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Optimal Blending Model for Paper Manufacturing With Competing Input Materials

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Abstract:

The stringent environmental regulations, growing shortages of raw materials have brought the concept of reverse supply chain into limelight. This paper presents a linear programming model for minimizing the cost of paper manufacturing by optimal blending of two competing input materials viz. wood as well as by recycling wastepaper. The objective function includes forward and reverse supply chain costs encountered during paper manufacturing from the manufacturer's viewpoint. The model also includes the economic implications of using the alternative input materials on the environment, paper quality along with the possible reuse of the non-relevant wastepaper collected during the segregation stage. The model can assist the manufacturer to determine the optimal amount of wood as well as wastepaper required to satisfy the market demand under a given situation. To gain further insight into the system behaviour, sensitivity analyses have been carried out which highlights various strategies under different conditions.

Key Words: Optimal blending, Supply chain, Paper manufacturing, Linear Programming

1. Introduction

Availability of natural resources and its exploitation have impact on economic development of a country. The quest of man for higher standard of living has multiplied the demands exorbitantly, and the earth's known resources have started depleting along with generation of solid waste at an alarming rate. The growing size and cost of the Municipal Solid Waste (MSW) (mainly of plastics, papers, metal scraps etc.) disposal problem is also

drawing attention all over the world. The costs, public apprehension, health and environmental concerns have made landfill and incineration methods undesirable. Incineration facilities are not very popular due to air pollution problems and landfills are predicted to become the most expensive disposal method because of diminishing availability of space and stringent environmental standards [Tilman and Sandhu (1998)]. Alternative management methods are being sought by recycling or reusing the ingredients of the municipal solid waste as far as possible.

The paper industry is one such sector which extensively depletes a natural resource i.e. forest cover by consumption of wood in an indiscriminate manner and adversely affecting the environment. Also the growth in the literacy as well as urbanization has led to increased use of paper products, consequently leading to increase in waste paper content in MSW. This clearly highlights the need for conservation of available resources i.e. forest cover by effective wastepaper management through recycling/reusing.

The focus on the recycling issue in recent years is mainly on (a) the municipal solid waste (or supply) of paper recycling and (b) on the policy aspects of the issue [Smith (1997)]. Samakovlis (2004) re-evaluates the hierarchy of paper waste management policies in a dynamic general equilibrium model. Grace et al. (1978) and Yohne (1979) performed economic studies of the supply and demand or trade, with waste paper. Recent publications on waste paper recycling concentrate on the high volatility of waste paper prices [Ackerman and Gallagher (2002)], examine inter-country differences in waste paper recovery and utilization rates [Bystrom and Lonnstedt (1995), Berglund et al. (2002)] or discuss whether wastepaper should be recycled or incinerated [Hanley and Slark (1994), Leach et al. (1997), Bystrom and Lonnstedt (1997), Samakovlis (2003)]. Various studies are reported on the environmental

issues in wastepaper recycling. Bloemhof-Ruwaard et al. (1996) developed an environmental life cycle optimization model for the European pulp and paper sector. The objective of the model was to minimize environmental impacts of virgin paper production, recycled paper production, wastepaper incineration and transport.

However, literature on the economic analyses for manufacturing paper by blending of two competing raw materials i.e. wood and recyclable wastepaper with due consideration to environment as well as the quality issues could not be traced. This paper attempts to fill the gap by developing a linear programming model to minimize system wide paper manufacturing cost by blending two competing raw materials. Further, the sensitivity analysis determines the effect of the quality of input raw material as well as the quality of the finished paper on the system. The effect of environmental costs (representing the degree of concern of manufacturer for the environmental) on the system has also been studied.

2. Functional Model of Indian Paper Industry

Figure 1 represents a functional supply chain model with various entities involved in manufacturing of paper in India by blending competing raw materials. The members involved in paper manufacturing (upto manufacturer) are consumer (a term used to represent a customer who sells the used paper for recycling to dealer), segregator, and supplier of segregated relevant wastepaper to manufacturer. The supplier of wood to the manufacturer is another important entity of the paper supply chain. The associated costs are collection cost of raw material, transportation, inventory (carrying, shortage and ordering), segregation and disposal, and manufacturing (raw material, power and miscellaneous) as well as reuse cost of certain varieties of wastepaper which are not fit for recycling. The wastepaper collected by the

