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The value of Information Sharing in a Supply Chain with Product Substitution

Muthusamy Ganesh[*], Srinivasan Raghunathan[#] and Chandrasekharan Rajendran[*]

[*]Department of Management Studies
Indian Institute of Technology Madras
Chennai, 600036
India

[#]School of Management
The University of Texas at Dallas
Richardson, TX 75083
USA

Corresponding Author Email: mganesh_iitm@yahoo.co.in
Phone: +91-44-2257 4550 /+91-44-2257 5553

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Abstract

The value of information sharing within a supply chain has been analyzed extensively by researchers. The extant literature has investigated only the case in which the supply chain manufactures and distributes a single product to customers. In this paper, we consider the case in which a supply chain distributes multiple products. These products are also substitutable in the sense that a consumer is willing to buy an alternate product when the customer's preferred product is out of stock. We show that such substitutability among products generally reduces the value of information sharing. This result occurs because of the demand pooling effect of substitution, which reduces demand variance even before information sharing. The reduction in the value of information sharing because of substitutability is higher when the number of substitutable products is higher and/or when the demands of products are less correlated. However, when information about the demands of only a sub set of products is shared, the value of information sharing under substitution is *higher* than that under no substitution under certain conditions. The key implication of our findings is that if substitution effects are ignored, then there is a risk of overestimating the value of information sharing. The overestimate can be very significant when either there is a large number of substitutable products or the demands of these products are less correlated and more independent.

Keywords: *Value of information sharing, Supply chain, Multiple products, Substitution*

1. Introduction

Information sharing among firms within a supply chain has been a cornerstone of recent innovations in supply chain management. It is well known that Wal-Mart and Proctor & Gamble (P&G) have been sharing Point-of-Sale and real-time inventory information for a long time now. Other companies such as Dell, Cisco, Dillard Department Stores, JC Penney, and Lucent Technologies have also initiated similar information sharing strategies (Dong and Xu, 2002). The primary benefit of sharing demand and inventory information is a reduction in the bullwhip effect and, consequently, a reduction in inventory holding and shortage costs within the supply chain (Forrester 1958, Sterman 1989, Lee et al. 1997a, 1997b)

The value of information sharing within a supply chain has been analyzed extensively by researchers. The extant literature has investigated only the case in which the supply chain manufactures and distributes a single product to customers. However, modern supply chains, even when there is a single manufacturer and a single retailer, often manufacture and distribute multiple products (or multiple varieties of a product) to satisfy diverse customer preferences. The ability to satisfy heterogeneous customer preferences by providing more product variety has been noted as a critical success factor in retailing (Kima et al., 2005). A study by the U.S. Federal Reserve Bank documented the dramatic increase in product variety in almost every industry during the 1980's and 90's (Annual Report of Federal Reserve Bank of Dallas, 1998). Table 1 offers some insights into the magnitude of the increase in product variety in some of the industries. Similar increases are also observed in automobile (new vehicle models rose from 140 to 260), entertainment (the number of TV channels increased from 5 to 185), and pharmaceutical (the number of over-the-counter pain relievers rose from 17 to 141) industries. The study also reported that not only the total number of varieties of a product in the market but also the number of product varieties offered by the same manufacturer increased dramatically during the 80's and 90's. One of the reasons why managing multiple varieties of a product, collectively known as a brand category, is challenging is that the varieties of the same product often are substitutable (Mahajan and Ryzin 2001). That is, when a variety that a customer is looking for is unavailable, the customer may buy another variety of the same product, which suggests that decisions taken regarding a variety has ramifications on decisions taken about other varieties. Furthermore, the demands of different varieties of the same product are likely to be correlated. The primary objective of this paper is to offer insights into how the value of information sharing within a two-level supply chain is affected when the supply chain distributes several substitutable products as opposed to a single product.

Table 1. Product Variety Statistics

FOOD PRODUCTS			HOUSE HOLD ITEMS			BEVERAGES		
Item	**1980**	**1998**	**Item**	**1980**	**1998**	**Item**	**1980**	**1998**
Meals	159	671	Laundry soaps, detergents	12	48	Milk, nondairy milk, yogurt drinks	26	255
Meat	42	234	Paper towels, napkins	11	126	Health drinks	4	70
Soup	119	291	Deodorizers, air fresheners	53	372	Soft drinks	26	252

Our analysis has led to the following significant findings. First, substitutability among products generally reduces the value of information sharing. This result occurs because of the demand pooling effect of product substitution. Demand pooling reduces the per product demand variance, and, hence, the value of information sharing. Second, the reduction in the value of information sharing because of substitutability increases when the number of substitutable products increases or when the correlation between product demands decreases. Third, we find, somewhat surprisingly, that when information about the demands of only a sub set of products is shared, the value of information sharing under substitution is *higher* than under no substitution when the correlation between demands is sufficiently high and the number of products for which demand is shared is sufficiently low. This result arises because of two impacts of demand correlation. First, demand correlation enables information about the demand of one product to be used to reduce the variance about the demand of other products. This effect of correlation is present under substitution and no-substitution cases. The second effect, which is present only under the substitution case, is that an increase in demand correlation increases the variance of the total or the pooled demand. When the number of products whose information is shared is small and the correlation is high, the latter effect dominates the former, resulting in a higher value of information sharing under substitution. As the number of products whose demand information is shared increases, the impact of the second effect reduces relative to that of the first under product substitution.

The key implication of our findings to supply chain management practice is that if substitution effects are ignored, then there is a risk of overestimating the value of information sharing. The overestimate can be very significant when either the there is a large number of substitutable products or when the demands of these products are independent.

The rest of the paper is organized as follows. We review the relevant literature in section 2. The modeling frame work is discussed in section 3. Sections 4 and 5 present the theoretical results of our analysis. Results from a numerical simulation are discussed in section 6. Finally, we conclude the paper with a summary in section 7.

2. Literature Review

Several types of information exchange have been considered for supply chains with two firms and stochastic demands.[1] There is substantial research on the inventory and shortage related benefits to the manufacturer when the retailer shares its information. Bourland et al. (1996) derived the benefits of information sharing when the review period of the manufacturer is not synchronized with that of the retailer. Metters (1997) showed that sharing information can reduce the bullwhip effect and increase profitability. Gavirneni et al. (1999) studied the value of information sharing for a finite capacity supplier facing demand from a single retailer. Chen et al. (2000) quantified the bullwhip effect for a single product two-stage supply chain consisting of a single retailer and a single manufacturer. They assumed that the demand followed an AR (1) process and that the retailer used a moving-average model for demand forecast and a simple order-up-to inventory policy for replenishment. In another paper, Chen et al. (2000) investigated the impact of forecast methods and demand patterns on the bullwhip effect. Lee, So, and Tang (2000) studied the benefit of demand information sharing when the underlying demand process faced by the retailer is a AR (1) process. Raghunathan (2001) showed that the results derived by Lee, So, and Tang overestimate the benefit of demand information sharing if the manufacturer uses the entire order history to do its forecast. Raghunathan and Yeh (2001) extended this model to multiple retailers and analyzed the optimal number of retailers in an information sharing arrangement. Moinzadeh (2002) showed that, in a decentralized system, channel coordination can be achieved by imposing an appropriate unit shortage cost at the supplier. Cachon and Fisher (2000) studied the value of sharing data in a model with one supplier, several identical retailers, and a stationary stochastic consumer demand. They conclude that accelerating the physical flow of goods through a supply chain is significantly more valuable than exchange of information.

[1] See Tayur et al. (1999) for a collection of papers on information sharing and contracting in supply chains.

