Heuristic to allocate intermediate buffer storage capacities in a production line subject to machine breakdowns

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Abstract

In this research proposal we consider a production line subject to random failures at each workstation and operating under a make-to-stock policy. Every time a workstation fails, a corrective maintenance activity is triggered to repair the workstation. In order to palliate the effect of the random failures in the performance of the system, intermediate buffers are placed in-between workstations. An inventory holding cost is associated to each buffer. The research objective in this work is to allocate capacity to each intermediate buffer in the line so that the average cost per time unit is minimized while the average service level is kept above a minimum pre-specified value. In this paper we assume that unsatisfied demand is lost and the service level is defined as the long term proportion of satisfied demand. A greedy simulation–based heuristic is presented to find a feasible solution to the problem.

Keywords: Inventory management, machine failures, simulation

Introduction

As stated in (Rezg et al. 2008), in order to minimize the impact of stopped production caused by repair activities inventory control decisions must be made based not only on the expected demand and desired service level, but also on the availability of the machines in the line. As of today academic literature has dealt mostly with production systems comprised of a single machine, with much less attention paid to more complex systems such as flow shops or job shops. In practice, however, production and maintenance professionals have to deal with much more complex systems for which there are no available models and tools to support the complex decisions they have to make. The aim of this research is to develop a more general model that helps practitioners in allocating intermediate buffer capacities in a more general production setting without imposing restrictions on the distribution of the random variables involved. In this paper we consider a production system organized as an assembly system with finite capacity buffers in-between workstations and at both ends of the line. The workstations are subject to random failures that trigger a corrective maintenance procedure. These corrective maintenance actions have random durations. At each buffer there is an inventory holding cost that increases monotonically with the stage of the process; this is, the earlier the stage in the production line the lower the inventory holding cost. The production system under study is operated under a push strategy and is subject to a random demand. If there is no inventory of finished product upon arrival of a customer order, the demand associated with that order is considered as lost sales. The research problem we address in this paper is to find the sizes of the buffers in-between workstation so that the total relevant cost is minimized while the probability of a stock out is kept below a given minimum value. In this work we exclude from the analysis the buffers at both ends of the production system (i.e. buffers containing raw materials and finished goods). We assume that these buffers are of infinite capacity and that raw materials are always available for processing. More formally, let $\Pi=\{M, D, B\}$ be an arbitrary production system (i.e. an assembly line) comprised of a set of machines (i.e. M), some stochastic process D that represents the demand of finished goods that is observed at the final buffer in the system (i.e. finished goods), and a set of intermediate buffers (i.e. B). We assume that the probability density functions of the process times, times between failures and repair times are known for each machine $m \in M$. We also assume that the stochastic process D that represents the demand in the system is known in advance. Let N and H be two vectors containing the sizes and unitary holding costs associated with buffer set B, and let n_i and h_i be the size and unitary holding cost of buffer $i \in B$ respectively. As we assume that vector H is known in advance, the decision variables we deal with in this research are the buffer sizes (i.e. vector N). Let $C(N)$ be the average cost per time unit if the system operates with buffer sizes in N, and let $\theta(N)$ be the average service level observed in the system, defined as the long term proportion of demand units that are satisfied from the finished good inventory if the system operates with buffer sizes in N. The objective of this research is to find a vector N^* so that $C(N^*)$ is minimized while $\theta(N^*)$ is kept above a minimum pre-specified value ϕ. In the remainder of this paper, the vector N will be referred to as the solution to the problem under study.

Literature Review

The simultaneous consideration of both inventory control and machine reliability has become an important research area in the recent years. Recent approaches for the single machine case include (Chelbi and Ait-Kadi 2004, Gharbi et al. 2007, Rezg et al. 2005, Rezg et al. 2008); where mathematical models are developed under the assumption of both lost sales and backlogs. In such references the authors propose analytical models to find the buffer size and the frequency of preventive maintenance activities so that the total average cost per time unit is minimized. More general approaches extend the problem to a production line with intermediate buffers are found in (Demir et al. 2010, Dolgui et al. 2002, Nahas et al. 2006, Nourelfath et al. 2005). In (Dolgui et al. 2002) the authors propose a genetic algorithm (GA) to allocate buffer capacities in a production line under the assumptions of deterministic processing times and exponentially distributed failure and repair times. Here the authors propose an aggregation approach to evaluate tentative allocation of buffer capacities and a GA to minimize the average steady-state inventory cost. In (Nourelfath et al. 2005) the authors present a variant of the problem in which for each workstation in the production line there are several versions available in the market, each with a different production rate, availability and price. The objective is to maximize the throughput of the production system maintaining the total cost below a certain given value. In (Nahas et al. 2006) a local search heuristic is used to maximize the throughput of a production line by approximating the flow of discrete parts in the production line by a continuous flow. More recently a tabu search solution approach was proposed in (Demir et al. 2010) to solve the same problem as in (Nahas et al. 2006) presenting better results with respect to computational cost.