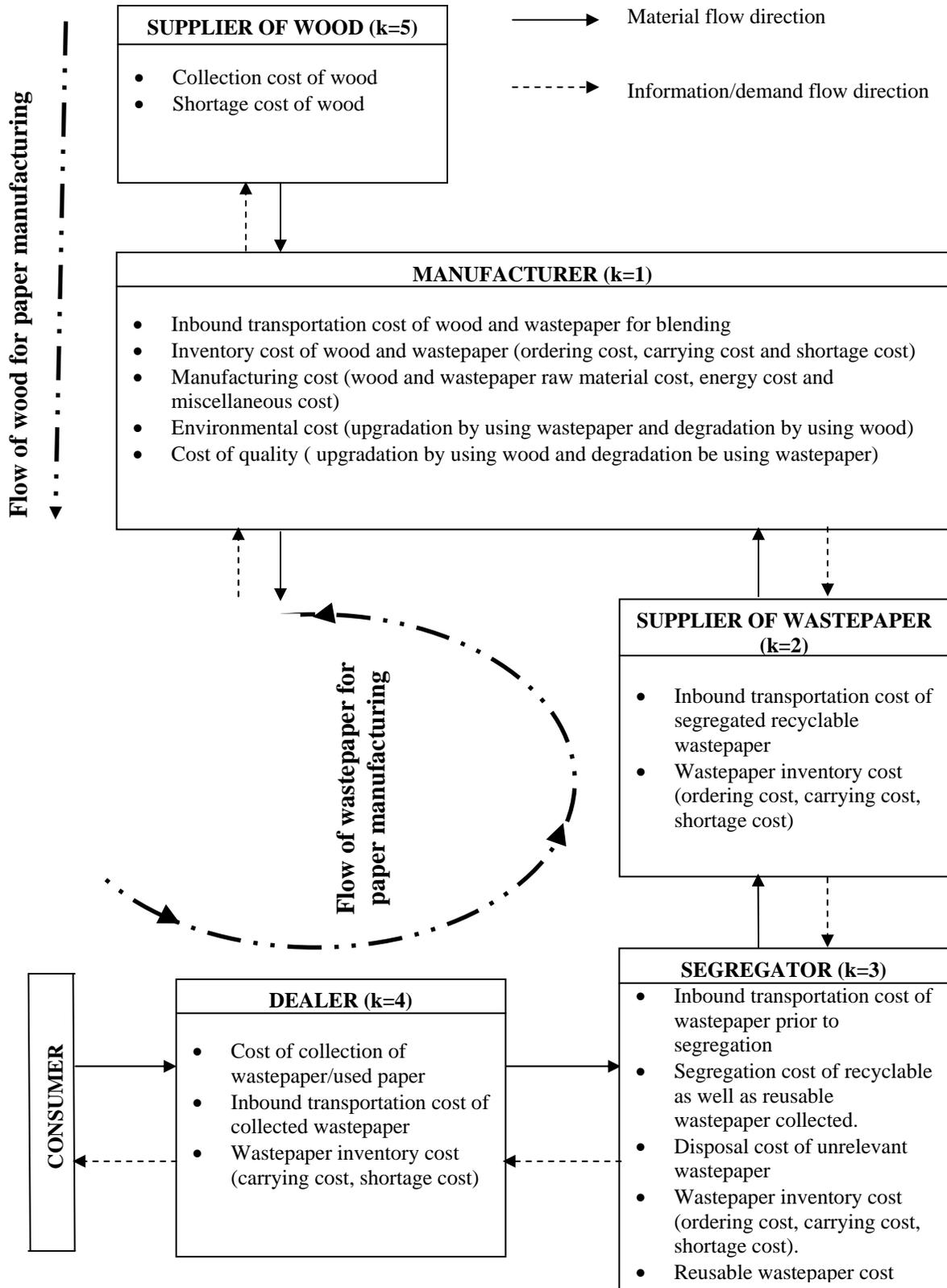


Figure-1: Schematic Diagram of Indian Paper Cycle from Manufacturer's Viewpoint (with detailed cost involved). 'k' indicates the echelon number in the entire supply chain

dealer from the consumer broadly consists of three varieties of wastepaper i.e. relevant recyclable wastepaper, reusable wastepaper (not fit for recycling but can be reused) and finally, irrelevant wastepaper (neither fit for recycling nor reusing). These costs have been shown at the required echelon the figure. An additional feature considered in the above model (at the requisite echelon) is the inclusion of cost of quality and the cost of impact on environment by the use of competing raw materials i.e. wood and relevant wastepaper for recycling. The figure also indicates the direction of the flow of raw materials (bold arrows) and demand information (dotted arrows) between the echelons.

