Employees’ Break-offs and the Birth of Industrial Clusters

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Empirical observation suggests that several industrial clusters originate from employees who break off and locate their new firms close to former employers. The reasons for such a choice are complex and include a variety of costs’ considerations. We present a two-player three-stage simultaneous game with interdependent decisions concerning break-offs, deterrent compensations, location, and profit maximizing production outputs. The structure of the game explains under what conditions a break-off is desirable, what location’s choice makes it optimal, and why the break-off process may lead to the birth of a cluster. We demonstrate how changes in a firm’s marginal production/congestion cost, the level of R&D investment in a region, and market size, all influence the likelihood of break-offs and their subsequent location decisions. Our results provide a rationale for why, in industries in which technology plays a significant role, an increase in R&D investment in the region may encourage the break-off firm to locate away from the incumbent. We also show that subsidies aimed at increasing manufacturing activities and at diffusing commercializable innovations can be useless in promoting business clustering, that they can be unnecessary for larger markets, and that their exact size is crucial in determining their effectiveness.

I. INTRODUCTION

Empirical evidence shows that many clusters originate from entrepreneurial break-offs (Zander 2004). In other words, from employees who separate from an incumbent firm to start their own businesses, and choose to locate them close to their former employers. The history of some of the world’s most renowned clusters points clearly in this direction. The establishment of

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Calzaturificio Voltan in 1898 gave rise to the footwear cluster in the Brenta’s region of Italy, as numerous workers set up their independent businesses nearby at the beginning of the 20th century (Boschma 1999, Bellandi 1989). Similarly, more than fifty firms can be traced back to Hybritech, the first biotech firm in San Diego, as former employees broke off and located their new firms in that area (DeVol et al. 2004). More recently, Toyota Motor Co. created strong connections with specialized car-parts suppliers established by former employees, leading to the development of Toyota city in Aichi-Hyun in the central part of Japan (Fruin and Nishiguchi 1993, Dyer and Nobeoka 2000). Finally, a knowledge-intensive software cluster has emerged in Dublin, Ireland, also as a result of break-off activities (O’Gorman and Kautonen 2004).

In recent years, several clusters in emerging economies have also resulted from a similar dynamics of break-offs and location decisions. Examples include the Bangalore software cluster in India (Lateef 1997), the surgical instrument cluster in Sialkot, Pakistan (Nadvi and Halder 2005), the Mexican furniture cluster in Chipilo, Puebla, and the software clusters in Mexico City, Monterrey, Guadalajara, and Aguascalientes, in Mexico (Pietrobelli and Rabellotti 2004). These examples provide evidence that new clusters may result from an intense process of break-offs by former employees who provide products related to those of their previous employers and compete in the same or connected markets. Two main cases can be identified.

In the first case, clustering break-off firms offer a variety of differentiated products which are imperfect substitutes for the products of former employers and compete in the same market. This is often the case in high-tech clusters where R&D investment is key, and where clusters tend to be driven by universities and research institutes (due to their intellectual and human capital endowments, Rothaermel and Ku 2008). In addition to Silicon Valley and the Boston Bio-tech belt, examples include the ICT service cluster in Zhongguancun, Beijing in China.
(Zhou and Xin 2003) and the Daejeon high-technology cluster in Korea (Sung et al. 2003). In the second case, break-off firms in a cluster offer a variety of differentiated products which are *imperfect complements* for the products of former employers. Examples include the ship-building cluster in Norway (Karlsen 2005), the metal-working cluster in the State of Espírito Santo, Brazil (Pietrobelli and Rabellotti 2004), and the automotive cluster in Ulsan, Korea, and in Setúbal, Portugal (Porter 1998a). In the last two examples, Hyundai Motor Co. and Volkswagen, respectively, play the role of hub for a network of car-part suppliers all located in geographical proximity.

The breath and quantity of examples confirms that the birth of a cluster can be often traced back to entrepreneurial break-offs and their locations’ choices. Yet, scholars are still unclear about what makes the owners of break-off firms locate close to former employers. While some research has addressed *spin-offs*’ location decisions, in which the incumbent firm makes the location choice, the literature has not yet explained the initial stages of clustering resulting from *break-offs* decisions, in which the break-off firm makes the location choice. Leyden et al. (2008), for example, have looked at a firm’s decision to locate on a university research park, but have not addressed the birth of the cluster. Furthermore, although the location choice literature has focused on overall benefits and costs of concentration under the assumption that a cluster already exists, it has not yet addressed the issue of agglomeration when no spillovers from pre-existing clustering exist.

The causes leading to the birth of an industrial cluster are, however, important because they have a significant impact on the strategic behavior of new and incumbent firms and, as a result, on the efficiency and structure of the markets involved. Moreover, the reasons causing agglomeration at the inception of a cluster, when economies of scale and scope have not yet set
in, are often different from those characterizing its later expansion. We thus fill current gaps in the literature by investigating when a prospective entrepreneur should break off from her current employer and, if breaking off, how far from that employer she should locate her new firm in the absence of an existing cluster.

Our paper complements and extends existing literature in two important ways. First, we provide an explanation for the location decisions of entrepreneurial break-offs. Second, consistently with empirical evidence, we show that, in many situations, the location decisions of entrepreneurial break-offs explain the initial stage (birth) of clusters. Specifically, our model allows us to identify the optimal tradeoff between the increase in congestion costs from clustering and the decrease in variable production costs from spillover benefits faced by the break-off firm, and to show how these costs and benefit depend on whether the break-off firm produces imperfect complements or substitutes for the incumbent’s product.

Understanding the break-off process and the resulting location decisions does not only have important implications for entrepreneurship and technology management research, but also for practice and policy. The clusters listed earlier illustrates the important role played by technology, and thus by the inventors, scientists and engineers who develop them in the formation of industrial parks. Promoting the formation of industrial clusters is one of the most common goals of entrepreneurial policy and one of the most difficult to realize. This is due, in part, to our incomplete understanding of the factors leading to clustering and to the mistaken assumption that the formation of industrial clusters is always desirable. Our framework provides significant new insight on the process leading to agglomeration and, in particular, on the conditions that make it desirable.
We develop our argument using a game-theory model describing the strategic moves of incumbent and break-off firms. The entrepreneur chooses whether to break-off and, if so, where to locate her firm. The incumbent firm, instead, chooses whether to accommodate entry or to deter it by compensating the prospective entrepreneur. In our framework, the strategic choices of both players are also shown to be dependent on the type of output produced by the break-off firm (i.e. imperfect substitute or imperfect complement) thereby rendering our results applicable to a broad range of situations.

