding increase in its secondary market.

5 Conclusions

In this paper, we model the dynamics of the Information Technology secondary market for products such as storage devices and servers, and study an OEM's incentives to support or deter it through the setting of a relicensing software fee. We provide insights into the trade-off between the first period benefits from maintaining an active secondary market (i.e. increasing the resale value which increases the original customer's valuation of the product) and the detrimental effect in the second period of cannibalizing the sales of new products (the competition effect). In doing so, we extend the existing theory on secondary markets by including competition at the OEM level and by explicitly modeling the incentives of a third-party (remanufacturer) to purchase and sell the used equipment. In addition, we complement the rapidly growing literature on remanufacturing by endogenizing the competitiveness of the secondary market through the heterogeneous purchase prices of the used product rather than assuming a fixed differentiating factor (i.e. procurement cost).

The primary question we answer is whether or not an OEM is better off eliminating or supporting the secondary market contingent on her competitive advantage, product characteristics, and consumers' preferences. For a monopolistic OEM, we find that consumers' awareness of the salvage value of the product is a key determinant of the market equilibrium. When consumers act strategically, taking into account the resale value effect in their purchase of new products, it is suboptimal for the OEM to completely shut down the secondary market. In contrast, when consumers act myopically, it may be optimal for the OEM to eliminate the secondary market by charging a high relicensing fee. We further explore the case of strategic consumers to identify how their preferences (expressed by their willingness-to-pay as well as to resell a used product) shape the incentives of the OEM to manipulate the size of the secondary market. Our results indicate that when consumers have a high reservation cost for reselling their used products, and consequently the procurement cost for the reseller is high, the OEM should maximize the secondary market by setting the relicensing fee equal to zero. However, as the procurement cost decreases, the relicensing fee increases since the OEM aims to exploit the increased marginal profits of used products. Similarly, the optimal relicensing fee is higher in the secondary markets where consumers value the used product close to the value of the new product.

Under a differentiated duopoly setting with two OEMs providing a high-end and low-end product respectively, we find that the high-end OEM always charges a higher relicensing fee than the low-end OEM. Interestingly, however, there is a non-monotonic relationship between the quality premium of the high-end OEM and its relicensing fee. In particular, there is a range of values for which the cannibalization effect between its new and used products drives the high-end OEM to increase its relicensing fee to deter the secondary market but as competition in the primary market becomes fiercer, the high-end OEM has to rely on the "resale value effect" and thus supports the secondary market. In addition, as the quality gap between the two OEMs decreases, the secondary market for the high-end OEM drops dramatically while the corresponding market of the low-end OEM rises steadily but at a lower rate. Thus, the overall size of the secondary market decreases. Overall, however, the secondary market under the two differentiated OEMs is always greater than the secondary market under a monopolistic OEM.

On the managerial side, our model highlights the strategic importance of supporting an active secondary market. This finding holds even for OEMs producing products on the high-end of the market. Thus, charging high relicensing fees that effectively shut down the secondary market, a strategy employed by some OEMs, appears to be myopic and suboptimal, at least in the presence of strategic consumers. Therefore, all OEMs should consider integrating their primary and secondary market strategies as the two are dependent upon each other. Indeed, companies that have been pioneers in implementing such strategies have experienced significant profits. For instance, IBM's global asset recovery systems division accounts for approximately 12% of IBM's profit (Johnson 2006). Furthermore, even companies that have been traditionally aggressive in deterring the secondary market for their products are gradually shifting to less strict policies. For example, the latest version of the software used to run SUN servers (Solaris 10) is now available for free on SUN's web site. Still, old habits are hard to break as SUN continues to charge significant fees for a subscription to obtain fixes, updates, and patches for their software; a fee that deters the secondary market.

Our analysis demonstrates that a well-informed consumer base can influence the OEM's strategy. Historically, customers of IT products did not take into account the future resale value in their initial purchases, a practice that could partially explain why some OEMs deployed policies to deter the secondary market for their products. Recent trends however, suggest that this is no longer the case. Today, there exists a large and increasing number of industry analyst firms who specialize in forecasting the resale value of IT equipment and offer comprehensive cost/benefit analysis over the lifecycle of the IT equipment. Perhaps more importantly, environmental advocates are suggesting that the existence of active secondary markets is the most efficient way of dealing with the growing concern of IT waste disposal. Thus, the combination of more strategic customers, the availability of predicted resell values for IT equipment, and environmental pressures are causing major IT equipment manufacturers to reevaluate their previous strategies for deterring the secondary market for their products.