To facilitate the model formulation, the following assumptions are postulated:

- Initial inventory and the maximum storage capacity for each variety of collected waste/raw material at each stage under consideration are known.
- The demands for the respective product/raw material at each echelon are known. In practice; these demands can be measured readily from order entries of each stage.
- The cost of internal distribution is ignored. Correspondingly, only the inbound transportation cost is taken into account in model formulation.
- There is no loss of any variety of at any echelon in system.
- There is enough demand for finished paper product i.e. whatever is produced at manufacturer stage is immediately sent to the market with no storage.
- The segregated wastepaper recovered for reuse is not stored but reused immediately with no storage or any other associated cost after segregation.
- No ordering cost at the dealer stage has been accounted as they collect the unsegregated wastepaper from society.

- Unit energy cost and miscellaneous cost are constant for any type of output product (finished paper) at manufacturer stage.

3. Optimal Blending Model for Paper Manufacturing

The objective of the proposed linear optimization model is to minimize the total system cost for paper manufacturing by blending the two competing raw materials i.e. wood and recyclable wastepaper subjected to various constraints. The model has the provision for varying degree of shortage of the product in demand at each echelon. The objective of the model is represented by equation (01). The concern of manufacturer for the quality of finished paper and environment has been reflected in the model by assigning appropriate environmental and quality opportunity costs component in the objective function. Wood in comparison to recyclable wastepaper has higher fiber strength, thus leading to improved quality characteristics. Hence, a negative quality cost is assigned in the model for the proportion of wood in the blend and a positive cost when wastepaper is used. In contrary, the use of wood depletes the natural resources as well as leads to ecological imbalance, whereas, the use of wastepaper improves the environment by reducing the content of solid waste. Thus, a negative environmental opportunity cost is assigned for the use of wastepaper and positive cost for the use of wood. A negative cost has also been assigned for reusing the proportion of wastepaper segregated at the segregator stage.

Various constraints considered in the formulation of the model are demand constraints, storage capacity constraints, wastepaper mix constraint, and non-negativity constraints (from equation 02-15). The model can solve for the optimal flow of raw materials

in the network and also determine the system cost for various degrees of shortages. The explanation of the notation used in model formulation is given in Appendix I.

The total cost of paper production by blending wood and wastepaper can be represented as:

Total cost =[collection cost of wastepaper by dealer from vendor customer]+[collection cost of wood at supplier of wood stage]+[transportation cost of wastepaper from dealer to manufacturing plant covering segregator, supplier stage and transportation cost of wood from supplier of wood to manufacturer stage]+[segregation cost of recyclable relevant wastepaper and reusable wastepaper at segregator stage]+[disposal cost at segregator stage]+[cost of reusing the reusable wastepaper]+[inventory carrying cost of raw materials (wood as well as wastepaper) for all relevant stages from dealer to manufacturer]+[ordering cost for wood and wastepaper at all echelons except supplier of wood and dealer stages]+ [shortage cost at all stages for required demand]+[manufacturing cost including raw material, energy and miscellaneous costs]+[quality cost at manufacturer]+[environmental cost assigned at manufacturer]

The mathematical representation of the proposed model is as follows:

Minimize

$$\begin{aligned}
& \left[(C_c \times Q_4) + (C_c^w \times Q_5) \right] + \left[\left(\sum_{k=1}^2 T_k^{RC} \times l_k \times Q_k^{RC} \right) + \left(\sum_{k=3}^5 T_k \times l_k \times Q_k \right) \right] + \left[(S_g^{RC} \times Q_3^{RC}) + (S_g^{RU} \times Q_3^{RU}) \right] + [D \times Q_3^D] \\
& - [C^{RU} \times Q_3^{RU}] + \left[\left(\frac{2b_w(0) + Q_5 - ((Q_0 \times W_f) / Y_w)}{2} \right) \times h_w + \left(\frac{2b_1^{RC}(0) + Q_1^{RC} - ((Q_0(1 - W_f)) / Y_{rc})}{2} \right) \times h_1^{RC} \right. \\
& \left. + \left(\frac{2b_4(0) + Q_4 - Q_3}{2} \right) \times h_4 + \left(\sum_{k=2}^3 \left(\frac{2b_k^{RC}(0) + Q_k^{RC} - Q_{k-1}^{RC}}{2} \right) \times h_k^{RC} \right) \right] + \left[\left(C_{O_3} \times \frac{d_4}{Q_3^*} \right) + \left(C_{O_w} \times \frac{d_5}{Q_w^*} \right) + \sum_{k=1}^2 \left(C_{O_k}^{RC} \times \frac{d_{k+1}^{RC}}{Q_k^{*RC}} \right) \right] \\
& + \left[(S_1 \times d_1 \times C_{S1}) + (S_4 \times d_4 \times C_{S4}) + (S_5 \times d_5 \times C_{S5}) + \sum_{k=2}^3 (S_k \times d_k^{RC} \times C_{Sk}) \right]
\end{aligned}$$