II. THEORETICAL BACKGROUND

Cluster formation refers to a process characterized by the progressive agglomeration of a number of firms in a specific geographic location. Silicon Valley in California and Route 128 in Massachusetts are well-known examples of clustering in the United States. Although numerous firms have populated these regions for a long time, the tendency for firms to concentrate in those areas still continues because of spillover benefits that increase their profitability (Cooper and Fotla 2000). The location choice literature has focused on overall benefits and costs of concentration under the assumption that a cluster has already been created, and has analyzed how firms’ location decisions further increase a cluster’s size (Porter 1998a, Folta et al. 2006, Krugman 1998). This literature, however, falls short from articulating how clusters are born since the conditions producing the birth of a cluster and those producing increasing degrees of agglomeration may be different. While the latter rely on a variety of established spillovers, the existence and intensity of such spillovers is significantly less certain at the cluster’s birth (Feldman 2001).

In general, economies of scale and scope and the resulting reduction in transaction costs have been identified as the main reasons for the existence of industrial clusters (Fujita et al. 1999,
Fujita and Thisse 2002). In particular, the literature on agglomeration looks at clustering emerging from an initially uniformly distributed population as the result of instabilities between agglomerative and degglomerative forces (Fujita et al. 1999). The location choice literature focuses on three sources of agglomerations which were identified already by Marshall (1920). Namely, labor pooling, specialized inputs, and knowledge spillovers. Other benefits, such as access to capital, proximity to customers, and psychological supports have been suggested more recently (Cooper and Fotla 2000). For example, Papageorgiou and Smith (1983) explain local instability by building a model in which agglomeration results from the relative size of congestion costs, which discourage clustering, and positive locational economies, which encourage clustering. In these models, movements toward and away from clustering are associated with the existence of non-linearities and critical threshold levels.

Within this context, two streams of research can be identified. The first research stream focuses on clusters fostered by endogenous interactions between location of demand and distribution of manufacturing firms (e.g., Krugman 1998), the five interrelated elements of the diamond model (e.g., Porter 1998b), strong social ties among firms (Saxenian 1994), the interdependence of multinational enterprises in a region (Enright 2000), and knowledge exchanges and institutions built by large firms (Bellandi 2001). The second research stream, instead, focuses on cluster development by entrepreneurial firms (Romanelli and Schoonhoven 2001). Often, new business ideas emerge when perspective entrepreneurs are employed, and cluster emerge when these new entrepreneurs choose to locate close to their former employers (Zander 2004). Audretsch and Feldman (1996), among others, suggest that R&D spillovers and proximity between entrepreneurial firms and universities promote the clustering of innovative activities in high-tech industries. Others, instead, have investigated the relationship between the
location of entrepreneurial firms and their performance (Fotopoulos and Louri 2000). Folta et al. (2006), for example, have shown that cluster size is positively related to technology firms’ performance, but only when that size is small. Minniti (2005) has shown that entrepreneurs tend to concentrate geographically as a result of a favorable social environment which provides role models and social cues.

Both streams of research share a focus on later stages of the agglomeration process, assuming that clustering has already emerged. As discussed earlier, proximity is indeed an important source of competitive advantage because an established geographic neighbor is likely to provide the entrepreneur with initial legitimacy and networking opportunities. Notably, spillovers of tacit knowledge remain localized even in this Internet age, as they are mostly available through face-to-face interactions (Leamer and Storper 2001). This phenomenon is salient especially in high-technology industries where knowledge plays a key role (Rothaermel and Ku 2008). In addition to agglomeration effects, however, we argue that the formation of entrepreneurial break-offs contributes significantly to clustering.

Traditionally, the formation of new firms has been explained looking at entrepreneurs’ occupational choices, and comparing the expected payoffs from staying with a current employer to those of breaking off and starting a new firm. While the occupational choice literature has explored factors that encourage entrepreneurs to create their own firms (Grossman 1984, Jovanovic 1994), it has not linked that decision to the location choice literature. Market conditions (Fotopoulos and Louri 2000, Reynolds et al. 1994, Choi and Shepherd 2004), the perception of more advanced technologies (Choi and Shepherd 2004), and public intervention (Tambunan 2005), all have been identified as having a positive influence on the rate of firm formation. When breaking off is selected, however, entrepreneurs are also confronted with the
decision of where to locate the new firms, and the relationship between the new firm and the incumbent is important. In some cases, this relationship contributes to the formation of clusters.

In industrial organization studies, the analysis of firms’ interaction has focused primarily on collusive behaviors and on how an incumbent can deter entry by threatening a price war (Spence 1977, Dixit 1980), and on the effects of limit pricing on a new firm’s entry decision (Milgrom and Roberts 1982). In our model, both incumbent and new firm make strategic decisions which are influenced by differences in their cost structures. Existing studies in this area have assumed either a symmetric (Dixit 1980) or an asymmetric (Milgrom and Roberts 1982) cost structure between incumbent and entrant. In our model we adopt an asymmetric cost structure because, due to liability of newness and smallness, the prospective entrepreneur faces higher marginal production costs than the incumbent (Aldrich and Fiol 1994). We complement and extend existing studies by modeling the incumbent’s reaction to deter (or not) entry through the payment of a strategic compensation. Further, in our model, the break-off decision is not made by the incumbent, thus allowing us to differentiate ourselves from the spin-off literature.

To summarize, our paper contributes to the literature by proposing a model that can identify the conditions under which a prospective profit-maximizing entrepreneur should break off from an incumbent firm, and what might encourage the resulting new firm to locate in close proximity to the incumbent thereby causing the potential birth of a cluster.

III. DECISION FRAMEWORK

Let us consider an incumbent, Firm I, producing product $x$ and employing individual $e$. Let us also assume that $e$ is considering breaking off from Firm I to start her own business, Firm E, that would produce $x'$, an imperfect substitute for $x$. Given this opportunity/threat of entry, what decisions would Firm I and the potential Firm E have to make? What is the sequence and
logic of these decisions? And how would they affect Firm I’s and Firm E’s profits? Since the decisions for Firm I and Firm E are interdependent, this situation can be effectively described by imagining them to be players in a two-player three-stage simultaneous game.

The first stage concerns the break-off decision and includes two strategies for each player. Employee $e$ has two choices: she may continue working for Firm I and receive a wage, or she may break-off and start Firm E. Firm I, on the other hand, knowing that $e$ may continue working or break-off, must decide whether to allow $e$ to break-off and start Firm E, or prevent entry by increasing $e$’s compensation and retain her as an employee. $e$ evaluates her new firm’s profit potential and compares it with the maximum amount Firm I may be willing to offer. Firm I considers the damage imposed by having a new competitor in the market and compares it to how much $e$ should be compensated to prevent the break-off. Each player knows the strategies and possible payoffs of the other player. However, neither observes what the other decides before making a move and their choices are made concurrently.