We conclude by briefly mentioning some robustness results that were not included in the paper. One of the major assumptions made in developing our model was that there is no other cost incurred by the third-party remanufacturer to make the old products available for the market other than the procurement cost from the customers of the new product in the first period. In other words, a used product purchased from a consumer can be sold as is. A unit remanufacturing cost can be easily incorporated in our model without affecting any of our qualitative insights. Interestingly, however, as the remanufacturing cost increases, both the third-party remanufacturer's and the OEM's profits decrease, exhibiting a convex relationship. This seemingly counter-intuitive effect is driven by the fact that any increase in the remanufacturing cost makes the reselling process less profitable, leading to lower demand for used products, and therefore, lower salvage values which affects the price of the new product in the first period. A second major assumption was that the relicensing fee is set in the first period and that consumers are aware of its value even though there are no used products for sell at that time. Due to the game theoretic nature of our model, allowing the OEM to set the relicensing fee in the second period has no impact on our results. If the OEM is allowed to set the relicensing fee in the second period, the OEM's second period profit is convex in the relicensing fee h, indicating the OEM should set it to its highest value. Yet, strategic consumers anticipate this strategy into their first period purchasing behavior by calculating the corresponding salvage value. Essentially, the OEM's total profits are maximized by signaling to the first period consumers a low relicensing fee (acting as an upper bound). Interestingly, the value that maximizes the OEM's total profits is the same as if the relicensing fee was set in the first period.

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Appendix

Proof of Lemma 1

The utilities for the new and used product in the second period will be:

$$U_2 = \theta - p_2$$

and

$$U_u = \delta\theta - p_u - h$$

where h is the license fee

Solving for the "indifferent" consumer, we get the following inverse demand equations

$$\theta_1 = \frac{p_2 - p_u - h}{1 - \delta}$$

$$\theta_2 = \frac{p_u + h}{\delta}$$

Which lead us to the following inverse demand functions:

price of new products : $p_2 = -q_2 + 1 - q_u\delta$

price of used products : $p_u = -q_u\delta + \delta - q_2\delta \ - h$

the respective profits will be :

$$Max_{q_2} \qquad \Pi_2(q_2|q_u) = (p_2 - c)q_2 + hq_u = (1 - q_2 - q_u\delta - c)\, q_2 + hq_u$$

$$s.t \qquad q_2 \geq 0$$

$$Max_{q_u,s}(q_u, s|q_2) \qquad \Pi_u = (p_u - s)q_u$$

$$q_u \leq q_1$$

$$q_u \leq s(\frac{1}{\gamma} + 1) - p_1^* \tag{1}$$

$$q_u \geq 0$$

The Lagrangian for the RM's problem is

$L(q_u, s, \lambda_1, \lambda_2) = \Pi_u(q_u, s) + \lambda_1(s(\frac{1}{\gamma} + 1) - p_1^* - q_u) + \lambda_2(q_1 - q_u) + \mu_1 q_u$

The conditions for optimality are :

$\frac{\partial L}{\partial q_u} = 0,$

$\frac{\partial L}{\partial s} = 0,$

$\lambda_1(s(\frac{1}{\gamma} + 1) - p_1^* - q_u) = 0$

$\lambda_2(q_1 - q_u) = 0$

$\mu_1 q_u = 0$

$\lambda_1 \geq 0, \lambda_2 \geq 0, \mu_1 \geq 0,$

Assume $s(\frac{1}{\gamma}+1) - p_1 - q_u > 0$,then $\lambda_1 = 0$. In this case, $\frac{\partial L}{\partial s} = -q_u \leqslant 0$.

However, if $q_u = 0$, there is no secondary market and thus, $s = 0$.Apparently, $s(\frac{1}{\gamma}+1)-p_1-q_u > 0$ will be a contradiction. Therefore, $\frac{\partial L}{\partial s} = -q_u < 0$.

Since this case can not meet the FOC ($\frac{\partial L}{\partial s} = 0$), from hereafter we assume that inequality (1) holds as an equality. Intuitively, the entrant would not be willing to buy more used units than the quantity she would launch in the secondary market.

Proof of Proposition 1

From Lemma 1 we know that, $q_u = s(\frac{1}{\gamma}+1) - p_1^* \Longrightarrow$

$$s = \frac{\gamma(q_u + p_1^*)}{1+\gamma} \tag{2}$$

Therefore, we can rewrite the entrant's problem as:

$$Max_{q_u} \qquad \Pi_u = (p_u - \frac{\gamma(q_u + p_1^*)}{1+\gamma})q_u$$

$$q_u \leq q_1^* \tag{3}$$

$$q_u \geq 0$$

while the OEM's objective is given by:

$$Max_{q_2} \qquad \Pi_2 = (-q_2 + 1 - q_u\,\delta - c)\,q_2 + h^* q_u$$

$$q_2 \geq 0$$

We solve the following problem by assuming that constraint (3) is not binding at the optimal solution. After determining the optimal first period price (p_1^*) and the corresponding optimal quantity (q_1^*) we verify our assumption by substituting this value to constraint (3) and showing that it always holds as a strict inequality.