Aviv (2001, 2002) investigated the value of collaborative forecasting and integrating retailer forecasts into the manufacturer's replenishment process. Gavirneni (2005) showed that when the wholesale price alternates between a high and a low level, information sharing benefits the manufacturer significantly. Zhao et al. (2002) considered a supply chain model with a single supplier and multiple retailers with capacity constraints. Raghunathan (2003) analyzed the value of demand information sharing in the context of a *N* retailer version of the Lee et al.(2000). Reddy and Rajendran (2005) developed a simulation model to investigate the three levels of information sharing, viz., no, full, and partial information sharing, by considering a single product multi echelon supply chain model. Chu and Lee (2005) studied how the incentive to share information is related to the cost of sharing in the specific context of a supply chain that manufactures and sells newsboy-type products. All of the above papers report that there are benefits to sharing demand information. They all consider supply chains with a single product. Our work differs from the previously cited work in that we consider a supply chain setting that deals with multiple substitutable products.

Few papers have investigated how information sharing affects pricing decisions within a supply chain. These papers do not investigate the inventory-related issues. Li (2002) analyzed a model that includes a manufacturer and several competing retailers. Li showed that retailers will not voluntarily share information and considers a contract in which the manufacturer pays a fee to retailers in return for their information. Zhang (2002) considered a model in which each retailer sells a different product developed from the same base product supplied by the manufacturer and allows these products to be either substitutes or complements. Li and Zhang (2005) analyzed the impact of three information sharing scenarios between retailers and the manufacturer, with varying degrees of confidentiality.

There is a significant body of literature on the effects of substitution among products (see, for example, Veinott 1965, Ignall and Veinott 1969, McGillivray and Silver 1978, Parlar and Goyal 1984, Parlar 1988, Pasternack and Drezner 1991, Moinzadeh and Ingene 1993, Drezner et al. 1995 Bitran and Dasu 1992, Bassok et al 1999, Rao et al 2004). However, the focus of this literature was not on information sharing.

3. Modeling Framework

We consider a two-level supply chain, consisting of a single manufacturer and a single retailer, which distributes N products. Consumer demands for these products occur at the retailer. The demand for a product follows a simple AR(1) process[2]. Hence,

$$D_{it} = d + \rho D_{i(t-1)} + \xi_{it}, \tag{3.1}$$

where $d > 0$, $-1 < \rho < 1$, and $i \in \{1,2, 3, \ldots, N\}$. For a given t, the random element of demand for product i, ξ_{it}, follows a normal distribution with mean zero and variance σ^2, and the correlation coefficient between ξ_{it} and ξ_{jt}, $i \neq j$, is ρ_r, $\frac{-1}{N-1} < \rho_r < 1$. Both σ^2 and ρ_r are independent of t and i. The condition $\frac{-1}{N-1} \leq \rho_r \leq 1$ guarantees that the covariance matrix of ξ_{it} is positive semi-definite. For a given i, ξ_{it} are *i.i.d*. We assume further that σ is significantly smaller than d, so that the probability of a negative demand for any product is negligible. We assume that the retailer uses the AR(1) model given by equation (3.1) in his forecasting and ordering process. That is, he uses this period's demand data to forecast the demands in the next period.

The manufacturer also uses a AR(1) model to forecast the order for any product from the retailer. Let Y_{it} be the manufacturer's forecast of order for product i during period t using the order received for product i during period (t-1). The manufacturer's forecasting model is

[2] Inventory models that assumed AR(1) demand process include Kahn (1987), Miller (1986). Lee, So, and Tang (1997), Raghunathan (2001), Raghunathan (2003).

$$Y_{it} = d_m + \rho_m Y_{i(t-1)} + \delta_{it} \tag{3.2}$$

We assume that the manufacturer's forecast of retailer order is unbiased, i.e., doesn't contain any systematic errors. We show later how d_m, ρ_m, and δ_{it} are related to the parameters in the retailer's forecasting model.

We consider a periodic review system in which each site reviews its inventory level and replenishes its inventory from the upstream site at the end of every period. We assume, for convenience, that the replenishment lead times are zero.[3] At the end of every time period t, after demand D_{it} has been realized, the retailer observes the inventory level of product i and places an order of size Y_{it} with the manufacturer to replenish the inventory of product i to meet the customer demand for product i during (t+1). Excess customer demand is backlogged at the retailer. The manufacturer ships the required order quantity Y_{it} to the retailer at the end of time period t, immediately after receiving the order, and places his own order with his supplier to meet the retailer's demand for the next period. If the manufacturer does not have enough stock to fill the orders, then we assume that the manufacturer will meet the shortfall by obtaining some units from an alternative source. Thus, the inventory system at the manufacturer resembles a system with backorders, and the manufacturer guarantees supply to the retailers.

We assume that no fixed ordering cost is incurred when placing the order, and that per unit inventory holding cost rate and per unit shortage cost rate are stationary over time. Further, we assume that the holding cost rate and the shortage cost rate are identical across products. This assumption is reasonable because the products in our model are simply different varieties of the same underlying basic product. Let h and p denote the unit holding and shortage cost per time period for the retailer respectively. Let H and P denote the unit holding and shortage cost per

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[3] See Lee et al. (2000) for the impact of manufacturer and retailer lead times on the benefits of information sharing. Our results will not change qualitatively for any constant leadtime.

time period for the manufacturer. We assume that the retailers and the manufacturer would also adopt the order-up-to policy, since such a policy minimizes the total holding and shortage costs over the infinite horizon. (See Heyman and Sobel 1994 and Kahn 1987). The model notation is summarized in Table 2.

Table 2. Summary of Model Notation

i	Product identifier
t	Time period identifier
D_{it}	Customer demand for product i during time period t
d	Base level for the customer demand
ρ	Auto correlation coefficient in the AR(1) model
ρ_r	Correlation coefficient across product demands during a time period
ξ_{it}	Random component of the customer demand for product i during time period t
N	Total number of products
H	Holding cost rate for the manufacturer
h	Holding cost rate for the retailer
P	Shortage cost rate for the manufacturer
p	Shortage cost rate for the retailer
σ	Standard deviation for the Customer Demand
Y_{it}	Retailer order quantity for the product i during time period t

In the absence of information sharing, the manufacturer receives only the order Y_{it} for product i from the retailer at the end of period t. The manufacturer does not know the retailer's forecasting model for the next period in this case. When the retailer shares his information with the manufacturer, the manufacturer knows the retailer's forecasting model and the realized demand D_{it} at the end of period t. Consequently, the manufacturer uses this information in his forecast under information sharing.

We derive the value of information sharing when the products are not substitutable first, followed by when the products are perfectly substitutable. We then compare the value in these two cases in order to analyze the impact of substitutability on the value of information sharing.

4. Ordering Decisions

4.1 No Substitution Case

In this case, consumers buy only their preferred products; they do not substitute. If a consumer does not find his preferred product, the demand is backlogged. Let $S_{i,t}$ denote the retailer's order-up-to level for product i for period t in order to meet the demand during period (t+1). At the end of period t, the retailer's order for product i, y_{it}, is given by the following.

$$y_{it} = D_{it} + \left(S_{it} - S_{i(t-1)} \right) \quad (4.1)$$

Notice that the order quantity replenishes the demand during period t plus the change being made in the order-up-to level from period $(t-1)$ to t. In order to find S_{it}, the retailer uses the following model at the end of period t.

$$\underset{S_{1t}, S_{2t}, \ldots, S_{Nt}}{Min} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \sum_{i=1}^{N} \left(p\left(x_{i(t+1)} - S_{it} \right)^{+} + h\left(S_{it} - x_{i(t+1)} \right)^{-} \right) g(x_{1(t+1)}, x_{2(t+1),}, \ldots, x_{N(t+1)}) dx_{1(t+1)} dx_{2(t+1)} \ldots dx_{N(t+1)}$$

where $x_{i(t+1)}$ is the demand of product i in time period (t+1), $g(x_{1(t+1)}, x_{2(t+1),}, \ldots, x_{N(t+1)})$ is the joint probability density function of demands during time period t, and $\left(x_{i(t+1)} - S_{it} \right)^{+} = Max\left\{ 0, x_{i(t+1)} - S_{it} \right\}$, $\left(S_{it} - x_{i(t+1)} \right)^{-} = Min\left\{ 0, S_{it} - x_{i(t+1)} \right\}$. The first order conditions for the above minimization model yield the familiar newsboy model for each product because the marginal distribution of a multi-variate normal distribution is a uni-variate normal distribution. Thus, the optimal order-up-to level for product i for period t is given by the following.

$$S_{it} = d + \rho D_{it} + k\sigma \quad (4.2)$$

where $k = \phi^{-1}\left(\frac{p}{p+h} \right)$ for the standard normal distribution function ϕ. Note that ρ_{Γ} does not play any role in retailer's ordering decision when products are not substitutable. This is intuitive because even though the demands of different products during a time period may be correlated, the demand distribution for a particular product in a time period depends only on the demand during the previous period and, in particular, does not depend on the demands of other products during the previous period. In other words, correlation among variates in a multi-variate normal distribution does not affect the marginal distribution of a variate.