In (Massim et al. 2010) an optimization algorithm is proposed to determine an appropriate buffer storage size in order to reduce manufacturing costs while maintaining a desirable production rate. In (Zequeira et al. 2004) and (Radhoui et al. 2009) a mathematical model is developed for a manufacturing system that considers random failures and the production of non-conforming parts. The model aims at deciding whether or not to undertake maintenance activities with the objective of ensuring a continuous supply of finished goods while minimizing the expected total cost per unit time. Optimal buffer allocation in an environment with deterministic (i.e. constant) production rates and geometrically distributed times to failure and to repair is considered in (Shi and Gershwin 2009). A mathematical model with non-linear constraints is proposed along with and an algorithm for solving the model. A simulated annealing approach for solving the buffer allocation problem in reliable production lines is presented in (Spinellis and Papadopoulos 2000). In this work the authors solve the problem with the objective of maximizing the average throughput of the line. Similarly, a tabu search approach is proposed in (Demir et al. 2012) for the problem of buffer allocation in a production line with unreliable machines with the objective of maximizing the throughput. In (Zequeira et al. 2008) periods between preventive maintenance activities are considered. In (Amiri and Mohtashami 2000) a simulation model was developed to solve the problem of buffer allocation in unreliable production systems under general probability density functions for the process times, the time between failures and the repair times. A GA approach combined with a line search technique was proposed for the objective of maximizing the throughput of the system. In this research we consider a problem similar to that considered in (Amiri and Mohtashami 2000). The differences between their work and ours is in that (1) we assume that the inventory holding cost is different for each buffer, and (2) the objective in our work is to minimize the average cost per time unit rather than maximizing the system's throughput. To the best of our knowledge there is no published work that considers the problem we address in this paper.

Solution Approach

The proposed solution approach is a simulation-based greedy heuristic that starts from an initial solution N_0 with arbitrary buffer sizes satisfying the only condition that these sizes are small enough so that the solution is unfeasible with respect to $\theta(N_0)$. At iteration k the proposed heuristic transforms N_k into N_{k+1} by increasing $n_{i*} \in N_k$ by Δ units. If we let $N_k\{i,\Delta\}$ be a new solution obtained from N_k by increasing n_i in Δ size units, then $i^*=argmin\{m_i|i \in N_k\}$ and m_i is computed as in equation (1).

$$m_i = \frac{C(N_k\{i,\Delta\}) - C(N_k)}{\theta(N_k\{i,\Delta\} - \theta(N_k)} \quad (1)$$

As the initial solution N_0 is unfeasible with respect to $\theta(N_0)$ (i.e. $\theta(N_0) < \phi$), at every iteration during the execution of the algorithm, one buffer size is increased by Δ units until at some iteration k the value of $\theta(N_k)$ is greater than or equal to ϕ for the first time. At this point the algorithm stops with a feasible solution to the problem. To select the buffer that will be increased in capacity, a marginal cost per service level unit is estimated for every buffer in the system. This marginal cost per service level unit is computed for every buffer in B (i.e. m_i) and computed as in equation (1). The buffer with the lowest marginal cost (i.e. i^*) is selected and its size is increased by Δ size units. This process continues until the objective service level value (i.e. ϕ) is reached. Let $x \leftarrow S(N)$ be a function that returns in x the expected value of the service level if the production system under study is operated with the buffer sizes in N, and let $y \leftarrow C(N)$ be a similar function that returns in y the expected value of the cost per time unit. The pseudocode for the proposed solution approach follows.

```
N* ← H(M, D, ϕ, Δ)
      k ← 0
      Nk ← {n1,n2,…}
      S0 ← S(N0), C0 ← C(N0)
      while Sk < ϕ do
            i* ← argmax{mi|i ∈Nk}
            Nk+1 ← Nk{i*,Δ}
            Sk+1 ← S(Nk+1), Ck+1 ← C(Nk+1)
            k ← k + 1
      end
      N* ← Nk
end
```

Computational Experiments

To test the performance of the proposed heuristic we used as a test instance the assembly line proposed in (Amiri and Mohtashami 2000) and depicted in Figure 1, where the circles represent the machines and the triangles represent the intermediate buffers.