Solving simultaneously for the quantities of the two competitors, we obtain the following Nash Equilibria (N.E) :

$$q_2^* = -\frac{2\,\delta + 2\,\delta\,\gamma + 2\,\gamma - \delta^2 - \delta^2\gamma + \delta\,h^* + \delta\,h^*\gamma + \delta\,\gamma p_1^* - 2\,\delta\,c - 2\,\delta\,\gamma c - 2\,\gamma c}{-4\,\delta - 4\,\delta\,\gamma - 4\,\gamma + \delta^2 + \delta^2\gamma} \tag{4}$$

$$q_u^* = \frac{-\delta\,\gamma + 2\,h^* + 2\,h^*\gamma - \delta - \delta\,c + 2\,\gamma p_1^* - \delta\,\gamma c}{-4\,\delta - 4\,\delta\,\gamma - 4\,\gamma + \delta^2 + \delta^2\gamma} \tag{5}$$

substituting q_u^* from (5),

(2) can be rewritten as :

$$s = \frac{\gamma(-\delta\,\gamma + 2\,h^* + 2\,h^*\gamma - \delta - \delta\,c - 2\,\gamma p_1^* - \delta\,\gamma c - 4\,\delta\,p_1^* - 4\,\delta\,\gamma p_1^* + \delta^2 p_1^* + \delta^2\gamma p_1^*)}{(-4\,\delta - 4\,\delta\,\gamma - 4\,\gamma + \delta^2 + \delta^2\gamma)(1+\gamma)}$$

First Period Analysis

The price in the first period, is given by

$$p_1 \;=\; 1 - q_1 + s \tag{6}$$

solving simultaneously (**??**) and (6) we get

$$p_1 \;=\; -\frac{(-q_1\,\delta^2\gamma + 4\,q_1\,\gamma + 4\,q_1\,\delta\,\gamma + \delta^2\gamma - 4\,\gamma + 2\,h\gamma - 5\,\delta\,\gamma - \delta\,\gamma c + 4\,q_1\,\delta - q_1\delta^2 - 4\,\delta + \delta^2)(1+\gamma)}{4\,\delta + 4\,\delta\,\gamma + 4\,\gamma + 2\,\gamma^2 - \delta^2 - \delta^2\gamma}$$

First period profits are consist of only the sales from new products :

$$\Pi_1(q_1, h) \;=\; (p_1 - c)q_1$$
$$st \qquad q_1 \geq 0, h \geq 0$$

and the cumulative profits over the two periods will be $\Pi = \Pi_1 + \Pi_2$

The OEM's overall objective will be :

$$\begin{aligned} Max_{q_1,h} \quad & \Pi(q_1, h) \\ \text{st} \quad & q_1 \geq 0 \\ & q_2^* \geq 0 \\ & q_u^* \geq 0 \\ & h \geq 0 \end{aligned}$$

Given, the second period equilibrium equations, an equivalent form is:

$$Max_{q_1,h} \qquad \Pi(q_1, h)$$

s.t

$$q_1 \geq 0 \tag{7}$$

$$h \geq \gamma q_1 + A \tag{8}$$

$$h \leq \gamma q_1 + B \tag{9}$$

$$h \geq 0$$

where $A = \frac{-\delta\,\gamma^2 - \gamma^2 + \gamma^2 c + \delta^2\gamma + 2\,\delta\,\gamma c - 3\,\delta\,\gamma - 2\,\gamma + 2\,\gamma c + \delta^2 - 2\,\delta + 2\,\delta\,c}{\delta\,(1+\gamma)}$ and $B = \frac{\delta(1+c)}{2} - \gamma$

Note that, $B - A = \frac{(4\delta+4\delta\gamma+4\gamma+2\gamma^2-\delta^2-\delta^2\gamma)(1-c)}{2\delta(1+\gamma)} > 0$ for $c < 1$,so $A < B$.

The Lagrangean is:

$L(q_1, h, \lambda_1, \lambda_2) = \Pi(q_1, h) + \lambda_1(\gamma q_1 + B - h) + \lambda_2(h - \gamma q_1 - A) + \mu_1 q_1 + \mu_2 h$

with the following conditions for optimality :

$$\frac{\partial L}{\partial q_1} = 0 \tag{10}$$

$$\frac{\partial L}{\partial h} = 0 \tag{11}$$

$$\lambda_1(\gamma q_1 + B - h) = 0 \tag{12}$$

$$\lambda_2(h - \gamma q_1 - A) = 0 \tag{13}$$

$$\mu_1 q_1 = 0 \tag{14}$$

$$\mu_2 h = 0 \tag{15}$$

$$\lambda_1 \geq 0, \lambda_2 \geq 0, \mu_1 \geq 0, \mu_2 \geq 0$$

Case 1. $\gamma q_1 + B - h = 0$ and $q_1 > 0$, then $\lambda_2 = 0, \mu_1 = 0, \mu_2 = 0$.

