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Optimal staffing policy for queuing systems with cyclic demands: waiting cost approach

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Abstract
Although the lost profit from lost business is quite difficult to estimate for the queuing systems with cyclic demands, this paper presents a creative and effective approach to formulate waiting cost including balking loss and reneging loss. The balking probability is defined as the balking index θ_1 multiplying by the expected queue length L_q and the reneging probability is defined as the reneging index θ_2 multiplying by the expected waiting time in queue W_q. Using the estimation of waiting cost allows decision maker to have the capability of determining the optimal number of servers for each planning period by minimizing the total cost including the service cost and the waiting cost.

Keywords: manpower planning; queuing theory; balking loss; reneging loss

1. Introduction

Recently, both of service and manufacture industries increase using contingent workers to fulfill the fluctuated demands of workforce. According to Pinker (2003), contingent workers help industry grew from 17.0 to 63.6 billion dollars between 1990 and 2000, and annual employment of contingent workers increased from .99 to 2.54 million. Brewster et al. (1997) also indicated that across Europe there is a substantial amount of contingent workers and that there has been a continuous increasing in its use. Firms rely more and more upon contingent workers to fill positions that were usually once considered the exclusive purview of full-time permanent workers. The prevalent using of these different staffing arrangements is driven by a firm's uncertainty in its demand for labor.

We can realize that firms make use of contingent workers for cutting costs and increasing the flexibility with respect to market fluctuations. Foote and Folta (2002) also presented the real options theory that can offer managers the ability to consider irreversibility and to make workforce investment decisions under conditions of minimum uncertainty and maximum flexibility.

The demands of labor for most of the service systems, such as fast-food restaurants and supermarkets, are varied from period to period. Thus, in order to minimize system cost for developing competitive advantages, managers need to determine staffing requirements, especially determine the work schedule for the contingent workers.

Martinich (2002) indicated the competition for customers in retail service industries, for instance, the fast-food, grocery and banking industries, is often fierce. This competition occurs not only in price, but especially in perceived customer service. According to Bennett (1990, p. B1), “no aspect of customer service is more important than the wait in line to be served”, and “the wait can destroy an otherwise perfect service experience.” Thus, even small reductions in waiting time will result in better quality of service and lead to enhance customer loyalty and increase sales.

An excellent review for the issue of time and consumer behavior from differing perspectives in the literature of economics, marketing and management science is given by Jacoby et al. (1976). In this paper, the economist, J. Mincer, is quoted as stating in his well known 1963 paper, “the properly defined price with which the consumer is faced is not p, the market selling price, but p = p+c where c is the opportunity cost of time.” (Jacoby et al., 1976, p.321) Although the importance of waiting time as a form of price has been recognized, it is difficult to account for this effect in making managerial decisions. Ittig (1994) mentioned two difficulties for estimating the opportunity cost of the waiting time. The one is that the waiting time may vary with each different customer, while the price is fixed. The other is that the monetary value of per unit of the waiting time during a shopping trip may vary widely from one customer to another, and may have difficulty to estimate. Therefore, it is more practical to consider using the concept of the waiting time as a form of the price to estimate the lost profit from lost business.

Commonly, managers of service systems in which the timing of customer demands for service is random and cyclic adjust staffing levels in order to provide a fixed level of service at all times. Examples include staffing of toll plazas (Edie, 1954), airline ground services (Stern and Hersh, 1980; Holloran and Byrne, 1986; Brusco et al., 1995), tele-retailing

(Andrews and Parsons, 1989), and banking (Brewton, 1989). In applications described in the literature, these staffing policies are typically determined by first dividing the workdays into "planning periods," such as hours, half-hours, etc. Thus, a series of stationary queuing models, most often M/M/S type models, is developed as one model for each planning period. Each of these models is independently solved for the minimum number of servers needed to meet the service level in that period. We refer this method of setting staffing policy as the stationary independent period by period (SIPP) approach. (Green, 2001) The assumptions for using a series of stationary queuing models to set staffing policy are: (i) delays in consecutive planning periods are statistically independent of one another; (ii) within each planning period the system achieves steady state; and (iii) the arrival rate does not change during the planning period. Although setting a fixed level of service to determine the minimum number of servers is used widely to set work schedules, but this approach does not minimize the total cost (service cost plus waiting cost) to determine the optimal staffing policies for queuing systems with cyclic demands. Thus, from the viewpoint of the managers, minimizing total cost for determining optimal staffing policy is considered more practical even the waiting cost in service systems, such as fast food restaurants, is quite difficult to estimate. Primarily, the waiting cost consists of the lost profit from the lost business. The lost business may occur immediately or in the future, because the customer judges the queue to be too long and does not join the queue or the customer is sufficiently irritated that he or she does not come again. This kind of waiting cost is hard to estimate, and it may need to revert to other criteria. (Hillier and Lieberman, 2000)