$$\begin{aligned}
& + \left[\left(C_P + C_{OM} + \left(\frac{C_R^w \times W_f}{Y_w} \right) + \left(\frac{C_R^{RC} \times (1 - W_f)}{Y_{rc}} \right) \right) \times Q_0 \right] + \left[\left(\left(\frac{C_{ED}^w \times W_f}{Y_w} \right) - \left(\frac{C_{ED}^{RC} \times (1 - W_f)}{Y_{rc}} \right) \right) \times Q_0 \right] \\
& + \left[\left(\left(\frac{C_R^{RC} \times (1 - W_f) \times C_q^{RC}}{Y_{rc}} \right) - \left(\frac{C_R^w \times W_f \times C_q^w}{Y_w} \right) \right) \times Q_0 \right] \quad \text{-- (01)}
\end{aligned}$$

Subject to:

$$\bullet \quad Q_4 + b_4(0) \geq (1 - S_4) \times d_4 \quad \text{-- (02)}$$

$$\bullet \quad Q_k^{RC} + b_k^{RC}(0) \geq (1 - S_k) \times d_k^{RC} \quad \forall k = 2,3 \quad \text{-- (03)}$$

$$\bullet \quad Q_5 + b_w(0) \geq (1 - S_1) \times \frac{d_1}{Y_w} \times W_f \quad \text{-- (04)}$$

$$\bullet \quad Q_1^{RC} + b_1^{RC}(0) \geq (1 - S_1) \times \frac{d_1}{Y_{rc}} \times (1 - W_f) \quad \text{-- (05)}$$

$$\bullet \quad Q_0 \geq (1 - S_1) \times d_1 \quad \text{-- (06)}$$

$$\bullet \quad Q_5 \geq (1 - S_5) \times d_5 \quad \text{-- (07)}$$

$$\bullet \quad Q_3^D \geq W_D \times Q_3 \quad \text{-- (08)}$$

$$\bullet \quad Q_3^{RU} \geq W_{RU} \times Q_3 \quad \text{-- (09)}$$

$$\bullet \quad Q_3 = Q_3^{RC} + Q_3^{RU} + Q_3^D \quad \text{-- (10)}$$

$$\bullet \quad Q_4 - Q_3 + b_4(0) \leq B_4 \quad \text{-- (11)}$$

$$\bullet \quad Q_k^{RC} - Q_{k-1}^{RC} + b_k^{RC}(0) \leq B_k^{RC} \quad \forall k = 2,3 \quad \text{-- (12)}$$

$$\bullet \quad Q_5 - \frac{Q_0 \times W_f}{Y_w} + b_w(0) \leq B_w \quad \text{-- (13)}$$

$$\bullet \quad Q_1^{RC} - \frac{Q_0 \times (1 - W_f)}{Y_{rc}} + b_1^{RC}(0) \leq B_1^{RC} \quad \text{-- (14)}$$

$$\bullet \quad Q_5, Q_4, Q_3, Q_3^{RC}, Q_3^{RU}, Q_3^D, Q_2^{RC}, Q_1^{RC}, Q_0 \geq 0 \quad \text{-- (15)}$$

Demand constraint

Wastepaper mix constraint

Storage constraint

Non-negativity constraint

Equation (02-03) represents the satisfaction of demand for the wastepaper at dealer, segregator and supplier of wastepaper stage, whereas, Equation (04-05) are the demand for raw materials i.e. wood and wastepaper respectively at the manufacturer stage to fulfill its demand for finished paper. Equations (06-07) represent the satisfaction of the demand for the finished paper and wood at manufacturer and supplier of wood stage respectively. Equations (08-10) signify the presence of relevant recyclable waste, reusable paper (not fit for recycling) in the mix of wastepaper's collected from consumers. It is assumed that the proportion of reusable wastepaper and disposable wastepaper is always ' W_{RU} ' and ' W_D ' respectively of total waste collection from consumers. Equations (11-14) denote the maximum available storage capacity at dealer, segregator, supplier of wastepaper and manufacturer stages for wastepaper and/or wood. Equation (15) corresponds to the estimates of the decision variables. All the estimates should be subjected to the non-negative domain in order to meet the basic requirements of feasible solutions.