Immediately after receiving the retailer's order for a product, the manufacturer ships the retailer's order and places its own order to bring its inventory to its order-up-to level, T_{it}. The manufacturer's order-up-to level depends on the information available to the manufacturer at the time it computes the order-up-to level, which, in turn, depends on whether the retailer shares its information with the manufacturer.

4.1.1 No Information sharing

Here the manufacturer receives only the retailer's order for each product; and the retailer does not share the actual demand for the product during any time period. Using equations (3.1), (4.1) and (4.2), we obtain

$$y_{i(t+1)} = d + \rho y_{it} + (1+\rho)\xi_{i(t+1)} - \rho\xi_{it} \tag{4.3}$$

Comparing (4.3) with (3.2), we find that the parameters in the manufacturer's forecasting model under no information sharing and those in the retailer's forecasting model are related in the following way: $d_m = d$, $\rho_m = \rho$, and $\delta_{it} = (1+\rho)\xi_{it} - \rho\xi_{i(t-1)}$. Although ξ_{it} is known to the retailer at the end of period t, it is not shared with manufacturer when the retailer orders y_{it}. Note that, from the manufacturer's perspective, $y_{i(t+1)}$ has a variance of $\sigma^2\left((1+\rho)^2 + \rho^2\right)$ under no information sharing. Let T_{it}^{nis} be the manufacturer's order-up-to level for product i at the end of period t to meet the retailer's demand during the period (t+1) under no information sharing.

Then, following an analysis similar to the one for the retailer, we get T_{it}^{nis} as given in the following equation.

$$T_{i,t}^{nis} = d + \rho y_{i,t} + \mathrm{K}\sigma\sqrt{\left((1+\rho)^2 + \rho^2\right)} \tag{4.4}$$

where, $K = \phi^{-1}\left(\frac{P}{P+H}\right)$

4.1.2 Full Information sharing

Under full information sharing, at the end of period *t*, the retailer shares with the manufacturer its demand data for all *N* products for that period, and, hence, the manufacturer knows ξ_{it} for all products for that period. So, ξ_{it} are no longer random variables from the manufacturer's perspective. In this case, let T_{it}^{fis} be the manufacturer's order-up-to level for product i under full information sharing. The expression for $T_{i,t}^{fis}$ is computed to be the following.

$$T_{it}^{fis} = d + \rho y_{i,t} - \rho\xi_{i,t} + \mathrm{K}\sigma\sqrt{(1+\rho)^2} \tag{4.5}$$

4.1.3 Partial Information sharing

In the partial information sharing case, we assume that, at the end of a period, the retailer shares its demand data for that period only for $n < N$ products. Without loss of generality, we assume that the retailer shares its demand data for the first *n* (i.e., $i \le n$) products. In this scenario, at the end of period *t*, the manufacturer knows ξ_{it} for $i \le n$. Now, based on this information, the manufacturer can update the distributions of ξ_{it}, $n+1 \le i \le N$. The following property of multivariate normal distributions is used to derive the distribution of ξ_{it}, $n+1 \le j \le N$ given ξ_{it}, $i \le n$.

Property 1: If $X \sim$ MN(μ, ϑ), where μ is a N-dimensional mean vector and ϑ is a NxN variance-covariance matrix, then ($X_{(2)}$ |$X_{(1)} = x_{(1)}$) ~ MN($\mu_{2.1}$, $\vartheta_{2.1}$), where $X = \begin{pmatrix} X_{(1)} \\ X_{(2)} \end{pmatrix}$, $\mu = \begin{pmatrix} \mu_{(1)} \\ \mu_{(2)} \end{pmatrix}$, $\Sigma = \begin{pmatrix} \vartheta_{11} & \vartheta_{12} \\ \vartheta_{21} & \vartheta_{22} \end{pmatrix}$, $\vartheta_{21} = \vartheta_{12}^{T}$, covariance($X_{(1)}$) = ϑ_{11}, covariance($X_{(2)}$) = ϑ_{22}, covariance($X_{(1)}$,$X_{(2)}$) = ϑ_{12}, $\mu_{2.1} = \mu_{(2)} + \vartheta_{2.1}\vartheta_{11}^{-1}(x_{(1)}-\mu_{(1)})$, and $\vartheta_{2.1} = \vartheta_{22} - \vartheta_{21}\vartheta_{11}^{-1}\vartheta_{12}$.

In our model of partial information sharing, we have the following.

$$\xi_{Nx1} = \begin{bmatrix} \xi_{1t} \\ .. \\ \xi_{nt} \\ \xi_{(n+1)t} \\ .. \\ \xi_{Nt} \end{bmatrix} \mu_{Nx1} = \begin{bmatrix} 0 \\ .. \\ 0 \\ 0 \\ .. \\ 0 \end{bmatrix} \Sigma_{NxN} = \begin{bmatrix} \sigma^2 & \rho_r\sigma^2 & .. & .. & .. & \rho_r\sigma^2 \\ \rho_r\sigma^2 & \sigma^2 & & & & .. \\ .. & & & & & .. \\ .. & & & & & .. \\ .. & & & & \sigma^2 & \rho_r\sigma^2 \\ \rho_r\sigma^2 & .. & .. & .. & \rho_r\sigma^2 & \sigma^2 \end{bmatrix}$$

$$\vartheta_{11_{nxn}} = \begin{bmatrix} \sigma^2 & .. & \rho_r\sigma^2 \\ .. & \sigma^2 & .. \\ \rho_r\sigma^2 & .. & \sigma^2 \end{bmatrix} \vartheta_{22_{(N-n)x(N-n)}} = \begin{bmatrix} \sigma^2 & .. & \rho_r\sigma^2 \\ .. & \sigma^2 & .. \\ \rho_r\sigma^2 & .. & \sigma^2 \end{bmatrix}$$

$$\vartheta_{11_{nx(N-n)}} = \begin{bmatrix} \rho_r\sigma^2 & .. & \rho_r\sigma^2 \\ .. & .. & .. \\ \rho_r\sigma^2 & .. & \sigma^2 \end{bmatrix} \vartheta_{21_{(N-n)xn}} = \vartheta^{-1}{}_{11_{nx(N-n)}} \begin{bmatrix} \rho_r\sigma^2 & .. & \rho_r\sigma^2 \\ .. & .. & .. \\ \rho_r\sigma^2 & .. & \sigma^2 \end{bmatrix}$$

Now applying property 1 to our distributions defined above, we get the conditional mean and conditional variance of ξ_{it}, $n+1 \le i \le N$ given ξ_{it}, $i \le n$, as the following.

$$\mu_{2.1} = \mu_{(2)} + \vartheta_{2.1}\vartheta_{11}^{-1}(x_{(1)}-\mu_{(1)}) = \begin{bmatrix} \rho_r \sum_{j=1}^{n} \frac{\xi_{jt}}{n} \\ .. \\ \\ .. \\ \rho_r \sum_{j=1}^{n} \frac{\xi_{jt}}{n} \end{bmatrix},\ \vartheta_{2.1\,(N-n)x(N-n)} = \begin{bmatrix} \sigma^2 \left(\frac{(1-\rho_r)(n\rho_r+1)}{(1+(n-1)\rho_r)} \right) \\ .. \\ \\ .. \\ \sigma^2 \left(\frac{(1-\rho_r)(n\rho_r+1)}{(1+(n-1)\rho_r)} \right) \end{bmatrix}$$