While making decision for the number of servers needed in the service system to meet time-varying demand, the balking probabilities are needed to estimate the amount of lost business in more practical consideration for the managers. (Liao, 2006) Balking is the act of not joining a queue because the prospective arriving customer judges the queue to be too long (Jones, 1999). Jones (1999) also mentioned that only 1% balking rate in drive-thru windows of U.S.-based Quick Service Restaurants (QSRs) can reduce QSR revenues by over $100 million per year for each $10 billion in sales. Zhou and Soman (2003) study consumers who are part of a queue and investigate their affective experiences and their decisions to leave the queue after having spent some time in it. They refer to the decision to leave as "reneging" which will be treated as a different action from the meaning of balking. Hence this research extends my previous work (Liao, 2006) to add another possible loss, reneging loss, to the formulation of the waiting cost. Assuming the data in Table 1 is collected from one fast food restaurant and is used for an example to show the service demand fluctuates in fifteen time periods. However, the manager uses only three servers in peak hours. Some of customers decided not to join a queue because they judge the queue to be too long or they may leave the queue after having spent some time in it, especially in peak hours. From Table 1, the total number of balking customers is 112 and reneging customers is 95. The balking probability is about (112/1424) = 0.079 = 7.9% and the reneging probability is about (95/1424) = 0.067 = 6.7%. Thus, the formulating of balking probability and reneging probability is needed to estimate the amount of lost business and evaluate the service configuration.

Recently, firms increase using contingent workforce in their service system, especially for the low-skilled required positions. In order to operate with minimum total cost, service cost plus waiting cost (balking loss and reneging loss), managers need to allocate appropriate contingent workforce for different periods that have cyclic demands. If you add more servers in the service systems, of course, the average queue length and waiting time will reduce, that is, the balking loss and the reneging loss will reduce, but the service cost will increase. Contrarily, if you use fewer servers in service systems, you can reduce the service cost, but the balking loss and the reneging loss will increase. In order to approach the optimal number of servers by minimizing total cost , the data in Table 1 and the SIPP M/M/S:(FCFS/∞/∞) queuing model are utilized for analyzing.

Table 1
Number of customers, servers, balking customers, and reneging customers

Time period	7–8	8–9	9–10	10–11	11–12	12–13	13–14	14–15	15–16	16–17	17–18	18–19	19–20	20–21	21–22	Total
Number of customers	77	34	30	6	220	358	121	57	37	39	38	53	147	91	116	1424
Number of servers	3	3	2	2	3	3	3	3	3	3	3	3	3	3	3	43
Number of balking customers	3	0	0	0	21	37	15	5	0	0	3	5	11	8	4	112
Number of reneging customers	2	0	0	0	20	35	16	3	0	0	0	3	7	6	3	95

2. Formulate balking loss and reneging loss

Balking is the act of not joining a queue because the prospective arriving customer judges the queue to be too long. (Jones, 1999) Hence, it is reasonable to assume the probability of balking is equal to $L_q \times \theta_1$, where L_q is the expected queue length (excludes customers being served), and θ_1 is the balking index. θ_1 is the average willingness of customers for balking behavior of some queuing system and the larger θ_1 means more likelihood customers will balk regardless of the value of L_q. For example, if one queuing system has L_q equal to 10 and θ_1 equal to 0.008, then, the balking probability is 0.08, that is, if this queuing system has expected ten customers stay in the queue will lead to eight balkers for every 100 customers. Moreover, if another queuing system has the same value of L_q but θ_1 equal to 0.012, then, the balking probability is 0.12, that is, if this queuing system has expected ten customers stay in the queue will lead to 12 balkers for every 100 customers.