Solving (10) and (11) gives

$\lambda_1 = -\frac{2c(\delta\gamma-\gamma+\delta)}{4\delta+4\delta\gamma+4\gamma+2\gamma^2-\delta^2(1+\gamma)}$

and $h = -\frac{1}{2}\frac{-\delta\gamma c+c\gamma-\delta c+\gamma^2-\delta-\delta\gamma+\gamma}{1+\gamma}$

We need $\lambda_1 \geq 0$ which holds only for $\delta < \frac{\gamma}{1+\gamma}$

since $4\delta + 4\delta\gamma + 4\gamma + 2\gamma^2 - \delta^2(1+\gamma) > 0$

Also, $h \geq 0$ holds only for $c < \frac{\delta(1+\gamma)-\gamma-\gamma^2}{\gamma-\delta-\gamma\delta}$.

However, when $\delta < \frac{\gamma}{1+\gamma}$, then$\frac{\delta(1+\gamma)-\gamma-\gamma^2}{\gamma-\delta-\gamma\delta} < 0$ and since $c < \frac{\delta(1+\gamma)-\gamma-\gamma^2}{\gamma-\delta-\gamma\delta}$., we conclude that $c < 0$ which is false.

Since, $\lambda_1 \geq 0$ and $h \geq 0$ can not be satisfied simultaneously, this case can not meet the conditions for optimality.

In other words, constraint (9), is not binding at the optimal solution, and thus, $q_u^* > 0$

Case 2. $(h - \gamma q_1 - A) = 0$,and $q_1 > 0$ then $\lambda_1 = 0, \mu_1 = 0, \mu_2 = 0$.

Solving (10) and (11) gives

$$\lambda_2 = \frac{-8\delta c + 8\gamma - 8c\gamma + \delta^2\gamma c - 3\delta^2\gamma - 6\delta\gamma c + \delta^2 c - 3\delta^2 + 8\delta(1+\gamma)}{\delta(-4\delta - 4\delta\gamma - 4\gamma - 2\gamma^2 + \delta^2(1+\gamma))} \tag{16}$$

$$h = \frac{1}{2}\frac{4\delta c - 4\gamma + 4\gamma c + 2\delta^2\gamma + 3\delta\gamma c + 2\delta^2 - 4\delta - 5\delta\gamma - \delta\gamma^2}{\delta(1+\gamma)} \tag{17}$$

We need $\lambda_2 \geq 0$ or equivalently

$$-8\delta c + 8\gamma - 8c\gamma + \delta^2\gamma c - 3\delta^2\gamma - 6\delta\gamma c + \delta^2 c - 3\delta^2 + 8\delta(1+\gamma) \leq 0 \tag{18}$$

since $\delta(-4\delta - 4\delta\gamma - 4\gamma - 2\gamma^2 + \delta^2(1+\gamma)) < 0$.

(18) is equivalent to $c \geq \frac{(1+\gamma)(3\delta^2-8\delta)-8\gamma}{-8\gamma+\delta^2(1+\gamma)-8\delta-6\delta\gamma}$

for $\delta > \frac{\gamma}{1+\gamma}$,$\frac{(1+\gamma)(3\delta^2-8\delta)-8\gamma}{-8\gamma+\delta^2(1+\gamma)-8\delta-6\delta\gamma} < 1$ and thus (18) is possible.

We also need $h \geq 0 \Rightarrow 4\delta c - 4\gamma + 4\gamma c + 2\delta^2\gamma + 3\delta\gamma c + 2\delta^2 - 4\delta - 5\delta\gamma - \delta\gamma^2 \geq 0 \Rightarrow$
$c \geq \frac{\delta\gamma^2-2\delta^2+4\delta+5\delta\gamma+4\gamma-2\delta^2\gamma}{(4\delta+4\gamma+3\delta\gamma)}$

Again for $\delta > \frac{\gamma(\gamma+2)}{2(1+\gamma)}$, $\frac{\delta\gamma^2-2\delta^2+4\delta+5\delta\gamma+4\gamma-2\delta^2\gamma}{(4\delta+4\gamma+3\delta\gamma)} < 1$ and the condition is possible.