If $e$ chooses to continue working for Firm I nothing happens and the game ends. If, on the other hand, $e$ chooses to break-off and starts Firm E, a location decision must take place. The location decision constitutes the second stage of the game. At this stage, Firm E is the only player facing a strategic choice since Firm I, being the incumbent, has already selected its location. $e$ knows Firm I’s location and, given the decision to break-off, must now choose where to locate Firm E. The location decision is, of course, crucial since location will have a direct effect on Firm E’s production costs and may make the difference between success and failure. Locating close to Firm I may provide a competitive advantage for Firm E since proximity may offer initial legitimacy, networking opportunities, and positive knowledge spillovers. On the other hand, proximity increases congestion costs and, as a result, has a negative effect on overall
firm performance. Ceteris paribus, Firm E’s location decision hinges on identifying the optimal tradeoff between congestion costs and spillover benefits that maximizes its expected profits. Noticeably, the cost structure and resulting competitiveness of Firm E can be significantly altered by the presence of public subsidies.

Assuming a break-off does take place, once the location decision is made, the presence of Firm E alters the competitive landscape, and Firm I and Firm E have to make production decisions taking into account each other’s presence. This is the third and last stage of the game and the two players make production decisions simultaneously without being able (or willing) to wait in order to observe what the other has decided to do.

At this point, one may wonder what could differ if Firm E’s output were to consist of something other than an imperfect substitute to product $x$. What if Firm E were to produce an imperfect complement? Would the game, strategic decisions, and relevant variables be any different? It turns out that the game remains the same even in those cases albeit with some quantitative calibration. Let us assume, for example, that while Firm I produces $x$, Firm E were to produce, $\hat{y}$, an input to $x$, and an imperfect complement for $y$, the version of that input developed in-house by Firm I. All three stages of the game would remain unchanged, as would the strategies leading to optimal decisions, since the process would still involve two competing players making interdependent decisions concerning break-offs, deterrent compensations, location, and profit maximizing outputs. Regardless of whether Firm E produces an imperfect substitute or complement, if the break-off takes place, the second stage of the game calls for choosing the location for the new entrant, Firm E. If the latter locates close to Firm I, we have the conditions for some agglomeration to take place.
This description illustrates how the break-off process may lead to the emergence of a cluster. The main question then becomes: Under what conditions will Firm E locate close enough to Firm I so as to start a possible cluster? As described earlier, the location of Firm E will depend on the precise tradeoff between the spillover benefits of proximity and the congestion costs that maximize Firm E’s expected profit. But how is that location identified exactly? And how would a change in any of the variables influencing Firm E’s profit affect that location’s choice? Intuitive reasoning cannot answer these questions. However, a formal version of the simple game we described can. The next section translates this intuitive narrative into a formal two-player three-stage simultaneous game.

IV. THE GAME

A. Demand and Product Types

Firm I is a monopolist. The entrepreneur decides whether to break off from the incumbent (Firm I) and enter the market by creating a new firm (Firm E). If entry takes place, the market becomes a duopoly. As more possible break-offs are considered, market structure becomes more competitive. Firm I and Firm E are both profit maximizers and the decision to break off depends on how tough competition with the incumbent is expected to be. The incumbent can deter that break-off by paying a non-negative compensation \( B \) to the prospective entrepreneur. Since Firm I is unwilling to experience negative profits, the compensation \( B \) is the maximum between 0 and Firm I’s optimal profit under monopoly minus its optimal profit under duopoly. \( B \) is thus endogenously determined. Endogenizing \( B \) is important because it allows our model to characterize what the exact size of the compensation should be in order to deter a break-off.
Both players make their respective decision without observing the other player’s actual choice. Thus, no player possesses an information advantage. If choosing to break-off, the entrepreneur has to select how far from the incumbent the resulting new firm should be located. The location decision depends on costs considerations and, in particular, on the tradeoff between spillover benefits from proximity and congestion costs. Once the location is selected, each firm chooses a production quantity \( q_i \), \( i \in \{I, E\} \), with profit \( \Pi_i(q_i, q_j) \), \( i, j \in \{I, E\} \) and \( i \neq j \). Firm I and Firm E face a demand \( q_i = M - 2p_i + \alpha p_j \), where \( i \neq j \), \( i, j \in \{I, E\} \) and \( \alpha \) equals \(-1\) for imperfect complements but \( 1 \) for imperfect substitutes. The coefficients of \( p_I \) and \( p_E \) in these demand functions represent the degrees of differentiation (i.e. degrees of complementarity and substitutability) between products.\(^2\)

B. Costs and Profits Functions

Let \( b \) represent the distance between Firm E and Firm I, which, without loss of generality, is normalized not to exceed 1. We assume the market size \( M \) to be independent from the geographic market space, that is, the market coverage of the firms in our framework can be extended to nations and even global markets. This flexibility gives our model broad applicability.\(^3\)

Firms face a market entry cost and a variable production cost. Since the important point is the cost difference between Firm I and Firm E, rather than their absolute cost levels, for

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\(^2\) The selection of values for these coefficients is done without loss of generality since our argument holds regardless of the degree of differentiation between products.

\(^3\) Because no restrictions on market size or on consumers’ geographical distribution are imposed, our model differs from Hotelling’s (1929) location model. Hotelling’s model assumes consumers of a local market to be evenly distributed on a linear space. This implies that a firm’s location choice is driven by demand-side considerations since each firm’s access to customers is different. In turn, this leads to a different level of maximized profit for different locations. Our model, instead, allows us to focus on the birth of a cluster as the result of supply-side variables abstracting from the relatively trivial case in which clusters stem from monopsony or the concentration of consumers’ demand in a specific location.
simplicity, and without loss of generality, Firm I is assumed to face 0 entry cost, since this cost has already been sunk, and 0 marginal production cost. Firm E, on the other hand, faces total production costs equal to \( C(q_E) = F + \left[ cq_E b / R \right] \), where \( F \) is a market entry cost, and \( cq_E b / R \) is a variable production cost, in which \( c \) is the marginal production cost given Firm E’s current technology, \( b \) represents the distance between the two firms, and \( R (> 0) \) accounts for the influence of R&D investment implemented inside the region (particularly important for the formation of industrial clusters characterized by high levels of technology). As the two firms get closer to each other (i.e., \( b \) decreases), spillover benefits decrease Firm E’s variable production cost (Fujita and Thisse 2002). Similarly, as R&D expenditure \( R \) increases, that cost further decreases. In fact, where R&D expenditure is significant, regional effects can benefit break-off firms since knowledge spillovers are spatially mediated in the form of intellectual capital and influenced by the increase in local R&D investment due to their stickiness (Rothaermel and Ku 2008, Audretsch and Feldman 1996).

We further note that, due to the liability of newness and smallness, Firm E is assumed to suffer a cost disadvantage with respect to Firm I (Aldrich and Fiol 1994). Noticeably, our model is consistent with a situation in which the prospective entrepreneur possesses lower marginal costs. This case, however, is somewhat trivial since the prospective entrepreneur would always have an incentive to break-off and exploit her cost advantage. By showing that Firm E has incentives to break-off even when Firm I enjoys a marginal cost advantage, we show that our results hold even in a “worst case scenario.”