Thus, the manufacturer's order-up-to levels are given by the following expressions.

$$\begin{aligned} T_{it}^{pis} &= d + \rho y_{it} - \rho\xi_{i,t} + K\sigma\sqrt{(1+\rho)^2},\ \text{if } i \le n \\ T_{it}^{pis} &= d + \rho y_{it} - \rho\rho_r \sum_{j=1}^{n} \frac{\xi_{jt}}{n} + K\sigma\sqrt{(1+\rho)^2 + \rho^2 \left(\frac{(1-\rho_r)(n\rho_r+1)}{(1+(n-1)\rho_r)} \right)},\ \text{if } i > n \end{aligned} \qquad (4.6)$$

4.2 Substitution Case

When the products are substitutable, though a customer may have preference for a specific product, he will be willing to buy an alternate product when his preferred product is out of stock. This is likely to occur when either the customers' preferences are weak or the customers' cost to search for the preferred product in another retailer is high. For our analysis, we assume that neither the retailer nor the manufacturer incurs any cost when customers buy an alternate or a non-preferred product. Further, we assume that all N products are substitutes of each other. These assumptions are made to isolate the effects of substitution. Without such symmetry assumptions, the results may be affected by factors other than substitution.

When products are substitutable, it is optimal for both the manufacturer and the retailer to treat the set of all products as a single entity and determine the order-up-to level for this entity as a whole. Once the total order (for the entire set of products) for a period is computed, the total order can be divided into orders for individual products based on any allocation scheme. Notice that the allocation scheme does not matter because of our assumption that all products are perfect substitutes of each other and the assumption that neither the retailer nor the manufacturer incurs any cost because of substitution. Substitution costs will simply force minimum order quantity for each product, while still maintaining the same overall order quantity. In case of symmetric substitution costs, it is easy to prove that the order-up-to-level will be identical for all products.

Let $S^{'}_{it}$ denote the retailer order-up-to level for product i under substitution The total demand for all products for the retailer is given by the following.

$$\sum_{i=1}^{N} D_{it} = Nd + \rho \sum_{i=1}^{N} D_{i(t-1)} + \sum_{i=1}^{N} \xi_{it}$$

Notice that $Var(\sum_{i=1}^{N} \xi_{it}) = \sigma^2 \left(N\left(1+\left(N-1\right)\rho_r\right)\right)$. Thus, the sum of order-up-to levels for all products for the retailer can be derived using news-boy analysis as the following.

$$\sum_{i=1}^{N} S^{'}_{it} = Nd + \rho \sum_{i=1}^{N} D_{i(t-1)} + k\sigma \sqrt{\left(N\left(1+\left(N-1\right)\rho_r\right)\right)} \quad (4.7)$$

We derive the sum of order-up-to levels for all products for the manufacturer in a manner similar to that in the no substitution case.

4.2.1 No Information sharing

The total retailer order is given by

$$\sum_{i=1}^{N} y_{i(t+1)} = Nd + \rho \sum_{i=1}^{N} y_{it} + \left(1+\rho\right) \sum_{i=1}^{N} \xi_{i(t+1)} - \rho \sum_{i=1}^{N} \xi_{it} \quad (4.8)$$

Thus, we have

$$\sum_{i=1}^{N} T^{'nis}_{it} = Nd + \rho \sum_{i=1}^{N} y_{it} + K\sigma \sqrt{\left(\left(1+\rho\right)^2 + \rho^2\right)\left(N\left(1+\left(N-1\right)\rho_r\right)\right)} \quad (4.9)$$

4.2.2 Full Information sharing

As explained earlier, in full information sharing, the retailer provides the manufacturer the demand for each period during each period. Thus, $\xi_{i,t}$ is known to the manufacturer when it computes its order-up-to level at time period *t*. Thus, we have

$$\sum_{i=1}^{N} T'^{fis}_{it} = Nd + \rho\sum_{i=1}^{N} y_{it} - \rho\sum_{i=1}^{N}\xi_{it} + K\sigma\sqrt{\left((1+\rho)^2\right)\left(N\left(1+\left(N-1\right)\rho_r\right)\right)} \quad (4.10)$$

4.2.3 Partial Information sharing

Using an analysis similar to that in the no substitution case, we get the following for the total order-up-to level for the manufacturer..

$$\sum_{i=1}^{N} T'^{pis}_{it} = Nd + \rho\sum_{i=1}^{N} y_{i,t} - \rho\sum_{i=1}^{n}\xi_{it} - \rho\left(\mathrm{N}-n\right)\rho_r \frac{\sum_{i=1}^{n}\xi_{it}}{n} + K\sigma \sqrt{\left\{\left((1+\rho)^2\right)\left(N\left(1+\left(N-1\right)\rho_r\right)\right)\right\} + \rho^2\left\{\frac{\left(N-n\right)\left(\left(1-\rho_r^{\,2}\right)+N\left(\rho_r-\rho_r^{\,2}\right)\right)}{1+\left(n-1\right)\rho_r}\right\}} \quad (4.11)$$

5. Impact of Substitution on the Value of Information Sharing

Since the retailer’s ordering decisions are unaffected by information sharing, the value of information sharing arises only from a reduction in the manufacturer’s cost.

If the manufacturer’s service level, as indicated by *K*, is sufficiently high, then the cost reduction comes primarily from inventory reduction. Silver and Petersen (1985, page 295), show that for any order-up-to-T_t system with Y_t being the demand in period *t*, the average inventory level per period can be approximated by

$$T_t - \mathrm{E}(Y_{t+1}) + \mathrm{E}(Y_t)/2 \quad (5.1)$$

Since information sharing does not change either $\mathrm{E}(Y_{t+1})$ or $\mathrm{E}(Y_t)$, the value of information sharing can be approximated by the savings from a reduction in the order-up-to level. We analyze the approximate value of information sharing in the no substitution and substitution cases next.

5.1 No Substitution Case

We know from equation (4.4) and (4.5) that

$$T_{it}^{nis} - T_{it}^{fis} = \left\{ \mathrm{K}\sigma\sqrt{\left((1+\rho)^2+\rho^2\right)} \right\} - \left\{ \mathrm{K}\sigma\sqrt{(1+\rho)^2} \right\} \qquad \text{Since} \quad \left[\mathrm{E}\left(\xi_{i,t}\right) = 0 \right]$$

So the value of full information sharing per period for one product is given by

$$H\left(T_{it}^{nis} - T_{it}^{fis}\right) = H\mathrm{K}\sigma\left\{ \sqrt{\left((1+\rho)^2+\rho^2\right)} - \sqrt{(1+\rho)^2} \right\}$$

The value of full information sharing for N products under no substitution is given by

$$H\left(T_{it}^{nis} - T_{it}^{fis}\right) = NH\mathrm{K}\sigma\left\{ \sqrt{\left((1+\rho)^2+\rho^2\right)} - \sqrt{(1+\rho)^2} \right\} \tag{5.2}$$

In the same manner, we can derive the value of partial information sharing using equation (4.4) and (4.6) as the following.

$$H\left(T_{it}^{nis} - T_{it}^{pis}\right) = N\sqrt{\left((1+\rho)^2+\rho^2\right)} - \left\{ n\sqrt{(1+\rho)^2} + \left(N-n\right)\sqrt{(1+\rho)^2+\rho^2\left(\frac{(1-\rho_r)(n\rho_r+1)}{\left(1+\left(n-1\right)\rho_r\right)}\right)} \right\} \tag{5.3}$$

The following properties hold for the value of information sharing under no substitution.