However, since $\frac{(1+\gamma)(3\delta^2-8\delta)-8\gamma}{-8\gamma+\delta^2(1+\gamma)-8\delta-6\delta\gamma} < \frac{\delta\gamma^2-2\delta^2+4\delta+5\delta\gamma+4\gamma-2\delta^2\gamma}{(4\delta+4\gamma+3\delta\gamma)}$, it is sufficient to have
$c \geq \frac{(1+\gamma)(3\delta^2-8\delta)-8\gamma}{-8\gamma+\delta^2(1+\gamma)-8\delta-6\delta\gamma}$ in which case $h > 0$

Case 3. $h - \gamma q_1 - A \neq 0$ and $\gamma q_1 + B - h \neq 0$ then $\lambda_1 = 0, \lambda_2 = 0, \mu_1 = 0, \mu_2 = 0$ (since $A \neq B$)

Solving (10) and (11) gives

$$\lambda_2 = \frac{3\delta^2+6\delta^2\gamma+8c\gamma+8\delta c+4\delta\gamma c-8\gamma-16\delta\gamma-8\delta-8\gamma^2-8\delta\gamma^2+4c\gamma^2-3\delta^2\gamma c-4\delta\gamma^2 c-3\delta^2 c+3\delta^2\gamma^2}{(1+\gamma)(3\delta^2\gamma+3\delta^2-8\delta\gamma-8\,\delta-8\gamma)}$$

$$h = \frac{1}{2}\frac{\delta^3\gamma c+3\delta^3\gamma+\delta^3 c+3\delta^3-3\delta^2\gamma^2-\delta^2\gamma c-11\delta^2\gamma-8\delta^2+8\gamma^2+8\delta\gamma^2}{3\delta^2\gamma+3\delta^2-8\delta\gamma-8\delta-8\gamma}$$

this case can also be feasible (at the optimality) as long as
for $c \leq -(\frac{3\delta^2+6\delta^2\gamma-16\delta\gamma-8\delta-8\gamma^2-8\gamma+3\delta^2\gamma^2-8\delta\gamma^2}{8\delta+4\delta\gamma+8\gamma-3\delta^2\gamma-4\delta\gamma^2-3\delta^2+4\gamma^2})$
and $c \leq \frac{3\delta^2\gamma^2-3\delta^3\gamma+11\delta^2\gamma-3\delta^3-8\gamma^2-8\delta\gamma^2+8\delta^2}{\delta^3+\delta^3\gamma-\delta^2\gamma}$

Case 4. $h-\gamma q_1-A=0$ and $\gamma q_1+B-h=0$,then $A=B$ which is false since $A<B$ so this case can not occur .

Case 5. $h=0$ and $q_1\neq 0$,then $\lambda_1=0,\lambda_2=0,\mu_1=0$ since $\delta>\gamma>\frac{\gamma}{1+\gamma}$ the case becomes feasible for $c\geq\frac{3\delta^2\gamma^2-3\delta^3\gamma+11\delta^2\gamma-3\delta^3-8\gamma^2-8\delta\gamma^2+8\delta^2}{\delta^3+\delta^3\gamma-\delta^2\gamma}$

To summarize the above Lagrangean analysis, we have the following conditions : for $c\leq\frac{3\delta^2\gamma^2-3\delta^3\gamma+11\delta^2\gamma-3\delta^3-8\gamma^2-8\delta\gamma^2+8\delta^2}{\delta^3+\delta^3\gamma-\delta^2\gamma}$ we fall into case 3 ($q_2>0$ and $h>0$), while for $c\geq\frac{3\delta^2\gamma^2-3\delta^3\gamma+11\delta^2\gamma-3\delta^3-8\gamma^2-8\delta\gamma^2+8\delta^2}{\delta^3+\delta^3\gamma-\delta^2\gamma}$ we fall into case 5.

Finally, for very high values of $c\geq\frac{(1+\gamma)(3\delta^2-8\delta)-8\gamma}{-8\gamma+\delta^2(1+\gamma)-8\delta-6\delta\gamma}$ case 2 ($q_2=0$) becomes also feasible.

Proof of Remark 1

We start by case 2. From our previous analysis,

$$h^*=\frac{1}{2}\frac{4\delta c-4\gamma+4\gamma c+2\delta^2\gamma+3\delta\gamma c+2\delta^2-4\delta-5\delta\gamma-\delta\gamma^2}{\delta(1+\gamma)} \tag{19}$$

$$\frac{\partial}{\partial\delta}\frac{1}{2}\frac{4\delta c-4\gamma+4\gamma c+2\delta^2\gamma+3\delta\gamma c+2\delta^2-4\delta-5\delta\gamma-\delta\gamma^2}{\delta(1+\gamma)} \tag{20}$$

Thus, $\dfrac{\partial h^*}{\partial\delta}=\frac{2\delta^2+2\gamma\delta^2+4\gamma+-4c\gamma}{2\delta^2+2\gamma\delta^2}>0$ and $h^*$is decreasing in δ.

To prove that $h^*$is decreasing in γ it is sufficient to show that the numerator of (19) is decreasing in γ.

By taking the first derivative, we get $\dfrac{\partial}{\partial\gamma}(4\delta c-4\gamma+4\gamma c+2\delta^2\gamma+3\delta\gamma c+2\delta^2-4\delta-5\delta\gamma-\delta\gamma^2)=$ $=4c-5\delta+3c\delta-2\gamma\delta+2\delta^2-4=4(c-1)+3\delta(c-1)-2\gamma\delta+2\delta(\delta-1)$ which is obviously negative for $c<1$.