Finally, when break-off is chosen, Firm I and Firm E face a congestion cost expressed by \( G(q_i) = gq_i / b \) where \( g \) is a constant marginal congestion cost given each firm’s current

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4 Firm I may possess a cost advantage over Firm E also as a result of economies of scale, a steep learning curve, or its ability to obtain raw materials at low-cost.
technologies (Folta et al. 2006). Congestion costs are negative spillovers from a new member (Leyden et al. 2008) and arise, for example, from increased costs of property, additional transportation costs, premium prices for material, labor, and utilities, as well as costs associated with the risk of knowledge leakages (Cooper and Fotla 2000, Folta et al. 2006, Galbraith and De Noble 1988, Shaver and Flyer 2000). Hence, locating close to an existing firm ($b$ decreases) spills over benefits by decreasing variable production cost, but it increases congestion cost as $g/b$ increases. Noticeably, $g/b^2$ (which plays a key role in the analysis) represents that cost per unit of production for each unit of distance (i.e., $g/b^2 = -\partial^2 G(q_i)/\partial b \partial q_i$). Each player’s objective is to maximize profit defined as revenues minus the sum of production and congestion costs, formally expressed as

$$\max_{q_i \geq 0} \Pi_i(q_i) = \max_{q_i \geq 0} \{ p_i q_i - C(q_i) - G(q_i) \}, \quad i \in \{I, E\}.$$  

We can now derive optimal decision rules about when the prospective entrepreneur should break off, and, if breaking off, how far from the incumbent the new firm should be located. This is the set of specific conditions that the verbal description in section III did not allow us to characterize.

V. ANALYSIS

A. Optimal Decision Rules

Following standard game theory, we use backward induction to determine the set of equilibrium decisions for the two players. Accordingly, we solve for the production decisions first, the location decision second, and last for the break-off and compensation decisions. In order to maximize profits, Firm I and Firm E select the optimal quantity each of them has to

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5 In other words, $g/b^2$ measures by how much the cost of producing one unit of output would change if Firm E were to move one mile, one yard, or any other “measure of distance” closer to (or further apart from) Firm I.
produce. Table I summarizes the optimal profits for Firm I and Firm E considering the case of imperfect substitutes and imperfect complements, respectively. Calculations leading to the optimal profits shown in Table I are presented in the Appendix.

### Table I
**Payoffs Are Players’ Optimal Profits** ($\Pi^*_E$, $\Pi^*_I$)†

(a) Firm I and Firm E produce imperfect substitutes

<table>
<thead>
<tr>
<th>Firm I</th>
<th>Deter Entry (Pay $B$)</th>
<th>Allow Entry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm E</td>
<td></td>
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</tr>
<tr>
<td>Break-off</td>
<td>$\Pi^*_E = \frac{6}{25} \left( M - \frac{g}{b} - \frac{4cb}{3R} \right)^2 - F$,</td>
<td>$\Pi^*_E = \frac{6}{25} \left( M - \frac{g}{b} - \frac{4cb}{3R} \right)^2 - F$,</td>
</tr>
<tr>
<td>$\Pi^*_I = \frac{6}{25} \left( M - \frac{g}{b} + \frac{ch}{3R} \right)^2$</td>
<td>$\Pi^*_I = \frac{6}{25} \left( M - \frac{g}{b} + \frac{ch}{3R} \right)^2$</td>
<td></td>
</tr>
<tr>
<td>Remain with Firm I</td>
<td>$\Pi^*_E = \frac{3}{200} \left{9M - \frac{4g}{b} + \frac{4cb}{3R} \right} \left{ M + \frac{4g}{b} - \frac{4cb}{3R} \right}$,</td>
<td>$\Pi^*_E = 0$,</td>
</tr>
<tr>
<td>$\Pi^*_I = \frac{3}{8} M^2 - B$</td>
<td>$\Pi^*_I = \frac{3}{8} M^2$</td>
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(b) Firm I and Firm E produce imperfect complements

<table>
<thead>
<tr>
<th>Firm I</th>
<th>Deter Entry (Pay $B$)</th>
<th>Allow Entry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Break-off</td>
<td>$\Pi^*_E = \frac{2}{3} \left( M - \frac{g}{b} - \frac{4cb}{5R} \right)^2 - F$,</td>
<td>$\Pi^*_E = \frac{2}{3} \left( M - \frac{g}{b} - \frac{4cb}{5R} \right)^2 - F$,</td>
</tr>
<tr>
<td>$\Pi^*_I = \frac{2}{3} \left( M - \frac{g}{b} + \frac{ch}{5R} \right)^2$</td>
<td>$\Pi^*_I = \frac{2}{3} \left( M - \frac{g}{b} + \frac{ch}{5R} \right)^2$</td>
<td></td>
</tr>
<tr>
<td>Remain with Firm I</td>
<td>$\Pi^*_E = B = \max \left{ 0, -\frac{1}{24} \left{ \frac{2}{3} M - \frac{4g}{b} - \frac{4cb}{5R}, \frac{M}{3} - \frac{4g}{b} - \frac{4cb}{5R} \right} \right}$,</td>
<td>$\Pi^*_E = 0$,</td>
</tr>
<tr>
<td>$\Pi^*_I = \frac{1}{24} M^2 - B$</td>
<td>$\Pi^*_I = \frac{1}{24} M^2$</td>
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</table>

† Noticeably, since the compensation $B$ is the difference between Firm I’s optimal profit under monopoly and its optimal profit under duopoly, $b$ (the distance between the two firms) appears in $B$. However, when $B$ is calculated in the monopoly case, $b=0$ in Firm I’s profit function since Firm E does not exist.

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6 Since we impose no restrictions on the size of the market, $M$, firms are price takers and compete on quantity produced (Cournot competition).
In the last stage of the game, realizing an expected positive profit is a necessary condition for entry to occur, that is \( \Pi_E^* > 0 \) is a participation constraint. Given the profit shown in Table I, and assuming the entrepreneur decides to break off, examining Firm E’s expected profit \( \Pi_E^* \) allows us to determine Firm E’s location selection. Specifically, everything else being equal, if the first-order derivative of \( \Pi_E^* \) with respect to \( b \) is negative, the optimal decision rule states that Firm E should come as close to Firm I’s as possible. Formally stated,

*Proposition 1*: Regardless of the type of output being produced, Firm E should locate closer to Firm I as long as the increased spillover benefits from clustering exceed the increased per-unit congestion cost. Specifically, in a cluster producing

(a) imperfect substitutes, if \( 4c/3R > g/b^2 \);

(b) imperfect complements, if \( 4c/5R > g/b^2 \).