Reneging is the act of leaving a queue after having spent some time in waiting (Zhou and Soman, 2003). Therefore, it is reasonable to assume that the probability of reneging is equal to $W_q \times \theta_2$, where W_q is the expected waiting time in queue (excludes service time), and θ_2 is the reneging index. θ_2 is the average willingness of customers for reneging behavior of some queuing system and the larger θ_2 means more likelihood customers will renege regardless of the value of W_q. For example, if one queuing system has W_q equal to 10 minutes and θ_2 equal to 0.005, then the reneging probability is 0.05, that is, if this queuing system has ten minutes expected waiting time in queue, then it will lead to five reneging customers for every 100 customers. Further, if another queuing system has the same value of W_q but θ_2 equal to 0.01, then the reneging probability is 0.1, that is, if this queuing system has ten minutes expected waiting time in queue, then it will lead to 10 reneging customers for every 100 customers. However, the values of θ_1 and θ_2 may vary with different fast food restaurants because they may have different competitive environments and competitive advantages. Competition is a rough-and-tumble process in which only the most efficient and effective restaurants win out. (Hill and Jones, 2007)

Balking probability plus reneging probability, $L_q \times \theta_1 + W_q \times \theta_2$, must be equal to or less than one. The number of balking customers can be calculated by $B(L_q, \theta_1, \lambda) = L_q \times \theta_1 \times \lambda$ and the number of reneging customers can be determined by $R(W_q, \theta_2, \lambda) = W_q \times \theta_2 \times \lambda$, where λ is the mean arrival rate in that planning period. If the mean purchase amount of customers, A, and the mean net profit rate of the fast food restaurant, R, are known, then the balking loss and the reneging loss can be formulated respectively as

$$\text{balking loss} = B(L_q, \theta_1, \lambda) \times A \times R = L_q \times \theta_1 \times \lambda \times A \times R \tag{1}$$

$$\text{reneging loss} = R(W_q, \theta_2, \lambda) \times A \times R = W_q \times \theta_2 \times \lambda \times A \times R \tag{2}$$

In practice, it is not too hard to estimate the mean purchase amount and the mean net profit rate, but managers may have trouble to estimate the balking index and the reneging index. However, we can collect data by directly observing and then find these two indexes. Based on Table 1, most of the balking and reneging customers are apparently shown in the time periods of 11−12, 12−13, 13−14, and 19−20. Hence, managers should focus on these four periods and find the balking index and the reneging index. For instance, in planning period 11−12 in Table 2, with the given $B(L_q, \theta_1, \lambda)$=21, L_q =15.1, λ= 220, the balking index θ_1

can be determined by $B(L_q, \theta_1, \lambda)= L_q \times \theta_1 \times \lambda$, that is, $\theta_1 = 21/(15.1\times220) = 0.0063$. By the same approach, the rest of the balking index θ_1 can be found, and the average θ_1 for these four time periods is 0.0081. Similarly, according to $R(W_q, \theta_2, \lambda) = W_q \times \theta_2 \times \lambda$, four different time periods of the reneging indexes and the corresponding average θ_2 are calculated and shown in Table 2. After those estimated parameters, θ_1, θ_2, λ, A, R, μ, and K are determined, the optimal number of servers can be approached (where μ is the mean service rate per server in that planning period, and K is the cost of hiring one server for one planning period.)

Table 2
The estimation of θ_1 and θ_2

Time period	Number of customers	Number of balking customers	L_q	θ_1	Number of reneging customers	W_q	θ_2
11-12	220	21	15.1	0.0063	20	4.15	0.0219
12-13	358	37	26.1	0.0040	35	4.32	0.0226
13-14	121	15	8.3	0.0149	16	4.14	0.0319
19-20	147	11	10.2	0.0073	7	4.18	0.0114
Average				0.0081			0.0220