4. Sensitivity analysis

To demonstrate the applicability of the proposed blending model, inputs were collected from Uttaranchal (a state in India, with considerable number of small and large-scale paper manufacturing plants). LINDO-32 (version 6.1) software was utilized to determine the optimum flow of raw materials as well as system wide cost in the paper supply chain for a given set of market situation/condition. The present section discusses the results of analysis being performed on the blending model for paper manufacturing (from competitive raw material sources). The strategies have been suggested with the objective of minimizing the system-wide costs for manufacturing of paper product under different market situation.

The section confers about the effects of yield of input raw materials and quality of finished paper on the system through yield and quality analysis respectively. The sensitivity of the system cost towards the shortages of the competing raw material and environmental opportunity cost has also been studied. The behavior of the model was compared with the anticipated intuitive behavior of the system.

4.1 Yield analysis of competing raw materials

Yield at the manufacturer stage is one of the important factors contributing to the quality of input raw material i.e. a high quality raw material will have a higher yield, whereas, a lower quality will have low yield. The input raw material yield (or its quality) also has an impact on the demand for raw materials between the echelons. The lower yield raw material will demand more raw materials from the system to satisfy the demand for the finished product at the manufacturer. For a desired quality of finished paper (with constant proportion of wood fiber i.e. W_f), yield sensitivity analysis of competing raw material (i.e. wood and

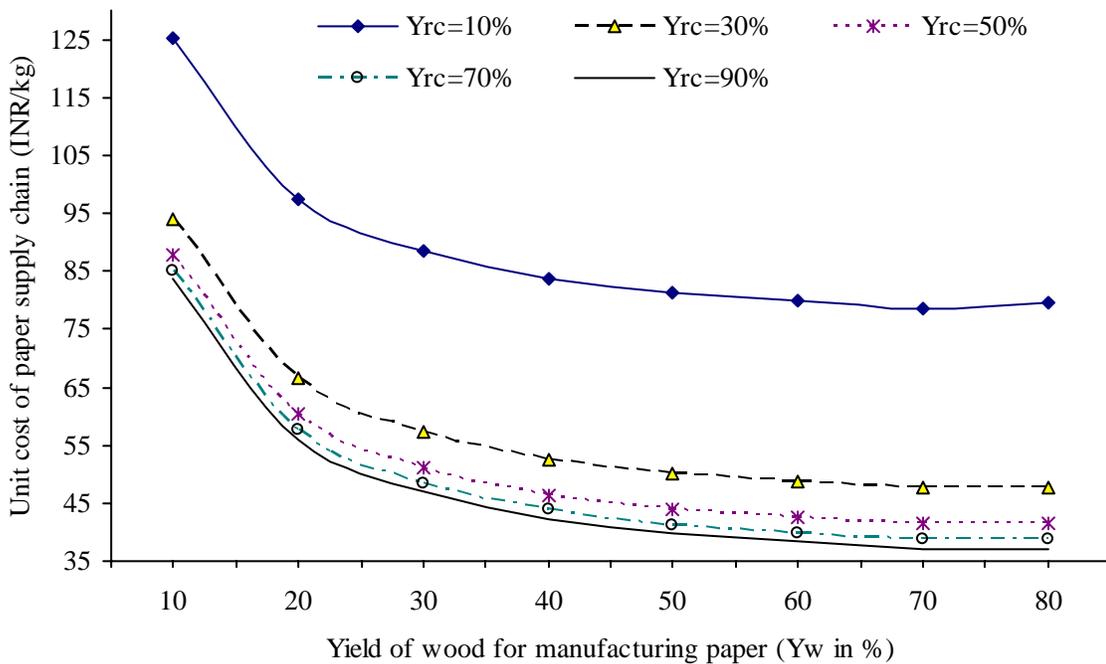


Figure 2: Yield analysis of competing raw material for paper production

relevant recyclable wastepaper) was conducted to determine the optimal yield of the input raw material in different market situation which leads to minimum system wide cost (Figure 2). Under all the conditions of constant yield of wastepaper (Y_{rc}), unit cost of paper manufacturing was found to decrease with the increase in yield of wood (Y_w). This can be attributed to the reduced quantity of input wood, improved quality of finished paper as well as the increase in positive cost component associated with environmental upgradation due to less consumption of wood. Similarly, trend was observed under constant Y_w and varying Y_{rc} .