In other words, the location decision depends on comparing the increase in spillover benefit due to clustering to the increase in marginal congestion cost due to clustering.\(^7\) When producing an imperfect substitute, for example, Firm E has an incentive to move closer to Firm I if, and only if, the per-unit congestion cost (for each unit of distance), \( g/b^2 \), is below the threshold, \( 4c/3R \). That threshold is proportional to an increase in the spillover benefits from clustering (since \( \partial^2 C(q_E)/\partial b\partial q_E = c/R \)). The same argument applies to the case of imperfect complements. The results from Proposition 1 are consistent with evidence suggesting that entrepreneurs tend to start their new firms within commuting distance of their previous workplace (Cooper and Fotla 2000).

For both types of clusters, when the increase in the per-unit congestion cost, \( g/b^2 \), is sufficiently low, Firm E locates close to Firm I and, as a result, we observe the birth of a

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\(^7\) Proofs of our propositions are provided in the Appendix.
potential cluster. Of course, what value of per-unit congestion cost is sufficiently low to produce clustering depends on Firm E’s type of output. Given the conditions provided by Proposition 1, ceteris paribus, a break-off is more likely to locate close to the incumbent when Firm I and Firm E produce imperfect substitutes than when they produce imperfect complements. The intuitive rationale for this result stems from cost-side considerations. Firm E selects where to locate with respect to Firm I in order to reduce its production cost thereby increasing its profits. However, this incentive is weaker when imperfect complements are produced since Firm I (or other end-customers) form the market for Firm E’s output. This result is intuitive and complements existing literature since benefits from spillovers are stronger for imperfect substitutes than imperfect complements.

Continuing with backward induction, we are now left with solving for the break-off and compensation decisions. In order to identify a sufficient condition for a break-off to take place, we compare Firm E’s expected profit $\Pi^*_E$ to the entrepreneur’s compensation $B$ from staying with Firm I. $B$ is the maximum amount of money that Firm I is willing to pay to deter a break-off in its monopoly market. If $\Pi^*_E$ is larger than $B$, then breaking off and starting Firm E is optimal for the entrepreneur; that is, the incentive compatibility constraint is satisfied. Satisfying both, the participation constraint associated with the production decision, and the incentive compatibility constraint associated with the break-off decision, guarantees a break-off. Formally stated,

**Proposition 2:** Given the entry cost $F$ is below a critical threshold, regardless of the type of output being produced, breaking off is optimal when a sufficiently large market size $M$ exists. The specific threshold levels for $F$ and $M$ depend on whether imperfect substitutes or complements are produced.
When Firm I and Firm E produce imperfect substitutes, and assuming a positive expected profit for Firm E (i.e., the participation constraint is satisfied), a prospective entrepreneur’s decision to break off will be based on how tough she expects Firm I’s competition will be after the break-off. Relatively low entry costs make entry less expensive while a sufficiently large market size improves the new firm’s expected profits. Noticeably, the critical thresholds on the market size, $M$, for imperfect complements is lower than the one for imperfect substitutes. In other words, it is easier for a break-off firm producing an imperfect complement to make a profit. The intuition behind this result is that when imperfect complements are produced, the incumbent has weaker incentives to discourage a break-off if the latter can provide a cost effective complement. In this case, a prospective entrepreneur should break off as long as market size is sufficient to support positive profits and the incumbent contributes to the size of this market rather than compete in it.

B. Accounting for Subsidies

In addition to its simplicity, one of the most desirable features of our model is flexibility. In fact, our framework can easily account for the effect of local subsidies, which are created by governments with the exact purpose of increasing firm formation and influencing location decisions. Subsidies are used frequently with the goal of creating jobs (Leyden et al. 2008), often through increased manufacturing activities (Holmes 1998), and for the diffusion of commercializable innovations and technologies (Leyden et al. 2008, Mathews 1997). They can take the form of grants for the purchase of equipment, low cost locations for factories, and common service facilities dedicated to newly formed businesses.

Although they may take a variety of forms, subsidies can be thought of as governments’ transfers that reduce the entry cost, $F$, faced by Firm E. In the presence of subsidies, when Firm I
and Firm E produce imperfect substitutes, a break-off should occur even with a small market size when $F$ is covered by subsidies exceeding a certain threshold. In fact, by exogenously reducing costs, a subsidy makes it possible for $\Pi_E^*$ to exceed $B$, thereby rendering a break-off attractive. Similarly, when Firm I and Firm E produce imperfect complements, an interval for the small market size $M$ can be identified so that a break-off should occur when the entry cost $F$ is covered by subsidies exceeding a critical threshold. Table II summarizes the optimal decision rules from Propositions 1 and 2 when also accounting for the presence of subsidies.  

<table>
<thead>
<tr>
<th>Stage</th>
<th>Decision</th>
<th>Subsidies</th>
<th>Imperfect substitutes or imperfect complements</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Break off</td>
<td>No</td>
<td>when $M \geq \bar{M}$ and $F &lt; \bar{F}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Yes</td>
<td>when $\underline{M} \leq M &lt; \bar{M}$ and $F$ is covered by subsidies</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2</th>
<th>Locate close to Firm I</th>
<th>Not applicable</th>
<th>Imperfect substitutes</th>
<th>Imperfect complements</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\frac{g}{b^2} &lt; \frac{4}{3} \frac{c}{R}$</td>
<td>$\frac{g}{b^2} &lt; \frac{4}{5} \frac{c}{R}$</td>
<td></td>
</tr>
</tbody>
</table>

Legend:
- $M$ market size
- $\underline{M}$ lower bound on market size
- $\bar{M}$ upper bound on market size
- $F$ entry cost
- $\bar{F}$ upper bound on the entry cost
- $g/b^2$ congestion cost per unit of distance

Mathematical expressions for these bounds depend on the type of output produced and are detailed in the Appendix.

Noticeably, in both cases of imperfect substitutes and complements, if the market is sufficiently small, subsidies become ineffective as the entrepreneur does not gain from participating in the industry.

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**TABLE II**

SUMMARY OF OPTIMAL DECISION RULES FOR FIRM E

8 Noticeably, in both cases of imperfect substitutes and complements, if the market is sufficiently small, subsidies become ineffective as the entrepreneur does not gain from participating in the industry.
C. Sensitivity Analysis

The optimal decision rules offered by Propositions 1 and 2 are, of course, affected by changes in key model parameters. In particular, changes in firm-specific advantages (marginal production cost $c$), the characteristics of a region (marginal congestion cost $g$), level of R&D investment ($R$) in a region, and market size ($M$), all influence the likelihood of break-offs and/or their subsequent location decisions.