3. SIPP M/M/S: (FCFS/∞/∞) model

The (M/M/S):(FCFS/∞/∞) queuing model assumes that all interarrival times are independently and identically distributed according to an exponential distribution, that all service times are independent and identically distributed according to another exponential distribution, that the number of servers is S (any positive integer), that all customers are first come first served, that there is no limit of customers on the entire system, and that the size of the calling source is infinite. (Hillier and Lieberman, 2000) The notations and equations are described as:

λ: mean arrival rate in that planning period,

μ: mean service rate per server in that planning period,

S: number of servers in that planning period,

L_q: expected queue length (excludes customers being served),

W_q: expected waiting time in queue (excludes service time),

P_n: probability of exactly n customers in queuing system,

$$P_0 = 1 / \left[\sum_{n=0}^{S-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^S}{S!} \frac{1}{1-\lambda/(S\mu)} \right], \tag{3}$$

$$L_q = \frac{P_0 (\lambda/\mu)^S \rho}{S!(1-\rho)^2}, \ where \quad \rho = \frac{\lambda}{S\mu}, \quad and \tag{4}$$

$$W_q = \frac{L_q}{\lambda}. \tag{5}$$

This research assumes the queuing systems of the fast food restaurants are SIPP M/M/S: (FCFS/∞/∞) models. Therefore, we can divide the workday into "planning periods" such as hours, and then a series of stationary queuing models is constructed, one model for each planning period. Each of these models is independently solved for the minimum number of servers needed to meet the minimum total cost which is equal to service cost plus balking loss and reneging loss. $TC(S)$ is the total cost for using S servers in the planning period, therefore the optimization problem of each planning period can be stated as

$$\text{Min TC}(S) = K \times S + [B(L_q, \theta_1, \lambda) + R(W_q, \theta_2, \lambda)] \times A \times R, \tag{6}$$

and if

$$\text{TC}(S\text{-}1) \geqq \text{TC}(S), \tag{7}$$

and

$$\text{TC}(S+1) \geqq \text{TC}(S), \tag{8}$$

hiring S servers is the optimal staffing policy for this planning period.

According to equation (7),

$K(S\text{-}1) + [L_{q(S\text{-}1)} \times \theta_1 + W_{q(S\text{-}1)} \times \theta_2] \times \lambda \times A \times R \geqq K \times S + [L_{q(S)} \times \theta_1 + W_{q(S)} \times \theta_2] \times \lambda \times A \times R$,

thus

$$(L_{q(S\text{-}1)} - L_{q(S)}) \times \theta_1 + (W_{q(S\text{-}1)} - W_{q(S)}) \times \theta_2 \geqq K/(\lambda \times A \times R). \tag{9}$$

According to equation (8)

$K(S+1) + (L_{q(S+1)} \times \theta_1 + W_{q(S+1)} \times \theta_2) \times \lambda \times A \times R \geqq K \times S + (L_{q(S)} \times \theta_1 + W_{q(S)} \times \theta_2) \times \lambda \times A \times R$, thus

$$K/(\lambda \times A \times R) \geqq (L_{q(S)} - L_{q(S+1)}) \times \theta_1 + (W_{q(S)} - W_{q(S+1)}) \times \theta_2. \tag{10}$$

Therefore, combining (9) and (10), we have

$$(L_{q(S)} - L_{q(S+1)})\, \theta_1 + (W_{q(S)} - W_{q(S+1)})\, \theta_2 \leqq K/(\lambda \times A \times R) \leqq (L_{q(S-1)} - L_{q(S)})\, \theta_1 + (W_{q(S-1)} - W_{q(S)})\, \theta_2. \tag{11}$$

By taking advantage of the equations, (3), (4), (5), and (11), each planning period can be independently solved for the optimal number of servers. Hence, fast food restaurants can achieve minimum total cost. In this research, Mathematica 5.1 software is used to approach the optimal number of servers.