(i) $\dfrac{\partial\left(T_{i,t}^{nis} - T_{i,t}^{fis}\right)}{\partial\rho_r} = 0$

(ii) $\dfrac{\partial\left(T_{it}^{nis} - T_{it}^{fis}\right)}{\partial N} = H\mathrm{K}\sigma\left\{ \sqrt{\left((1+\rho)^2+\rho^2\right)} - \sqrt{(1+\rho)^2} \right\} \quad > 0$

(iii) $\dfrac{\partial\left(T_{it}^{nis} - T_{it}^{pis}\right)}{\partial n} = -\left(1+\rho\right) + \left[\dfrac{-\rho^2\left(N-n\right)(\rho_r^{\,3}-\rho_r^{\,2})}{2\left(1+(n-1)\rho_r\right)^2\sqrt{(1+\rho)^2+\rho^2\left(\frac{(1-\rho_r)(1+n\rho_r)}{\left(1+\left(n-1\right)\rho_r\right)}\right)}} + \sqrt{(1+\rho)^2+\rho^2\left(\frac{(1-\rho_r)(1+n\rho_r)}{\left(1+\left(n-1\right)\rho_r\right)}\right)} \right] > 0$

The above three properties show that the value of information sharing under no substitution is unaffected by the correlation among product demands and is increasing in the total number of products and number of products whose demands are shared.

5.2 Substitution Case

The value of information sharing under substitution can be derived in a manner similar to that under no substitution using equations (4.9), (4.10), and (4.11). We simply show the final expressions for the value of full information sharing and partial information sharing below.

$$H\left(\sum_{i=1}^{N} T'^{nis}_{it} - \sum_{i=1}^{N} T'^{fis}_{it}\right) = HK\sigma\sqrt{\left(N\left(1+\left(N-1\right)\rho_r\right)\right)}\left\{\sqrt{\left(\left(1+\rho\right)^2+\rho^2\right)}-\sqrt{\left(\left(1+\rho\right)^2\right)}\right\} \quad (5.4)$$

$$H\left(T'^{nis}_{it} - T'^{pis}_{it}\right) = HK\sqrt{\left(\left(1+\rho\right)^2+\rho^2\right)\left(N\left(1+\left(N-1\right)\rho_r\right)\right)} - HK\sqrt{\left\{\left(\left(1+\rho\right)^2\right)\left(N\left(1+\left(N-1\right)\rho_r\right)\right)\right\}+\rho^2\left\{\frac{\left(N-n\right)\left(\left(1-\rho_r\right)^2+N\left(\rho_r-\rho_r^{\,2}\right)\right)}{1+\left(n-1\right)\rho_r}\right\}} \quad (5.5)$$

The following properties hold for the value of information sharing under full substitution.

(i) $$\frac{\partial\left(T'^{nis}_{it} - T'^{fis}_{it}\right)}{\partial\rho_r} = HK\sigma\left\{\sqrt{\left(\left(1+\rho\right)^2+\rho^2\right)}-\sqrt{\left(\left(1+\rho\right)^2\right)}\right\}\frac{1}{2\sqrt{\left(N\left(1+\left(N-1\right)\rho_r\right)\right)}}\left(N^2-N\right)>0$$

(ii) $$\frac{\partial\left(\sum_{i=1}^{N} T^{nis}_{it} - \sum_{i=1}^{N} T^{fis}_{it}\right)}{\partial N} - = HK\sigma\left\{\sqrt{\left(\left(1+\rho\right)^2+\rho^2\right)}-\sqrt{\left(\left(1+\rho\right)^2\right)}\right\}\frac{\left(1+2N\rho_r-\rho_r\right)}{2\sqrt{\left(N\left(1+\left(N-1\right)\rho_r\right)\right)}}>0$$

(iii)

$$\frac{\partial\left(T^{nis}_{it} - T^{pis}_{it}\right)}{\partial n} = \frac{\rho^2\left\{\left[\left(1+\left(n-1\right)\rho_r\right)\left(1-\rho_r\right)^2\right]+\left[\left(\left(N-n\right)\left(\left(1-\rho_r\right)^2+N\left(\rho_r-\rho_r^{\,2}\right)\right)\right)\rho_r\right]\right\}}{2\left(1+\left(n-1\right)\rho_r\right)^2\sqrt{\left\{\left(\left(1+\rho\right)^2\right)\left(N\left(1+\left(N-1\right)\rho_r\right)\right)\right\}+\rho^2\left\{\frac{\left(N-n\right)\left(\left(1-\rho_r\right)^2+N\left(\rho_r-\rho_r^{\,2}\right)\right)}{1+\left(n-1\right)\rho_r}\right\}}}>0$$

The above properties show that the value of information sharing under substitution is increasing in demand correlation, the number of substitutable products, and on the number of products whose demand information is shared.

5.3 Comparison of the Value of Information Sharing Under Substitution and No Substitution for Full Information Sharing

Comparing equation (5.2) and (5.4), we get

$$NH\mathrm{K}\sigma\left\{\sqrt{\left((1+\rho)^2+\rho^2\right)}-\sqrt{(1+\rho)^2}\right\}\geq HK\sigma\sqrt{\left(N\left(1+\left(N-1\right)\rho_r\right)\right)}\left\{\sqrt{\left((1+\rho)^2+\rho^2\right)}-\sqrt{\left((1+\rho)^2\right)}\right\}\Rightarrow$$

$$N\geq\sqrt{\left(N\left(1+\left(N-1\right)\rho_r\right)\right)}\Rightarrow N^2\geq\left(N\left(1+\left(N-1\right)\rho_r\right)\right)\Rightarrow\rho_r\leq 1$$

So, the value of full information sharing for no substitution is always higher than substitution under full information sharing.

5.4 Comparison of the Value of Information Sharing Under Substitution and No Substitution for Partial Information Sharing

We are unable to show whether the value of partial information sharing is greater under substitution or under no substitution. Further, we are unable to analytically derive meaningful conditions under which value of information sharing is higher under substitution (or no substitution). Consequently, we used to a numerical example to perform the comparative analysis. For the numerical example, we used the following parameter values.

$\sigma = 2$, $\rho = 0.7$, H =1, P =25 and N =10

The results for the numerical example are shown in Table 3. The first (second) row in each cell represents the value of information sharing under substitution (no substitution). The results illustrate that the value of partial information sharing can be greater under substitution or under no substitution depending on parameter values. The numbers in bold indicate that the value of information sharing is higher under substitution in those cells. Specifically, we find that the value of information sharing under substitution is higher when the fraction of products for which information is shared is sufficiently small and the correlation between products demands is sufficiently high. We offer more insights into the conditions under which either of these cases occurs in Section 6.

Table 3. Value of partial information sharing under substitution and no substitution

$r \setminus \rho_r$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.2	0.0849 0.2769	0.7194 1.0490	1.1853 1.2326	**1.6685** 1.5056	**2.1551** 1.8503	**2.6377** 2.2539	**3.1122** 2.7073	**3.5766** 3.2034	**4.0299** 3.7371	**4.4717** 4.3043
0.4	0.1710 0.5539	1.2287 2.0480	1.7941 2.2449	2.3021 2.5008	2.7624 2.7931	**3.1837** 3.1099	**3.5731** 3.4447	**3.9358** 3.7934	**4.2762** 4.1536	**4.5974** 4.5235
0.6	0.2585 0.8308	1.6083 3.0168	2.1649 3.1686	2.6359 3.3520	3.0488 3.5522	3.4198 3.7629	3.7587 3.9810	4.0722 4.2047	4.3651 4.4332	4.6409 4.6658
0.8	0.3475 1.1078	0.5373 1.1203	2.4144 4.0481	2.8419 4.1426	3.2156 4.2434	3.5515 4.3480	3.8589 4.4551	4.1440 4.5644	4.4110 4.6754	4.6629 4.7880
1.0	0.4379 4.9021	0.6036 4.9021	2.5939 4.9021	2.9818 4.9021	3.3247 4.9021	3.6355 4.9021	3.9216 4.9021	4.1883 4.9021	4.4390 4.9021	4.6763 4.9021