A change in the marginal production cost $c$ affects the left-hand side of the conditions in Proposition 1. For example, ceteris paribus, an increase in $c$ yields a larger left-hand side for each and every condition, making the sufficient conditions for locating in proximity of the incumbent more likely to be satisfied. From Proposition 2, in either large markets without the support of subsidies or in small markets with the support of subsidies, the lower the marginal production cost for Firm E, the higher the likelihood of breaking-off, ceteris paribus. This is the case because, as shown in the proof of Proposition 2 in the Appendix, the sufficient conditions for breaking-off are weaker and, as a result, more likely to be satisfied. However, regardless of the type of output produced, the break-off firm will be more likely to locate closer to the incumbent if, and only if, ceteris paribus, there is an increase in the marginal production cost, $c$. Thus, the marginal production cost moderates positively the relationship between the likelihood of breaking-off and that of cluster birth. Indeed, regardless of the type of output, an increase in the marginal cost of production, $c$, is expected to encourage the break-off firm to locate in close proximity of the incumbent because spillover benefits reduce total variable costs. Shaver and Flyer (2000) and Wright et al. (2008) provide support for this finding by arguing that low-profile firms with less-advanced technology, lower growth, or a small number of patents are motivated to cluster.
An increase in the marginal congestion cost, \( g \), on the other hand, has just the opposite effect. From Proposition 1, an increase in \( g \) yields a larger right-hand side for each and every condition, making these conditions less likely to be satisfied. From Proposition 2, it can be shown that, in either large markets without the support of subsidies or small markets with the support of subsidies, the lower the marginal congestion cost, the higher the likelihood of breaking-off, ceteris paribus, because the break-off firm can anticipate more profit. Thus, congestion costs moderate negatively the relationship between the likelihood of breaking-off and that of cluster birth. In fact, Fotopoulos and Louri (2000) and Campi et al. (2004) find that congestion costs have a negative effect on the clustering of firms leading to relocation or death. Also, Folta et al. (2006) recently demonstrated that diseconomies of agglomeration take place when congestion costs increase due to an increase in cluster size.

In general, in either large markets without the support of subsidies or small markets with the support of subsidies, the higher the likelihood of breaking-off, the higher the likelihood of cluster birth, ceteris paribus. Of course, in the presence of subsidies a break-off is expected to be more likely for the two cluster types even when the market is small. In fact, and regardless of the type of output, the existence of subsidies increases the likelihood of break-offs, since subsidies correspond, de facto, to a decrease in fixed costs and may generate a positive profit. Tambunan (2005) provides support to this result noticing that public intervention, including the provision of subsidies for a common service facility and export training, contributed to the development of the wooden furniture cluster in Jepara, Indonesia, by helping the cluster to gradually develop export markets.

In large markets, or in small markets with the support of subsidies, the larger is R&D investment \( R \) in a region, the higher the likelihood of breaking off, ceteris paribus. In addition,
R&D investment negatively moderates the relationship between the likelihood of breaking-off and that of cluster birth. In fact, for industry in which technology plays a significant role, an increase in R&D investment in the region is expected to encourage the break-off firm to locate away from the incumbent because the entrant firm can benefit from spillovers from these investments by simply being in the region. This finding complements Zeller (2001), who argues that firm-specific relational assets (as opposed to region-specific R&D investment), such as the accumulation and transmission of tacit knowledge, might be key to fostering spatial concentration of biotech firms. This finding is also in line with Leyden et al. (2008), who argue that firm-level R&D investment is a major determinant of firms’ location selections on university research parks.

Finally, in addition to $c$, $g$ or $R$, the break-off decision is also likely to be affected by a change in the market size $M$, since the conditions in Proposition 2 depend on this parameter. The larger the size of the market, the higher the likelihood of break-offs, ceteris paribus. In the absence of subsidies, we find that a larger market size yields a higher likelihood of breaking off when imperfect substitutes or imperfect complements are produced because the new firm’s prospects are more favorable. This is consistent with Fotopoulos and Louri (2000) and Choi and Shepherd (2004) who argue, respectively, that the increase in population density and more knowledge of customer demand have a positive influence on firm formation. It is also consistent with Davidsson and Wiklund (2001) who argue that an increase in the demand from population and income growth at the regional level has a positive effect on the rate of firm formation. Noticeably, however, if the market size is sufficiently small, this positive relationship might not hold, even in the presence of subsidies. Table III provides a summary of these results.
TABLE III
SENSITIVITY ANALYSIS

<table>
<thead>
<tr>
<th>Decision</th>
<th>Subsidies</th>
<th>Market size bounds</th>
<th>Increases in</th>
<th>Type of output produced</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No</td>
<td></td>
<td>$M \geq \bar{M}$</td>
<td>$C$</td>
<td>less likely</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$G$</td>
<td>less likely</td>
</tr>
<tr>
<td>Break off</td>
<td></td>
<td></td>
<td>$R$</td>
<td>more likely</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$M$</td>
<td>more likely</td>
</tr>
<tr>
<td>Yes</td>
<td></td>
<td>$M \leq M &lt; \bar{M}$</td>
<td>$C$</td>
<td>less likely</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$G$</td>
<td>less likely</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$R$</td>
<td>more likely</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$M$</td>
<td>more or less likely</td>
</tr>
</tbody>
</table>

Legend:
- $M$: market size
- $\underline{M}$: lower bound on market size
- $\bar{M}$: upper bound on market size
- $c$: marginal production cost
- $g$: marginal congestion cost
- $R$: the effect of R&D investment in a region

Mathematical expressions for these bounds depend on the type of output produced and are detailed in the Appendix.

VI. CONCLUSION

A significant amount of research exists on industrial clusters and agglomeration economies. To date, however, not much attention has been paid to the way clusters may come into being. In this paper we investigate an employee’s decision to break-off from her employer, her choice of where to locate the new firm, and the resulting possibility of clustering. We use a game-theory approach, with simultaneous moves and complete information, and show how the birth of clusters depends on cost considerations and on the type of goods produced by firms. Thus, our paper fills a gap in the literature at the interface of firm emergence and location choice.
Our framework can be extended in several ways. First, we examine only the case of two players. The game, however, could be extended to include one incumbent firm and multiple new firms. In this new game, the timing of break-off would be important because the incumbent might cap the total amount of compensations to be distributed. A scenario with multiple incumbent firms and multiple prospective firms could be also modeled. The distribution on a geographic space of incumbents would then play a role in break-off decisions and their resulting location selection. Second, our framework assumes complete information. One can imagine that a former employee possesses complete information on the resulting new firm’s production costs, but the incumbent does not. At best, the incumbent may possess a good estimate of a probability distribution function for those costs. Incomplete information could be included in our model by assuming, for instance, that the new firm’s marginal production cost is one of two types, \( c \) with probability \( \theta \) and 0 (i.e., negligible) with probability \( 1 - \theta \), and that there is an asymmetry of information between that new firm and the incumbent. This would permit an investigation of how the incumbent’s beliefs (\( \theta c \)) on the new firm’s expected production cost influences the break-off and location decisions. Third, employees may not be risk neutral, and, of course, more conservative break-off behaviors are expected when risk increases. Higher perceived risk translates into higher entry costs, which would increase the size of subsidies and decrease the threshold on the entry cost required to encourage a break-off but would not affect the location decision. Also, albeit this is a theoretical paper and applied tests are beyond its scope, our framework lends itself well to empirical testing and our propositions could be leveraged for future analyses of clustering.\(^9\)