4. Approaching the optimal number of servers

From Table 1, the number of customers between 7pm and 8pm is 147, $\lambda = 147$, and the mean service rate per server for this fast food restaurant is estimated to be 40 customers/hour, $\mu = 40$, the cost per server per hour is 90 dollars (through out this paper, all the dollars is New Taiwan Dollars, NTD), $K = 90$, from Table 2, the balking index is estimated to be 0.0081, $\theta_1 = 0.0081$ (that is, if $L_q = 1$, the probability of balking = 0.0081, $L_q = 2$, the probability of balking = 2×0.0081 = 0.0162, and so on), the mean purchase amount of customers is estimated to be 100 dollars, A=100, and the mean net profit rate of this restaurant is estimated to be 0.5, $R = 0.5$. From Table 3, if a manager hires four servers for this planning period, according to equations (3) and (4), $L_q = 9.34$, that is, the balking probability of those 147 customers is equal to $L_q \times \theta_1 = 9.34 \times 0.0081 = 0.0757 = 7.57\%$, this means about 7.57% of those 147 customers will balk, therefore the number of balking customers, $B(L_q, \theta_1, \lambda) = L_q \times \theta_1 \times \lambda$, is equal to $0.0757 \times 147 = 11.13$ which indicates about 11 customers will balk. Therefore, balking loss equals to $B(L_q, \theta_1, \lambda) \times A \times R = 11.13 \times 100 \times 0.5 = 557$. Moreover, from Table 2, the reneging index is estimated to be 0.022, $\theta_2 = 0.022$ (that is, if $W_q = 1$, the probability of reneging = 0.022, $W_q = 2$, the probability of reneging = 2×0.022 = 0.044, and so on). From Table 3, if a manager hires four servers for this planning period, according to equation (5), W_q = 3.81minutes, that is, the reneging probability of those 147 customers is equal to $W_q \times \theta_2 = 3.81 \times 0.022 = 0.0838$ = 8.38%, this means about 8.38% of those 147 customers will renege, therefore the number of reneging customers, $R(W_q, \theta_2, \lambda) = W_q \times \theta_2 \times \lambda$, is equal to $0.0838 \times 147 = 12.32$ which indicates about 12 customers will renege. Therefore, reneging loss equals to $R(W_q, \theta_2, \lambda) \times A \times R = 12.32 \times 100 \times 0.5 = 616$, service cost = $K \times S$ = $90 \times 4 = 360$, and the total cost = service cost + balking loss + reneging loss = 360 + 557 + 616 = 1533.

However, if one more server is added to this service system, that is $S = 5$, from Table 3, L_q is reduced dramatically to 1.21, and the balking probability is also reduced dramatically to 0.98%, since the balking probability = $L_q \times \theta_1 = 1.21 \times 0.0081 = 0.0098$. This means only 0.98% of those 147 customers will balk, which the number of balking customers is 1.44 rather than 11.13. Moreover, from Table 3, W_q is also reduced dramatically to 0.49, and the reneging probability is also reduced dramatically to 1.08%, since the reneging probability = $W_q \times \theta_2 = 0.49 \times 0.022 = 0.0108$. This means only 1.08% of those 147 customers will renege, which the number of reneging customers is 1.58 rather than 12.32. Then, the balking loss, the reneging loss, and the service cost of 5 servers are 72, 79, and 450 respectively. Consequently, the total cost = service cost + balking loss + reneging loss = 450 + 72 + 79 = 601. It is obviously that using 5 servers instead of using four servers only costs 90 dollars more in service cost, but it costs 1022 dollars much less in balking loss and reneging loss.

Furthermore, if six servers are used in the service system, from Table 3, L_q is reduced to 0.33, and the balking probability is also reduced to 0.27%. Therefore, the number of balking customers is reduced to 0.40. Moreover, from Table 3, W_q is reduced to 0.14, and the reneging probability is also reduced to 0.31%. Therefore, the number of reneging customers is reduced to 0.46. However, compare to the case of using five servers, the service cost increases 90 dollars, but the balking loss plus reneging loss decreases 108 dollars. Therefore, the total cost for using six servers, 583, is still less than the total cost for using five servers, 601. It is obviously that, when S is switched to 7, 8, and 9 one by one, the increases of the service cost become significant because the loss in balking and reneging is very low. Consequently, the total cost is rising.