6. Simulation Analysis

In the last section, we theoretically derived and analyzed the approximate value of information sharing under no substitution and substitution scenarios. The approximate value accounted for only the savings in the inventory holding cost, and it ignored the shortage cost. In this section, we analyze the "true" value of information sharing that includes both inventory holding and shortage costs through a numerical simulation. The objectives of our simulation were two-fold. First, we wanted to verify that results from our theoretical analysis of the approximate value of information sharing hold for the actual value of information sharing also. Second, we wanted to obtain a sense of the magnitude of the effect of substitution on the value of information sharing. For our simulation, we fixed the following parameters for all simulation scenarios:

$$\sigma = 2, \rho = 0.7, h = 1, H = 1, p = 25, P = 25, d = 40$$

We varied the following parameters within the respective ranges shown.

$$N \in \{5, 10, 25, 50\}$$

$$r \in \{0.2, 0.4, 0.6, 0.8\}, \text{ where } \left(r = \frac{n}{\mathrm{N}} \right)$$

$$\rho_{\mathrm{r}} \in \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$$

For each scenario, we analyzed the no substitution and substitution cases and conducted the simulation for 10,000 periods each case. Table 4 and table 5 show, respectively, the sum of inventory holding cost and shortage cost for all periods and all products for each of the scenarios simulated.

Table 4. Value of information sharing for No substitution

N	Correlation Coefficient / Strategies		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
50	***PIS***	r = 0.2	59155	62391	69618	84634	104578	133117	163754	197363	225977	266094
		r = 0.4	117373	120674	123498	135492	149023	174675	198566	224763	248009	280286
		r = 0.6	179193	178367	178636	188167	197227	216258	232783	250326	266008	289360
		r = 0.8	234300	231174	231748	238194	243708	254681	264304	275011	284201	300289
	FIS	r=1.0	293165	286290	286284	291434	290797	297518	299627	299959	303100	308713
25	***PIS***	r = 0.2	29930	31207	36509	44382	53704	67701	81836	98176	115561	122285
		r = 0.4	59155	59340	63031	70175	76391	87136	97852	111630	123135	127209
		r = 0.6	89348	89641	90329	96548	98360	107083	115290	125416	133197	133700
		r = 0.8	117373	116759	116395	121587	121218	127712	131546	136803	141461	138575
	FIS	r=1.0	147981	144723	143784	147974	144091	148507	148685	149651	150992	147974
10	***PIS***	r = 0.2	11414	11293	13388	17121	20222	25955	30521	36059	43462	50413
		r = 0.4	23487	23249	25852	28705	31283	35660	38946	43022	49372	53315
		r = 0.6	35591	34485	36200	39692	40647	44702	46341	49136	52899	55732
		r = 0.8	47522	46125	47892	50024	49680	53165	52838	55057	56794	57534
	FIS	r=1.0	59155	57217	58282	60643	58879	60574	59411	60122	59911	59799
5	***PIS***	r = 0.2	5627	5695	6386	7734	9913	11604	14359	16595	19488	24754
		r = 0.4	11414	11034	12442	14582	15959	18299	19314	21578	24361	26944
		r = 0.6	17805	16892	18446	20538	20592	23367	23606	24735	26960	28418
		r = 0.8	23487	22967	24542	26003	25356	26904	26987	27206	28857	28876
	FIS	r=1.0	29930	29024	30521	32190	30742	31752	30669	30477	30814	30535
PIS – Partial Information Sharing; ***FIS*** – Full Information Sharing												

Table 5. Value of information sharing for Full Substitution

N	Correlation Coefficient / Strategies		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
50	***PIS***	r = 0.2	7063	50660	84040	118580	152950	183250	210210	242740	278700	308100
		r = 0.4	13613	71530	106720	140460	170850	199510	222830	250800	284540	311820
		r = 0.6	21291	83340	118620	151110	180180	208210	228890	254780	286350	313570
		r = 0.8	28399	90000	123680	154100	183500	211580	231740	257090	287970	313660
	FIS	r=1.0	36737	92510	123610	152560	182110	211810	232140	257670	289540	315200
25	***PIS***	r = 0.2	4702	22061	38697	57280	75390	92050	103080	118040	135870	151620
		r = 0.4	9867	32795	50331	68180	85010	99890	109760	121910	138830	153160
		r = 0.6	15162	40886	58532	76330	93140	107180	114730	125560	141150	154140
		r = 0.8	20607	46077	62499	79190	95800	109100	116020	126620	142620	155680
	FIS	r=1.0	26392	50382	64652	81090	97230	110670	117350	127320	143210	156030
10	***PIS***	r = 0.2	2982	8639	14306	20094	26280	32195	38121	44323	50848	58853
		r = 0.4	6517	14483	20891	27243	32735	38672	43909	48544	53811	60975
		r = 0.6	10532	19552	26544	32597	37776	42613	46403	50451	55008	61083
		r = 0.8	13702	22328	29034	35218	39891	44076	47380	50687	54719	61239
	FIS	r=1.0	16506	24114	29794	35585	40610	44573	47884	51217	55333	61561
5	***PIS***	r = 0.2	2850	5236	6442	8674	10618	13552	16392	19654	23371	27291
		r = 0.4	5242	8881	10309	13648	15720	18714	20627	24191	26669	29503
		r = 0.6	8413	12698	14494	17348	19293	21932	23728	25863	27725	30249
		r = 0.8	10995	14408	16299	18747	20911	22818	24713	26770	28320	30444
	FIS	r=1.0	14118	17882	19960	21715	23430	24694	26273	27950	29329	30665

We computed the value of information sharing for each scenario under the no substitution and no substitution cases as the following:

Value of information sharing =

Total cost without information sharing – Total cost with information sharing

Table 6 shows a summary of our results from numerical simulations. The data in table 6 clearly show that the value of information sharing is significantly lower when products are substitutable than when they are not. These data confirm our overall theoretical result that substitution reduces the value of information sharing.

Table 6. Average Value of Information Sharing Under Substitution and No substitution

Strategies/ Measure	**No Substitution**		**Substitution**	
	No Information sharing	***Full Information sharing***	***No Information sharing***	***Full Information sharing***
Average Total Cost of manufacturer	18532581	17199607	11954641	11076790
Average value of information sharing	1332974		877851	

We further computed how close the value of information sharing under simulation was to the value obtained through analytical expressions given in Section 5. These results are given in appendix I and II. The results show that the error is less than 2% in the no substitution case. The errors for the substitution case were higher (up to 36%) because, in the substitution case, the order-up-to level decisions are made based on the total demand of all products which has a higher variance, but, in the no substitution case, the order-up-to level decision is made based on the demand for a single product, which has a lower variance.

6.1 Impact of Demand Correlation on the Value of Information Sharing

Figure 1 shows the impact of correlation between product demands during a period on the value of full information sharing for N = 10. We observed a similar pattern for other values of *N* also. The figure shows that the correlation coefficient does not affect the value when products are not substitutable. However, the value of information increases as demands become more correlated in the substitution case, thus decreasing the difference between the value of information sharing under substitution and no substitution. This result can be explained as follows. In the case of non-substitutable products, since ordering decisions for each product, under no information sharing and full information sharing, are made based on the product's individual demand only, correlation among product demands does not play any role in the ordering decisions. Consequently, correlation does not affect the costs under information sharing and no information sharing, and hence, the value of information sharing. However, in the case of substitutable products, both the manufacturer and the retailer make ordering decisions based on the total demand for the entire group of products. Note that when products are fully substitutable, neither the manufacturer nor the retailer cares about the sale of any single product, but only cares about the total demand as a whole. The variance of the total demand increases as the correlation among individual product demands increase. Since the value of information sharing arises primarily from a reduction in variance, a higher variance leads to a higher reduction from information sharing, which explains a higher value of information sharing in the case of substitutable products.