\(^9\) The set of parameters in our propositions can be operationalized in several straightforward ways. Some authors have used the number of firms (Folta et al. 2006, Shaver and F. Flyer 2000) or number of current employees (Holmes 1998) per unit area (state, city, county or metropolitan statistical area) as proxy for the likelihood of cluster formation and the likelihood of break-offs. Others have used Euclidian geographic distances (Nachum and Wymbs...
Finally, our framework provides significant insights for practitioners and policy makers. Regional and local policies are often aimed at promoting the formation of businesses or at attracting businesses in a particular region. Unfortunately, however, those attempts are often marred by failure. The reason for such negative outcomes rests, to a large extent, on the mistaken assumption that business clustering is always possible, always desirable, and that what works in one region may be exported elsewhere. Our paper contributes to correct those beliefs by showing that clustering is neither always possible nor desirable, and that several of the key variables determining the success of a cluster are endogenous to the system, that is, they have to be tailored carefully to the specific region. Chang et al. (forthcoming), for example, develop an empirical test of Minniti’s (2005) model and show that the development of entrepreneurial clusters is often path dependent and, as a result, policies that have been effective in one region may not work well in another. Among other things, they support our conclusion that, lacking certain conditions, subsidies are useless in promoting business clustering, that they are unnecessary for larger markets, and that, even when potentially useful, their exact size is crucial in determining their effectiveness. Moreover, as shown in Table II, the desirability of subsidies is highly dependent on market size and they may be needed only when the market size is below its upper bound, \( M \).

2005) or Herfindahl-type concentration indexes (Zaheer and Manrakhan 2001), and entrepreneurship scholars have used the number of new firms created in a geographic area (Campi et al. 2004, Johnson 2004) or its growth rate (Folta et al. 2006, Reynolds et al. 1994) to operationalize firm formation. Surveys have been used to determine the likelihood of opportunity exploitation leading to firm formation with a seven-point Likert scale (Choi and Shepherd 2004), although these types of measurement may be less desirable because break-offs are undistinguishable from spin-offs. Other key parameters in our model include market size, which has been operationalized by measuring consumers’ purchasing power in a geographic area as growth in income (Reynolds et al. 1994), per capita income (Rothaermel et al 2006), and growth rate of consumption (Mariotti and Piscitello 1995). The level of R&D investment in a region has been assessed by R&D expenditures (Audretsch and Feldman 1996) or the ratio of R&D staff to total number of industrial employees (Mariotti and Piscitello 1995). Marginal production cost has been operationalized by surveying the development of enabling technology and managerial capability of new firms (Choi and Shepherd 2004). Finally, marginal congestion cost has been captured by the length of commuting times, wage levels (Folta et al. 2006), and the prices of real estate such as industrial property (Galbraith and De Noble 1988), or, alternatively, by measuring the density of a population (Campi et al. 2004, Mariotti and Piscitello 1995).
At the microeconomic level, in the absence of subsidies, a prospective entrepreneur producing imperfect substitutes is advised to break off from the incumbent only when the market size is sufficiently large and entry cost is sufficiently low. For an incumbent firm, instead, paying a compensation to deter a break-off can be profitable when the goods are imperfect substitutes, but might be unnecessary when the goods are imperfect complements because the breaking-off is advantageous to both players (as identified in the Appendix in the proof of Proposition 2). There is still much to be learned about cluster formation. We hope others will join us in this task.

APPENDIX

DERIVATION OF OPTIMAL PROFITS PRESENTED IN TABLE I. In the cluster producing imperfect substitutes, entry cost $F$ and marginal production cost $c$ are 0 for Firm I. From the demand functions, $q_I = M - 2p_I + p_E$ and $q_E = M - 2p_E + p_I$ for Firm I and Firm E, respectively, prices in terms of production quantities are $p_I = M - \frac{2}{3}q_I - \frac{1}{3}q_E$ and $p_E = M - \frac{2}{3}q_E - \frac{1}{3}q_I$. Consequently, $\Pi_I = \left\{ M - \frac{2}{3}q_I - \frac{1}{3}q_E \right\} q_I - g q_I / b$ and $\Pi_E = \left\{ M - \frac{2}{3}q_E - \frac{1}{3}q_I \right\} q_E - F - cq_E b / R - g q_E / b$. For each firm, the best response function is derived from differentiating these equations with respect to $q_I$ and $q_E$, respectively, and equating them to 0. Solving simultaneously the resulting system of two equations yields $q_I^* = \frac{3}{5} \left\{ M - \frac{g}{b} + \frac{cb}{3R} \right\} (> 0)$ and $q_E^* = \frac{3}{5} \left\{ M - \frac{g}{b} - \frac{4cb}{3R} \right\} (> 0)$. Optimal profits are thus $\Pi_I^* = \frac{6}{25} \left\{ M - \frac{g}{b} + \frac{cb}{3R} \right\}^2$ and $\Pi_E^* = \frac{6}{25} \left\{ M - \frac{g}{b} - \frac{4cb}{3R} \right\}^2 - F$ (Firm E may experience negative profit). In the absence of Firm E, Firm I’s profit is $\Pi_I = \left\{ M - \frac{2}{3}q_I \right\} q_I$, which leads to an optimal production quantity $q_I^* = \frac{3}{4}M (> 0)$ and an optimal monopoly profit of $\Pi_I^* = \frac{3}{8}M^2$. Similar derivations are made for the cluster producing imperfect complements and available upon request to the authors.