4.2 Quality analysis of finished paper on the paper supply chain

The quality of fiber produced from wood is superior in comparison to the quality of fibre produced from wastepaper. Hence, the quality of the finished paper product greatly depends on the proportion of wood fibre present in a unit of finished paper (W_f). Sensitivity analysis was performed on the model to study the impact of W_f on the system's supply chain cost under a given quality of input competing raw materials (in terms of yield). The outcome

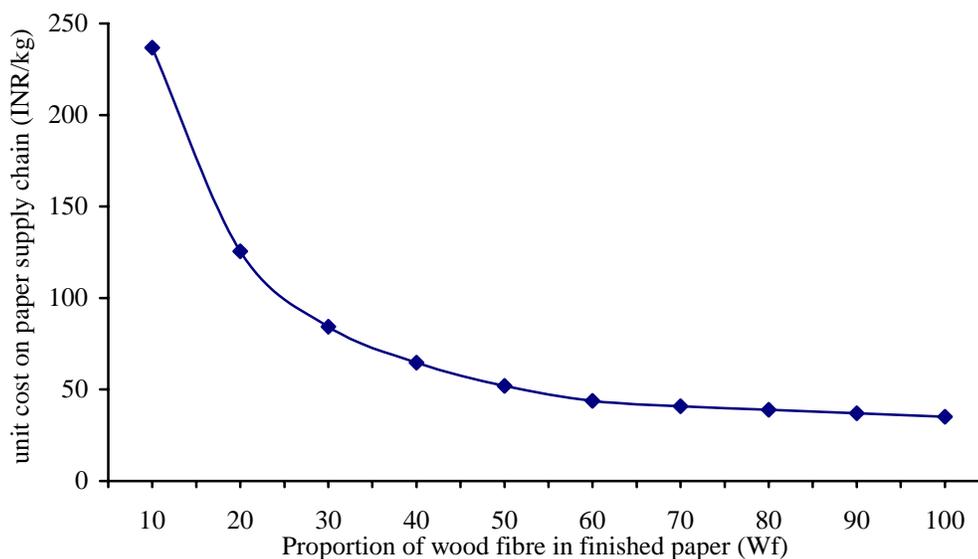


Figure 3: Quality analysis of finished paper in terms of wood fibre content

of the analysis is shown in figure 3. It is seen that with increase in the proportion of wood fiber (W_f), the unit cost of the supply chain decreases. This can be attributed to decrease in the quantity of wastepaper flow as well as associated costs in the system, and increase in quality of finished paper product. The decrease in cost compensates for the increase in environmental degradation costs (due to increased use of wood and reduced use of wastepaper).

4.3 Environmental cost sensitivity analysis

Constant degradation of the environment (due to increase in solid waste and reduction in natural resources due to human usage) has forced governments to enforce strict environmental regulation, and the consumers also demand manufacturers to follow environmental friendly methods. Hence, the proposed model also considers the environmental component in its objective function. This is an opportunistic cost highlighting the concern of the manufacturer towards the environment. The behavior of the system towards the environmental concern was analyzed by varying the unit environmental cost of using wood