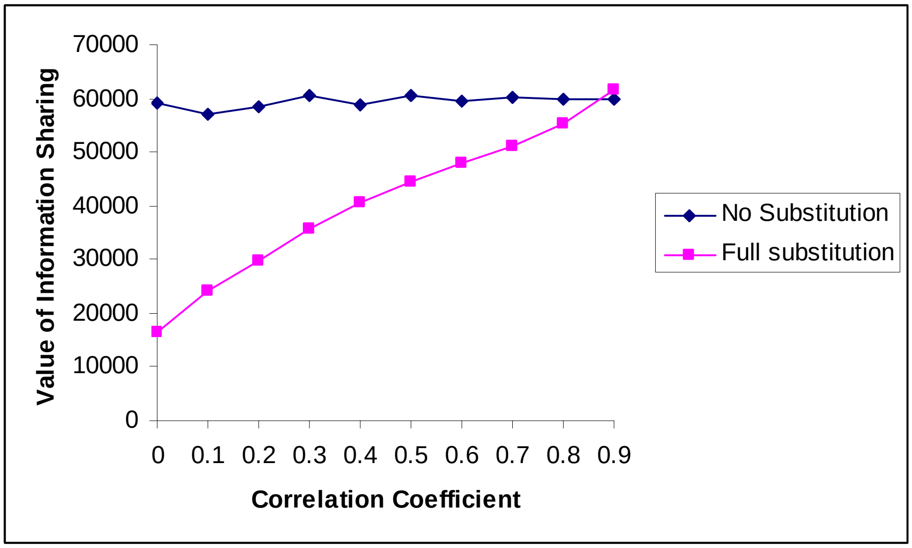

Figure 1.Value of Information Sharing under No Substitution and Substitution

6.2 Impact of Number of Products on the Value of Information Sharing

Figure 2 shows the impact of number of products on the value of full information sharing. For this analysis, we fix the value of $\rho_r = 0.5$. The figure shows that the value of information sharing increases as the number of products increases whether or not the products are substitutable. This result also confirms our theoretical result. An interesting observation from Figure 2 is that the marginal increase in value when *N* increases is higher when the products are not substitutable than when they are. This suggests that the reduction in the value of information sharing because of substitutability is likely to be significantly higher when the number of products is large. This result can be explained as follows. In the case of non-substitutable products, the observation that the increase in value is linear in *N* is intuitive because the value of information sharing for one product is unaffected by other products. However, the value of information sharing is concave in *N* when products are substitutable. This result arises from the fact that the standard deviation of total demand, which determines the inventory holding and shortage costs, is a concave function (specifically, the standard deviation varies as the square root of *N*) of *N*. Consequently, the value of information sharing is also a concave function of *N*.

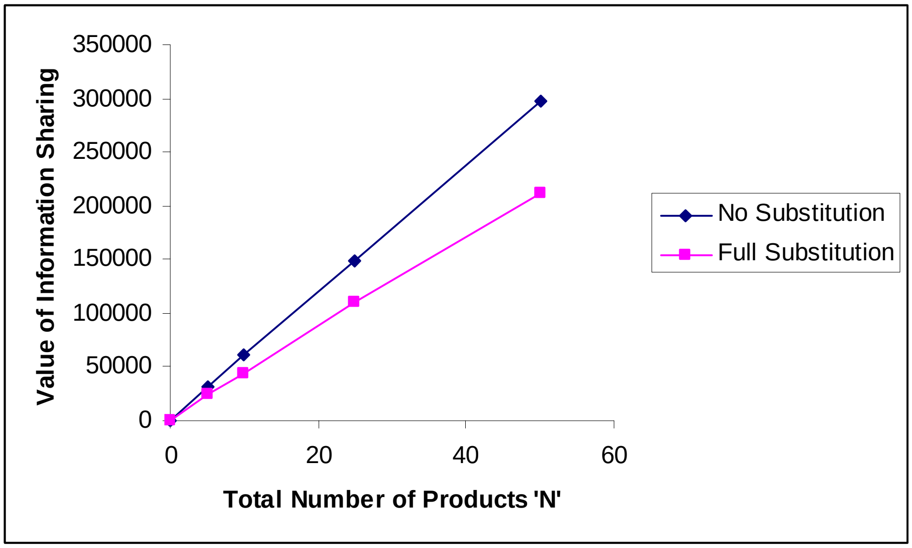

Figure 2. Effect of Number of Products N on the Value of Information Sharing

6.3 Impact of Partial Information Sharing

In sections 6.1 and 6.2, we assumed that when information is shared, the retailer shares his demand information for all products. However, because of cost and complexity considerations, the information sharing arrangement may involve only a sub set of the products. For instance, Raghunathan (2003) shows, in a different context, that it may be optimal for the manufacturer to obtain information only from a sub set of retailers rather than from all retailers. Similar considerations might apply even in the context of a single retailer selling multiple products. Our primary objective in this analysis is to determine how the number of products for which information is shared affects the value of information sharing. Table 7 shows the results for various values of r and ρ_r when N is fixed at 10. As in table 3, the first (second) row in each cell represents the value of information sharing under substitution (no substitution). The finding that the value of information sharing increases as the number of products for which information is shared increases under both substitution and no substitution cases is intuitive; more information reduces demand variance more, which leads to a higher value. However, the more interesting finding is that when the number of products for which information is shared is low, the value of information sharing under substitution is ***higher*** than under no substitution under certain conditions. We find that this is more likely to occur when the demand correlation is high. This is somewhat counter-intuitive because one would expect substitution to mitigate the variance because of demand pooling effect and hence result in a smaller value of information sharing. The seemingly counter-intuitive finding is the outcome the interplay of two effects of demand correlation that impact the value of information sharing. First, demand correlation enables information about the demand of one product to be used to reduce the variance about the demand of other products. This effect of correlation is present under substitution and no-substitution cases. The second effect, which is present only under the substitution case, is that demand correlation increases the variance of the total or pooled demand. When the number of products whose information is shared is small, the latter effect dominates the former, resulting in a higher value of information sharing under substitution. As the number of products whose demand information is shared increases, the impact of the second effect reduces relative to that of pooling under product substitution. When demand correlation is high, the second effect weakens only when the number of products whose demand is shared is also high.

Further, we also find that the value of information sharing is a concave function of the number of products whose demand data are shared when the products are substitutable. This finding shows that the incremental value of information for an additional product is smaller as r increases, which suggests that when demands are positively correlated, information behave as substitutes.

Table 7 Comparison of Value of Partial Information Sharing under Substitution and No Substitution

$r \setminus \rho_r$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.2	2982 11414	8639 11293	**14306** 13388	**20094** 17121	**26280** 20222	**32195** 25955	**38121** 30521	**44323** 36059	**50848** 43462	**58853** 50413
0.4	6517 23487	14483 23249	20891 25852	27243 28705	**32735** 31283	**38672** 35660	**43909** 38946	**48544** 43022	**53811** 49372	**60975** 53315
0.6	10532 35591	19552 34485	26544 36200	32597 39692	37776 40647	42613 44702	**46403** 46341	**50451** 49136	**55008** 52899	**61083** 55732
0.8	13702 47522	22328 46125	29034 47892	35218 50024	39891 49680	44076 53165	47380 52838	50687 55057	54719 56794	**61239** 57534
1.0	16506 59155	24114 57217	29794 58282	35585 60643	40610 58879	44573 60574	47884 59411	51217 60122	55333 59911	**61561** 59799

7. Conclusions

In this paper, we analyzed the impact of consumer product substitution on the value of information sharing in supply chains. We showed that substitutability among products generally reduces the value of information sharing. Further, the reduction in the value of information sharing because of substitutability is higher when the number of substitutable products is higher and/or when the demands of products are less correlated. We also showed that there are instances in which the value of information sharing is actually higher under substitution than under no substitution. For instance, when information about the demands of only a sub set of products is shared, the value of information sharing under substitution can be higher when the demand correlation is sufficiently high and the number of products whose demand information is shared is sufficiently low.