PROOF OF PROPOSITION 1. For part (a), $\partial \Pi_E^* / \partial b = \frac{12}{25} \left\{ M - \frac{g}{b} - \frac{4cb}{3R} \right\} \left( \frac{g}{b^2} - \frac{4cb}{3R} \right)$. Since $q_E^* = \frac{3}{5} \left\{ M - \frac{g}{b} - \frac{4cb}{3R} \right\}$ must be positive, $\partial \Pi_E^* / \partial b$ is negative if and only if $g / b^2 < 4c / 3R$. Similarly, $\partial \Pi_E^* / \partial b = \frac{1}{3} \left\{ (M - \frac{g}{b} - \frac{4cb}{3R}) \left( \frac{g}{b^2} - \frac{4cb}{3R} \right) \right\}$ for parts (b) and the result follows.
PROOF OF PROPOSITION 2 (accounting for subsidies). When Firm I and Firm E produce imperfect substitutes the difference between Firm I’s optimal profits under monopoly and duopoly is \( B = \frac{3}{200} \left( 9M - \frac{4g}{b} + \frac{4ch}{5R} \right) \left( M + \frac{4g}{b} - \frac{4ch}{3R} \right) \), which is positive since production quantities (i.e., \( q_i^* \) and \( q_E^* \)) must be positive (Firm I thus has an incentive to pay a compensation to deter the break-off and keep its monopoly position). Now,

\[
\Pi^*_E - B = \frac{3}{50} \left( M - \frac{g}{b} - \frac{4ch}{3R} \right) \left( M - \frac{3g}{R} - \frac{17ch}{3R} \right) - \frac{3}{40} \left( 9M - \frac{4g}{b} + \frac{4ch}{3R} \right) \frac{g}{b} - F,
\]

and \([\Pi^*_E - B] + F \geq 0\) if and only if \( 0 \leq M \leq \frac{16}{7} \left( \frac{2g}{b} + \frac{ch}{R} \right) - \frac{40}{7} \sqrt{\frac{1}{2} \left( \frac{g}{b} \right)^2 + \frac{8c}{R} + \frac{1}{18} \left( \frac{ch}{R} \right)^2} \) or

\[
M \geq \frac{16}{7} \left( \frac{2g}{b} + \frac{ch}{R} \right) + \frac{40}{7} \sqrt{\frac{1}{2} \left( \frac{g}{b} \right)^2 + \frac{8c}{R} + \frac{1}{18} \left( \frac{ch}{R} \right)^2}.
\]

Since \( q_E^* > 0 \) yields \( M > \frac{g}{b} + \frac{4ch}{3R} \), only the latter interval on \( M \) applies. That is, \( M \) must be above a critical threshold. A break-off should thus occur when \( F < \frac{3}{50} \left( M - \frac{g}{b} - \frac{4ch}{3R} \right) \left( M - \frac{3g}{R} - \frac{17ch}{3R} \right) - \frac{3}{40} \left( 9M - \frac{4g}{b} + \frac{4ch}{3R} \right) \frac{g}{b} \).

But when \([\Pi^*_E - B] + F < 0\), the interval on \( M \) that applies is

\[
\frac{g}{b} + \frac{4ch}{3R} \leq M \leq \frac{16}{7} \left( \frac{2g}{b} + \frac{ch}{R} \right) + \frac{40}{7} \sqrt{\frac{1}{2} \left( \frac{g}{b} \right)^2 + \frac{8c}{R} + \frac{1}{18} \left( \frac{ch}{R} \right)^2},
\]

and a break-off should occur if subsidies that exceed

\[
- \frac{3}{50} \left( M - \frac{g}{b} - \frac{4ch}{3R} \right) \left( M - \frac{3g}{R} - \frac{17ch}{3R} \right) + \frac{3}{40} \left( 9M - \frac{4g}{b} + \frac{4ch}{3R} \right) \frac{g}{b}
\]

are available to cover the entry cost \( F \).

When Firm I and Firm E produce imperfect complements the difference becomes

\[
- \frac{1}{24} \left( \frac{7}{3} M - \frac{4g}{b} - \frac{4ch}{3R} \right) \left( \frac{1}{5} M - \frac{4g}{b} - \frac{4ch}{3R} \right) \left( M - \frac{4g}{b} - \frac{4ch}{3R} \right) \right) \right) \frac{g}{b}
\]

which, with \( q_i^* > 0 \), is negative if and only if \( M > \frac{12g}{b} + \frac{12ch}{3R} \) (Firm I may thus have no incentive to pay a compensation). As a result,

\[
\Pi^*_E - B = \frac{2}{3} \left( \frac{1}{3} M - \frac{g}{b} - \frac{4ch}{3R} \right)^2 - F \text{ when } M \text{ is above a critical threshold, and a break-off should occur if } F < \frac{2}{3} \left( \frac{1}{3} M - \frac{g}{b} - \frac{4ch}{3R} \right)^2,
\]

which is the participation constraint. When \( M \leq \frac{12g}{b} + \frac{12ch}{3R} \),

\[
\Pi^*_E - B = \frac{1}{6} \left( \frac{1}{3} M - \frac{g}{b} - \frac{4ch}{3R} \right) \left( \frac{23}{12} M - \frac{5g}{R} - \frac{17ch}{3R} \right) - \frac{1}{8} \left( \frac{7}{12} M - \frac{4g}{b} - \frac{4ch}{3R} \right) \frac{g}{b} - F,
\]

and \([\Pi^*_E - B] + F \geq 0\) if and only if \( 0 \leq M \leq \frac{144}{69} \left( \frac{2g}{b} + \frac{ch}{R} \right) - \frac{72}{23} \sqrt{\frac{1}{2} \left( \frac{g}{b} \right)^2 + \frac{8c}{R} + \frac{1}{18} \left( \frac{ch}{R} \right)^2} \) or

\[
\frac{144}{69} \left( \frac{2g}{b} + \frac{ch}{R} \right) + \frac{72}{23} \sqrt{\frac{1}{2} \left( \frac{g}{b} \right)^2 + \frac{8c}{R} + \frac{1}{18} \left( \frac{ch}{R} \right)^2} \leq M \leq \frac{12g}{b} + \frac{12ch}{3R}.
\]

Since \( q_E^* > 0 \) yields \( M > \frac{3g}{b} + \frac{12ch}{3R} \), only the latter interval on \( M \) applies. In other words, again, the market size \( M \) must be above a critical threshold. A break-off should thus occur when
\[ F < \frac{1}{6} \left( \frac{M - \frac{g}{b} - \frac{4ch}{5R}}{\frac{17ch}{5R}} \right) \frac{\frac{1}{12} M - \frac{5g}{b} - \frac{17ch}{5R}}{\frac{7}{8} \left( \frac{4g}{b} - \frac{4ch}{5R} \right) \frac{g}{b} }. \]

But when \([\Pi_k - B] + F < 0\), the interval on \(M\) that applies is

\[
\frac{3g}{b} + \frac{12ch}{5R} < M < \frac{144}{69} \left( \frac{2g}{b} + \frac{ch}{R} \right) + \frac{27}{25} \sqrt{1 \left( \left( \frac{g}{b} \right)^2 + \frac{8c}{R} + \frac{1}{50} \left( \frac{ch}{R} \right)^2 \right)},
\]

and a break-off should occur if subsidies that exceed

\[
-\frac{1}{6} \left( \frac{M - \frac{g}{b} - \frac{4ch}{5R}}{\frac{17ch}{5R}} \right) \frac{\frac{1}{12} M - \frac{5g}{b} - \frac{17ch}{5R}}{\frac{7}{8} \left( \frac{4g}{b} - \frac{4ch}{5R} \right) \frac{g}{b}} \text{ are available to cover the entry cost } F.
\]

REFERENCES