Thus,$h^*$is decreasing in γ.

Finally,$\dfrac{\partial h^*}{\partial c}=\frac{1}{2\delta+2\gamma\delta}(4\gamma+4\delta+3\gamma\delta)>0$

Similarly, we prove the properties for case 3, where

$h^*=\frac{1}{2}\frac{\delta^3\gamma c+3\,\delta^3\gamma+\delta^3c+3\,\delta^3-3\,\delta^2\gamma^2-\delta^2\gamma c-11\,\delta^2\gamma-8\,\delta^2+8\,\gamma^2+8\,\delta\,\gamma^2}{3\,\delta^2\gamma+3\,\delta^2-8\,\delta\,\gamma-8\,\delta-8\,\gamma}$

Note, however, that in this case $\dfrac{\partial h^*}{\partial c}=\frac{\delta^2(\delta-\gamma+\gamma\delta)}{6\delta^2-16\delta-16\gamma\delta-16\gamma+6\gamma\delta^2}<0$

Proof of Proposition 2

The utilities for the new and used product in the second period will be:

$U_2=\theta-p_2$ and $U_r=\delta\theta-p_u-h$ where h is the license fee

Solving for the "indifferent" consumer, we get the following inverse demand equations

$$\theta_1=\frac{p_2-p_u-h}{1-\delta}$$

$$\theta_2=\frac{p_u+h}{\delta}$$

price of new products : $p_2 = -q_2 + 1 - \delta \sum_{i=1}^{N} q_u^i$

price of used products : $p_u = \delta - q_2\delta \ - h - \delta \sum_{i=1}^{N} q_u^i$

the respective profits will be :

$$
\begin{aligned}
&\text{OEM}: \ Max_{q_2} \quad \Pi_2 = \left(-q_2 + 1 - \delta \sum_{i=1}^{N} q_u^i - c\right) q_2 + h \sum_{i=1}^{N} q_u^i \\
&\text{s.t} \quad q_2 \geq 0 \\
&\text{and for the } i^{th} \text{ RM} : \ Max_{q_u^i} \quad \Pi_r = (p_u - s) q_u^i \\
&\text{s.t} \quad \sum_{i=1}^{N} q_u^i \leq s(\tfrac{1}{\gamma} + 1) - p_1 \\
&\qquad \sum_{i=1}^{N} q_u^i \leq q_1 \\
&\qquad q_u \geq 0
\end{aligned}
$$

Let $Q_u^{-i} = \sum_{j=1, j\neq i}^{N} q_u^j$

then we can rewrite the OEM's and RM's problems as :

$$
\begin{aligned}
&\text{OEM}: \ Max_{q_2} \quad \Pi_2 = \left(-q_2 + 1 - \delta q_u^i - \delta Q_u^{-i} - c\right) q_2 + h(q_u^i + Q_u^{-i}) \\
&\text{s.t} \quad q_2 \geq 0 \\
&\text{and for the } i^{th} \text{ RM} : \ Max_{q_u^i} \quad \Pi_u = (\delta - q_2\delta \ - h - \delta q_u^i - \delta Q_u^{-i} - s) q_u^i \\
&\text{s.t} \quad q_u^i + Q_u^{-i} \leq s(\tfrac{1}{\gamma} + 1) - p_1 \\
&\qquad q_u^i + Q_u^{-i} \leq q_1 \\
&\qquad q_u \geq 0
\end{aligned}
$$

The FOC with respect to q_2 and q_r^i

$$
\begin{aligned}
-2q_2 + 1 - \delta q_u^i - \delta Q_u^{-i} - c = 0 \\
\delta - q_2\delta \ - h - 2\delta q_u^i - \delta Q_u^{-i} - s = 0
\end{aligned}
$$

however, since we assume N symmetric remanufacturers $Q_u^{-i} = (N-1) q_u^i$

and the FOC can be rewritten as

$$
-2q_2 + 1 - \delta N q_u^i - c = 0 \tag{21}
$$

$$
\delta - q_2\delta \ - h - \delta(N+1) q_u^i - s = 0 \tag{22}
$$

In addition, the market clearing price s will satisfy

$$
N q_u^i = s(\frac{1}{\gamma} + 1) - p_1 \tag{23}
$$

solving simultaneously (21) ,(22) ,and (23)

we get the equilibrium quantities

$q_2 = -\frac{\gamma N+\delta+\delta\gamma+\delta N+\gamma N\delta-\delta^2 N-\delta^2 N\gamma+\delta Nh+\delta N\gamma h+\delta N\gamma p_1-c\gamma N-\delta c-\gamma\delta c-N\delta c-\gamma N\delta c}{-2\gamma N-2\delta-2\delta\gamma-2\delta N-2\gamma N\delta+\delta^2 N+\delta^2 N\gamma}$