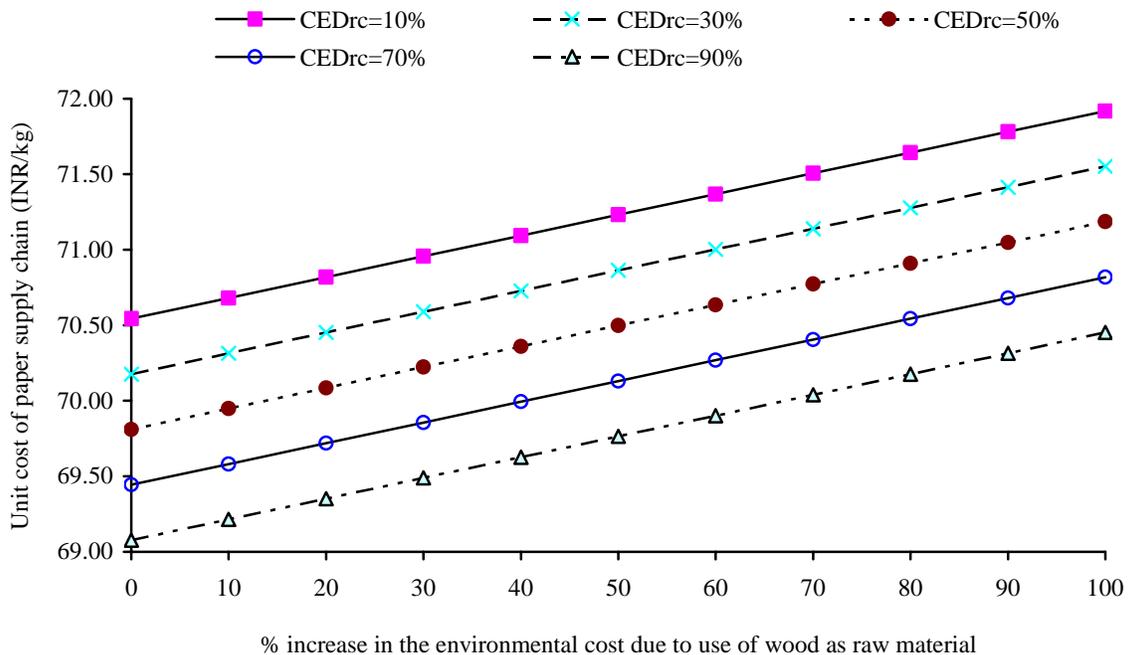


Figure 3: Environmental cost sensitivity analysis

* Note: CEDrc = % increase in unit opportunity environmental cost

(C_{ED}^W) and wastepaper (C_{ED}^{RC}) as competing raw material. The outcome of the analysis is shown in figure 4. It is noticed that with the increase in C_{ED}^W at constant C_{ED}^{RC} the cost of the paper supply chain increases. In contrast, with increase in C_{ED}^{RC} at constant C_{ED}^W the cost of the paper supply chain decreases. Hence, the manufacturer having higher concern for the environment should increase the proportion of wastepaper and reduce that of wood for manufacturing paper.

5. Conclusion

Reverse logistics is an area of growing importance. In this paper a linear cost optimization blending model was formulated with two competing raw material sources i.e. wood and wastepaper for manufacturing of finished paper product. The objective function of the blending model included commonly accounted costs of collection, transportation, inventory, manufacturing, segregation and disposal along with the opportunity quality and environmental cost. The model thus takes care of the globally relevant topic of manufacturing eco-friendly quality products. The results of the sensitivity analysis on the model are in line with the intuitive behavior of the system. It is seen from the yield analysis (section 4.1) that with improved quality of input raw materials (in terms of yield increases) the system wide cost of manufacturing paper by blending decreases. Hence, the manufacturer should opt for better input raw materials. The quality analysis (section 4.2) indicates that increasing proportion of wood fibre in the finished paper decreases the system wide cost even at the cost of degradation to the environment. Thus, it is upto the manufacturer to convey its degree of environmental concern to the government and society by assigning higher environmental

costs in the model. Higher emphasis on using increased content of recycled paper (by assigning higher environmental) decreases the system wide cost, whereas, higher emphasis on using wood (by assigning lower environmental cost) increases the system wide cost.

It envisaged that the quantitative analysis presented in this paper can play a vital role in the decision process of a paper manufacturer for determining an optimal blend of quality and quantity of competing input raw materials. The choice for the supply chain strategy could be justified based on the quantitative reasons. However, limitations of the present study pertain to ignoring the effect of internal distributions on system wide supply chain cost and assumption of “no loss condition” of any variety. Difficulty in obtaining the estimates of the environmental cost is also a major limitation of the study. The linear model presented can possibly be extended to include non-linearities, stochasticity of parameters and multiplicity of objectives in future studies on this topic.

Appendix I: Nomenclature of the proposed optimal blending model

| | |
|------------|--|
| C_c | Unit collection cost of wastepaper at dealer stage (INR/T) |
| C_c^w | Unit collection cost of wood (INR/T) |
| Q_4 | Quantity of unsegregated wastepaper collected by dealer stage |
| Q_5 | Quantity of wood supplied by supplier of wood stage(k=5) to the manufacturer stage(k=1) |
| T_k^{RC} | Cumulative unit transportation cost of relevant recyclable waste moving from (k+1) to k stage (INR/km/T) |
| T_k | Cumulative unit inbound transportation cost for unsegregated wastepaper at ‘k’th stage (INR/km/T) |
| l_k | Total distance traveled by raw material wastepaper before entering ‘k’th stage from (k+1) stage |
| l_5 | Total distance traveled by wood from k=5 stage before entering k=1 stage. |
| Q_5 | Quantity of wood moving out of supplier of wood stage and entering manufacturer stage |
| Q_k | Quantity of unsegregated wastepaper moving out of ‘k’th stage and entering (k-1) stage |