Our findings have significant implications for practicing managers. The key implication of our findings is that if substitution effects are ignored, there is a significant risk overestimating the value of information sharing. Consequently, managers assessing the returns from supply chain management and collaboration software that aim to improve visibility among supply chain partners through information sharing should be cautious about the claims made based on results from single product simulations. The risk can be especially significant when either the there is a large number of substitutable products and when the demands of these products are independent or not strongly correlated.

We believe that this paper is the first, in the growing literature on supply chain information sharing, to investigate the effect of consumer substitution on the value of supply chain information sharing. Being the first study, we made several simplifying assumptions in our analysis. For instance, we assumed that the lead time for the manufacturer to supply to the retailer is zero. We also considered a two-level supply chain. We made these assumptions because the impacts of lead time and the number of levels in the supply chain have been considered by prior literature that does not consider substitution. We do not anticipate that lead time and number of supply chain levels will have a different impact when substitution is considered. We assumed an AR (1) model for the demand forecasting process. This model has been extensively used by prior literature. Future research should address other demand forecasting models with substitution. Furthermore, we assumed that the AR(1) model parameters are known to both manufacturer and the retailer. However, in reality, these parameters are likely to be estimated based on historical data. Our model assumes that all products are symmetric; that is, demand models and costs are identical across products. This was done to isolate the effects of substitution so that factors such as demand sizes and costs do not contaminate our findings. However, in reality, we are likely to find substitutable products with different market shares (i.e., demands). Future research can look into how the impact of substitution is affected by market shares of products.

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APPENDIX I

Error Percentage for No-Substitution

N	Correlation Coefficient / Strategies		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
50	***NIS***	r=0.0	1.41	1.41	1.41	1.41	1.40	1.42	1.42	1.43	1.45	1.44
	PIS	r=0.2	1.35	1.05	1.02	1.02	1.05	1.06	1.06	1.07	1.07	1.05
		r=0.4	1.31	1.10	1.08	1.08	1.09	1.10	1.10	1.11	1.10	1.08
		r=0.6	1.25	1.13	1.11	1.10	1.10	1.11	1.12	1.12	1.12	1.10
		r=0.8	1.19	1.16	1.13	1.12	1.12	1.13	1.14	1.14	1.14	1.10
	FIS	r=1.0	1.11	1.15	1.14	1.13	1.12	1.13	1.13	1.14	1.14	1.11
25	***NIS***	r=0.0	1.47	1.45	1.43	1.43	1.43	1.45	1.45	1.46	1.46	1.43
	PIS	r=0.2	1.40	1.06	1.03	1.05	1.07	1.08	1.07	1.08	1.07	1.06
		r=0.4	1.34	1.09	1.05	1.07	1.10	1.09	1.09	1.11	1.09	1.07
		r=0.6	1.31	1.14	1.11	1.12	1.14	1.13	1.12	1.13	1.14	1.09
		r=0.8	1.26	1.18	1.16	1.16	1.16	1.14	1.15	1.16	1.15	1.12
	FIS	r=1.0	1.21	1.22	1.19	1.19	1.18	1.16	1.17	1.18	1.18	1.18
10	***NIS***	r=0.0	1.43	1.38	1.34	1.37	1.36	1.37	1.38	1.38	1.40	1.41
	PIS	r=0.2	1.41	1.04	1.00	1.04	1.05	1.04	1.07	1.09	1.07	1.03
		r=0.4	1.33	1.06	1.02	1.06	1.06	1.05	1.08	1.10	1.09	1.07
		r=0.6	1.20	1.00	0.95	0.98	0.96	0.98	1.02	1.03	1.03	1.02
		r=0.8	1.17	1.03	0.99	1.02	1.03	1.02	1.05	1.08	1.07	1.06
	FIS	r=1.0	1.10	1.05	1.00	1.04	1.03	1.03	1.07	1.08	1.09	1.08
5	***NIS***	r=0.0	1.32	1.31	1.28	1.37	1.36	1.38	1.39	1.40	1.42	1.41
	PIS	r=0.2	1.28	0.95	0.96	1.00	0.97	0.99	0.99	1.06	1.06	1.02
		r=0.4	1.26	1.04	1.00	1.05	1.09	1.06	1.12	1.10	1.11	1.05
		r=0.6	1.16	1.05	1.03	1.06	1.08	1.09	1.15	1.14	1.12	1.09
		r=0.8	1.10	1.08	1.05	1.09	1.10	1.10	1.15	1.14	1.13	1.13
	FIS	r=1.0	0.94	1.01	0.96	1.01	1.02	1.03	1.10	1.09	1.10	1.10

NIS* –**No Information Sharing, ***PIS – Partial Information Sharing; ***FIS*** – Full Information Sharing

APPENDIX II
Error Percentage for Full-Substitution

N	Correlation Coefficient / Strategies		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
50	***NIS***	r=0.0	34.92	18.24	14.28	12.21	10.91	9.98	9.37	8.76	8.29	7.91
	PIS	r=0.2	35.21	18.96	14.99	12.90	11.56	10.60	9.87	9.29	8.80	8.38
		r=0.4	35.61	19.23	15.16	13.01	11.64	10.66	9.92	9.33	8.83	8.40
		r=0.6	35.95	19.34	15.20	13.03	11.65	10.66	9.92	9.33	8.83	8.40
		r=0.8	36.33	19.42	15.23	13.05	11.66	10.67	9.92	9.33	8.83	8.40
	FIS	r=1.0	36.66	19.44	15.23	13.04	11.65	10.66	9.91	9.32	8.82	8.39
25	***NIS***	r=0.0	27.31	17.09	13.66	11.82	10.67	9.80	9.15	8.57	8.16	7.75
	PIS	r=0.2	27.54	17.52	14.23	12.40	11.21	10.31	9.65	9.11	8.65	8.24
		r=0.4	27.86	17.78	14.41	12.53	11.30	10.37	9.68	9.13	8.66	8.24
		r=0.6	28.24	17.99	14.52	12.60	11.35	10.41	9.72	9.16	8.68	8.26
		r=0.8	28.59	18.11	14.59	12.65	11.38	10.43	9.74	9.17	8.68	8.27
	FIS	r=1.0	28.93	18.19	14.64	12.67	11.40	10.44	9.74	9.18	8.69	8.27
10	***NIS***	r=0.0	18.11	13.93	11.85	10.49	9.65	8.93	8.40	8.06	7.68	7.28
	PIS	r=0.2	18.46	14.29	12.23	10.99	10.09	9.44	8.92	8.49	8.13	7.78
		r=0.4	18.66	14.48	12.39	11.12	10.20	9.50	8.97	8.52	8.14	7.78
		r=0.6	18.70	14.51	12.37	11.07	10.15	9.43	8.90	8.45	8.09	7.74
		r=0.8	19.05	14.70	12.50	11.18	10.22	9.49	8.95	8.50	8.12	7.76
	FIS	r=1.0	19.31	14.80	12.57	11.22	10.24	9.51	8.96	8.50	8.13	7.76
5	***NIS***	r=0.0	12.27	10.86	9.83	8.82	8.26	8.02	7.57	7.13	6.94	6.75
	PIS	r=0.2	12.51	11.14	9.911	9.17	8.62	8.14	7.80	7.50	7.24	6.99
		r=0.4	12.77	11.18	10.13	9.38	8.82	8.31	7.95	7.62	7.34	7.05
		r=0.6	12.90	11.28	10.22	9.46	8.86	8.36	7.98	7.64	7.34	7.04
		r=0.8	13.02	11.41	10.31	9.52	8.90	8.37	7.99	7.65	7.37	7.05
	FIS	r=1.0	13.07	11.20	10.21	9.44	8.8	8.30	7.94	7.59	7.30	7.01

NIS **–**No Information Sharing, ***PIS*** – Partial Information Sharing; ***FIS*** – Full Information Sharing