$q_u = \frac{-\delta-\delta\gamma+2h+2\gamma h-\delta c-\gamma\delta c+2\gamma p_1}{-2\gamma N-2\delta-2\delta\gamma-2\delta N-2\gamma N\delta+\delta^2 N+\delta^2 N\gamma}$

$s = \frac{\gamma\left(-\delta N+2Nh-2\delta p_1-N\delta c+\delta^2 Np_1-2\delta Np_1\right)}{-2\gamma N-2\delta-2\delta\gamma-2\delta N-2\gamma N\delta+\delta^2 N+\delta^2 N\gamma}$

From hereafter we proceed as in the case of one RM:

Since the OEM maximizes over q_1 and h, first we take the first derivative $\frac{\partial\Pi(q_1,h)}{\partial q_1}$ and set it equal to zero so that we derive the $q_1^*(h)$.

We can now substitute this value to the expression for the OEM's profits and apply the FOC with respect to h (since the function is concave in h)

The FOC yield :

$$h^* = \frac{(\delta^3 Nc+3\delta^3 N+3\delta^3 N\gamma+\delta^3 N\gamma c-4\delta^2 N-7\delta^2 N\gamma-3\delta^2\gamma^2 N-\delta^2 N\gamma c-4\delta^2\gamma-4\delta^2+4\gamma^2 N\delta+4\delta\gamma+4\delta\gamma^2+4\gamma^2 N)}{2(3\delta^2 N+3\delta^2 N\gamma-4\gamma N\delta-4\delta N-4\delta\gamma-4\delta-4\gamma N)}$$

From, the last expression :

$$\frac{\partial h^*}{\partial N} = -2\,\frac{\delta^3 c\left(-\gamma^2-\gamma+\delta+2\,\delta\,\gamma+\delta\,\gamma^2\right)}{\left(3\,\delta^2 N+3\,\delta^2 N\gamma-4\,\gamma N\delta-4\,\delta\,N-4\,\delta\,\gamma-4\,\delta-4\,\gamma N\right)^2}$$

$$\frac{\partial^2 h^*}{\partial N^2} = 4\,\frac{\delta^3 c\left(-\gamma^2-\gamma+\delta+2\,\delta\,\gamma+\delta\,\gamma^2\right)\left(3\,\delta^2+3\,\delta^2\gamma-4\,\delta\,\gamma-4\,\delta-4\,\gamma\right)}{\left(3\,\delta^2 N+3\,\delta^2 N\gamma-4\,\gamma N\delta-4\,\delta\,N-4\,\delta\,\gamma-4\,\delta-4\,\gamma N\right)^3}$$

Since we are in the case of positive license fees $\delta > \frac{\gamma}{1+\gamma} \Leftrightarrow \delta(1+\gamma) > \gamma \Leftrightarrow$
$\delta(1+\gamma)^2 > \gamma(1+\gamma) \Leftrightarrow \left(-\gamma^2-\gamma+\delta+2\,\delta\,\gamma+\delta\,\gamma^2\right) > 0 \Rightarrow \frac{\partial^2 h^*}{\partial N^2} < 0$

Also, we can readily see that

$\left(3\,\delta^2+3\,\delta^2\gamma-4\,\delta\,\gamma-4\,\delta-4\,\gamma\right) < 0$

$\left(3\,\delta^2 N+3\,\delta^2 N\gamma-4\,\gamma N\delta-4\,\delta\,N-4\,\delta\,\gamma-4\,\delta-4\,\gamma N\right) < 0$

and therefore $\frac{\partial^2 h^*}{\partial N^2} > 0$. Thus, we proved that h^* is convex decreasing in N.

Also,

$$\frac{\partial\Pi^*}{\partial N} = 4\,\frac{\delta\,c^2\left(\delta^2\gamma^2+2\,\delta^2\gamma+\delta^2-2\,\delta\,\gamma^2-2\,\delta\,\gamma+\gamma^2\right)}{\left(3\,\delta^2 N+3\,\delta^2 N\gamma-4\,\gamma N\delta-4\,\delta\,N-4\,\delta\,\gamma-4\,\delta-4\,\gamma N\right)^2}$$

$\frac{\partial^2\Pi^*}{\partial N^2} = -8\,\frac{\delta\,c^2\left(\delta^2\gamma^2+2\,\delta^2\gamma+\delta^2-2\,\delta\,\gamma^2-2\,\delta\,\gamma+\gamma^2\right)\left(3\,\delta^2+3\,\delta^2\gamma-4\,\delta\,\gamma-4\,\delta-4\,\gamma\right)}{\left(3\,\delta^2 N+3\,\delta^2 N\gamma-4\,\gamma N\delta-4\,\delta\,N-4\,\delta\,\gamma-4\,\delta-4\,\gamma N\right)^3}$