| | |
|----------------|---|
| Q_k^{RC} | Quantity of recyclable (relevant) wastepaper in moving from 'k'th stage to (k-1) stage. |
| Q_3^{RU} | Quantity of reusable wastepaper (not fit for recycling) in mixture moving from dealer to segregator stage |
| Q_3^D | Quantity of disposable wastepaper (not fit for recycling as well as reuse) in mixture moving from dealer to segregator stage |
| S_g^{RC} | Unit segregation cost incurred for segregating relevant recyclable wastepaper from the unsegregated waste collected from dealer stage (INR/T) |
| S_g^{RU} | Unit segregation cost incurred for segregating reusable wastepaper from unsegregated waste collected (INR/T) |
| D | Unit disposal cost of irrelevant wastepaper after regenerating from recyclable & reusable wastepaper's |
| C^{RU} | Unit cost of using reusable wastepaper collected after segregation. |
| $b_w(0)$ | Initial inventory (at t=0) for wood at the manufacturer stage |
| $b_4(0)$ | Initial inventory (at t=0) for unsegregated wastepaper at dealer stage (k=4) |
| Q_0 | Quantity of finished paper moving out of manufacturer stage to fulfill its demand. |
| W_f | Desired proportion of wood fibre in the finished product |
| Y_w | Yield percentage for wood at manufacturer to get a unit weight of desired finished paper |
| Y_{rc} | Yield percentage of recyclable wastepaper at manufacturer to get unit weight of desired finished paper |
| h_w | Unit inventory carrying cost for wood at manufacturer stage (INR/T) |
| h_k^{RC} | Unit inventory carrying cost for relevant recyclable wastepaper at 'k' stage (INR/T) |
| h_4 | Unit inventory carrying cost for unsegregated wastepaper at dealer stage (INR/T) |
| $b_k^{RC}(0)$ | Initial inventory (tt=0) of relevant recyclable wastepaper at k stage. |
| C_{O_3} | Ordering cost of unsegregated wastepaper at segregator stage. |
| C_{O_w} | Ordering cost of wood at the manufacturer stage |
| $C_{O_k}^{RC}$ | Ordering cost of relevant recyclable wastepaper at k stage |
| d_4 | Demand for total unsegregated wastepaper by segregator stage from dealer stage |
| d_5 | Demand for wood by manufacturer stage from supplier of wood stage |
| d_{k+1}^{RC} | Demand for relevant recyclable wastepaper by the 'k' stage from (k+1) stage. |
| Q_3^* | Economic order quantity (EOQ) for unsegregated wastepaper at the segregator stage |

| | |
|---------------|---|
| Q_w^* | EOQ for wood at manufacturer stage |
| Q_k^{*RC} | EOQ for relevant recyclable waste paper at 'k' stage |
| S_k | Percentage shortage in material demanded at the 'k' stage |
| d_1 | Demand for finished paper at manufacturer stage |
| C_{Sk} | Unit shortage cost of the material (wastepaper and finished paper) at the k stage. |
| C_{Sw} | Unit shortage cost of wood at the supplier of wood stage. |
| C_P | Unit energy cost for manufacturing paper (INR/T) |
| C_{OM} | Unit miscellaneous cost for manufacturing paper (INR/T) |
| C_R^w | Unit cost of wood used as raw material (INR/T) |
| C_R^{RC} | Unit cost of relevant recyclable wastepaper used as raw material |
| C_{ED}^w | Unit environmental degradation cost due of use of wood for producing paper |
| C_{ED}^{RC} | Unit environmental up gradation cost due to use of relevant wastepaper for producing paper |
| C_q^{RC} | It is a measure of the impact of use of recyclable wastepaper on quality cost and is estimated as a proportion cost of wastepaper used. |
| C_q^w | It is a measure of the impact of use of wood on quality cost and is estimated as a proportion cost of wood used. |
| W_D | Proportion of nonrelevant wastepaper (neither fit for reusing or recycling) collected with unsegregated waste |
| W_{RU} | Proportion of reusable wastepaper (not fit for recycling) collected with unsegregated waste |
| B_4 | Maximum storage capacity of unsegregated wastepaper at the dealer stage. |
| B_k^{RC} | Maximum storage capacity of relevant recyclable wastepaper at 'k' stage. |
| B_w | Maximum storage capacity of wood at the manufacturer stage. |

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