Table 3
The approach of optimal number of servers in the time period of 19-20

S	K	L_q	θ_1	W_q	θ_2	λ	$B(L_q, \theta_1, \lambda)$	$R(W_q, \theta_2, \lambda)$	A	R	service cost	balking loss	reneging loss	total cost
4	90	9.34	0.0081	3.81	0.0220	147	11.13	12.32	100	0.5	360	557	616	1533
5	90	1.21	0.0081	0.49	0.0220	147	1.44	1.58	100	0.5	450	72	79	601
6	**90**	**0.33**	**0.0081**	**0.14**	**0.0220**	**147**	**0.40**	**0.46**	**100**	**0.5**	**540**	**20**	**23**	**583**
7	90	0.10	0.0081	0.04	0.0220	147	0.12	0.13	100	0.5	630	6	6	642
8	90	0.03	0.0081	0.01	0.0220	147	0.04	0.03	100	0.5	720	2	2	724
9	90	0.01	0.0081	0.00	0.0220	147	0.01	0.00	100	0.5	810	1	0	811

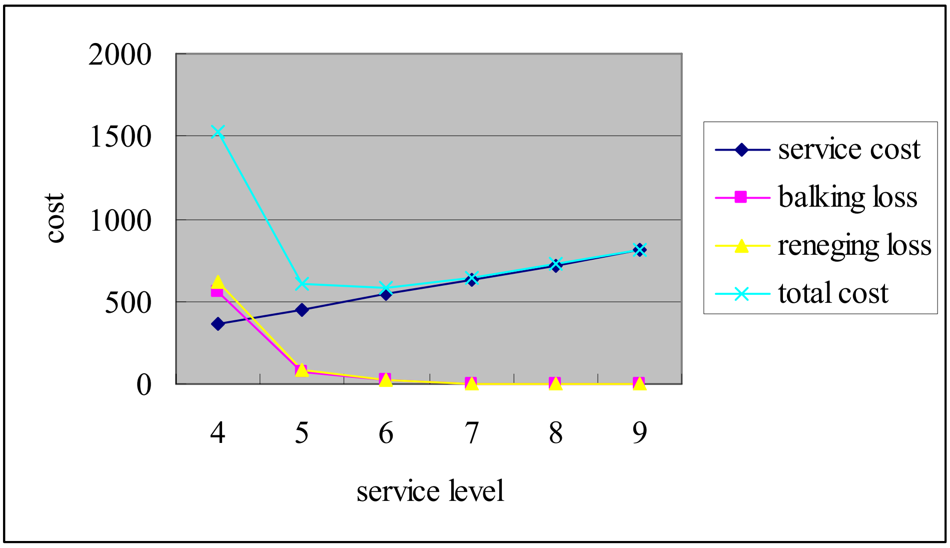

Fig. 1. Cost analysis

As a result, from Table 3 and Fig. 1, the service cost is increasing from S = 4 to S = 9, but the balking loss and reneging loss are decreasing. The total cost is decreasing from S = 4 to S = 6 and increasing from S = 6 to 9, thus S = 6 gives the optimal number of servers of this planning period. Therefore, using 6 servers instead of using 3 servers in the period 19-20 of Table 1, the restaurant can save [3 × 90 + (11 + 7) × 100 × 0.5] – 583 = 587 dollars.

5. Conclusions

This research is focus on formulating balking loss and reneging loss as waiting cost, and on using this concept to find the optimal number of servers of each planning period for queuing systems with varying demands by minimizing the total cost which includes the service cost, the balking loss, and the reneging loss. One example data was used to test the practicality and the benefit of using this optimal staffing policy, and the result shows that this approach is not only innovative but effective tool to aid managers to determine the optimal number of servers for each planning period of the time-varying demands queuing systems. By using this approach, restaurants can benefit from minimizing the total cost to increase the return on invested capital, enhancing the loyalty of customers to reduce the customer defection rate. The benefit of customers is that they do not need to stay a long time in the waiting line. Moreover, utilizing this optimal staffing policy, firms can build their competitive advantages in such a dramatically competing industry.

The lost profit from lost business is quite difficult to estimate. Therefore, the most significant contribution of this work is that the balking probability is defined as the balking index θ_1 multiplying by the expected queue length L_q and the reneging probability is defined as the reneging index θ_2 multiplying by the expected waiting time in queue W_q. With those balking probability and reneging probability, the balking loss and reneging loss can be estimated, and decision makers can minimize the total cost to determine the optimal staffing policy.

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