Similarly,

$\left(\delta^2\gamma^2+2\,\delta^2\gamma+\delta^2-2\,\delta\,\gamma^2-2\,\delta\,\gamma+\gamma^2\right) = \delta(1+\gamma)[\delta(1+\gamma)-2\,\delta\,\gamma]+\gamma^2 > 0$

$\left(3\,\delta^2+3\,\delta^2\gamma-4\,\delta\,\gamma-4\,\delta-4\,\gamma\right) < 0$

$\left(3\,\delta^2 N+3\,\delta^2 N\gamma-4\,\gamma N\delta-4\,\delta\,N-4\,\delta\,\gamma-4\,\delta-4\,\gamma N\right) < 0$

so, $\frac{\partial^2\Pi^*}{\partial N^2} < 0$

By substituting the optimal license fee, to the second period equation we derive the corresponding expressions and the proofs regarding the properties are similar .

Along the same lines, we can prove that $\frac{\partial q_r^*}{\partial N} < 0$ with $\frac{\partial^2 q_r^*}{\partial N^2} > 0, \frac{\partial Q_r^*}{\partial N} > 0$ with $\frac{\partial^2 Q_r^*}{\partial N^2} < 0$, and $\frac{\partial s}{\partial N} > 0$ and $\frac{\partial^2 s}{\partial N^2} < 0$

$$\frac{\partial q_r^*}{\partial N} = 2\,\frac{\left(3\,\delta^3\gamma^2 + 6\,\delta^3\gamma + 3\,\delta^3 - 7\,\delta^2\gamma^2 - 11\,\delta^2\gamma - 4\,\delta^2 + 4\,\gamma^2\right)c}{\left(3\,\delta^2 N + 3\,\delta^2 N\gamma - 4\,\gamma N\delta - 4\,\delta\,N - 4\,\delta\,\gamma - 4\,\delta - 4\,\gamma N\right)^2}$$

$$\frac{\partial^2 q_r^*}{\partial N^2} = -4\,\frac{\left(3\,\delta^3\gamma^2 + 6\,\delta^3\gamma + 3\,\delta^3 - 7\,\delta^2\gamma^2 - 11\,\delta^2\gamma - 4\,\delta^2 + 4\,\gamma^2\right)c\left(3\,\delta^2 + 3\,\delta^2\gamma - 4\,\delta\,\gamma - 4\,\delta - 4\,\gamma\right)}{\left(3\,\delta^2 N + 3\,\delta^2 N\gamma - 4\,\gamma N\delta - 4\,\delta\,N - 4\,\delta\,\gamma - 4\,\delta - 4\,\gamma N\right)^3}$$

$$\frac{\partial Q_r^*}{\partial N} = 8\,\frac{\left(-\gamma^2 - \gamma + \delta + 2\,\delta\,\gamma + \delta\,\gamma^2\right)\delta\,c}{\left(3\,\delta^2 N + 3\,\delta^2 N\gamma - 4\,\gamma N\delta - 4\,\delta\,N - 4\,\delta\,\gamma - 4\,\delta - 4\,\gamma N\right)^2}$$

$$\frac{\partial^2 Q_r^*}{\partial N^2} = -16\,\frac{\left(-\gamma^2 - \gamma + \delta + 2\,\delta\,\gamma + \delta\,\gamma^2\right)\delta\,c\left(3\,\delta^2 + 3\,\delta^2\gamma - 4\,\delta\,\gamma - 4\,\delta - 4\,\gamma\right)}{\left(3\,\delta^2 N + 3\,\delta^2\gamma - 4\,\gamma N\delta - 4\,\delta\,N - 4\,\delta\,\gamma - 4\,\delta - 4\,\gamma N\right)^3}$$

$$\frac{\partial s}{\partial N} = 8\,\frac{\left(\delta + \delta\,\gamma - \gamma\right)g\delta\,c}{\left(3\,\delta^2 N + 3\,\delta^2 N\gamma - 4\,\gamma N\delta - 4\,\delta\,N - 4\,\delta\,\gamma - 4\,\delta - 4\,\gamma N\right)^2}$$

$$\frac{\partial^2 s}{\partial N^2} = -16\,\frac{\left(\delta + \delta\,\gamma - \gamma\right)\gamma\delta\,c\left(3\,\delta^2 + 3\,\delta^2\gamma - 4\,\delta\,\gamma - 4\,\delta - 4\,\gamma\right)}{\left(3\,\delta^2 N + 3\,\delta^2 N\gamma - 4\,\gamma N\delta - 4\,\delta\,N - 4\,\delta\,\gamma - 4\,\delta - 4\,\gamma N\right)^3}$$