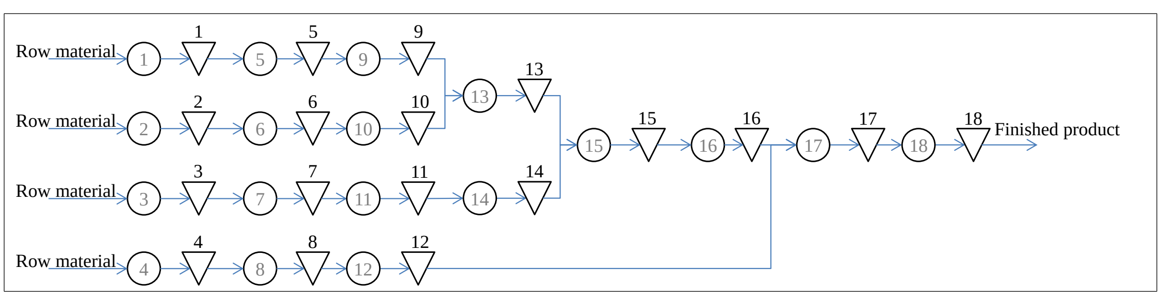

Figure 1. Production system used for testing

The system used for testing is comprised of 17 buffers and 18 machines. The density functions of the process times, times between failures and repair times are presented in Table 1. The letters E, G and W represent the exponential, gamma and weibull density functions respectively.

Table 1. Description of the machines in the production system used for testing

Station	Process time	Time between failures	Repair time
1	W(20,10,2)	W(1000,70,20)	G(1000,250,80)
2	W(25,11,4)	W(9000,200,18)	G(1200,250,80)
3	W(30,11,4)	W(9500,200,18)	G(200,250,80)
4	W(20,10,6)	W(9000,200,18)	G(1250,250,80)
5	W(35,4,3)	W(15000,130,20)	G(400,300,52)
6	W(37,9,6)	W(17000,240,90)	G(2354,300,52)
7	W(36,10,6)	W(18000,240,90)	G(2000,325,52)
8	W(40,9,6)	W(1700,240,90)	G(2000,300,52)
9	W(30,11,9)	W(11000,350,18)	G(500,300,52)
10	W(45,16,9)	W(9000,180,40)	G(700,300,52)
11	W(46,12,9)	W(9000,180,40)	G(756,300,52)
12	W(48,11,12)	W(9400,140,70)	G(730,300,52)
13	E(40)	W(12000,200,80)	G(1500,300,52)
14	W(44,17,9)	W(8650,180,40)	G(700,300,52)
15	W(27,12,10)	W(8100,300,25)	G(2659,140,60)
16	E(43)	W(8230,250,30)	G(2768,160,65)
17	W(26,12,10)	W(7700,324,25)	G(2546,140,60)
18	W(33,14,10)	W(7900,400,25)	G(2434,143,60)

As per the buffers, the instance proposed in (Amiri & Mohtashami n.d.) includes a buffer size that we used as the initial solution (i.e. N_0). Since the problem addressed in the above mentioned reference does not consider different holding cost for the buffers, we assigned an arbitrary unitary holding cost for each buffer that increases with the stage of the buffer in the system. A summary of the relevant buffer information is presented in Table 2. For the stochastic process D that models the observed demand at the end of the production system we assumed that customer orders arrive according to a homogeneous Poisson process of rate λ and that each arriving customer order if for one unit. For testing purposes the rate λ was established arbitrarily as λ=0.00702 customer orders per time unit.

Table 2. Description of the buffer set used for testing

Buffer *i*	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**
n_i	200	180	250	170	200	240	160	210	150	150	220	220	140	140	230	200	150
h_i	1	1	1	1	2	2	2	2	3	3	3	3	4	4	5	6	7

The proposed algorithm was implemented in Visual Basic for Applications® (VBA) and executed on a machine equipped with four processors Intel® Core i7 and 6 GB of memory running Windows 7 at 64 bits. As per the parameters of the algorithm we used ϕ=0.95 and Δ=10 size units. The computational time required by the algorithm to terminate for this test instance was of 41640 seconds (i.e. 11.56 hours). The evolution of the service level as the heuristic evolves searching for a feasible solution is depicted in Figure 2. As it can be seen in the figure, the expected value of the service level is not exactly ϕ=0.95 upon termination. This is because the test for whether or not the system has reached the pre-specified service level value is a statistical test. The algorithm stops when there is no statistical evidence that the true service level value is strictly below ϕ. In a similar manner, Figure 3 depicts the evolution of the average cost per time unit as the heuristic evolves in time.

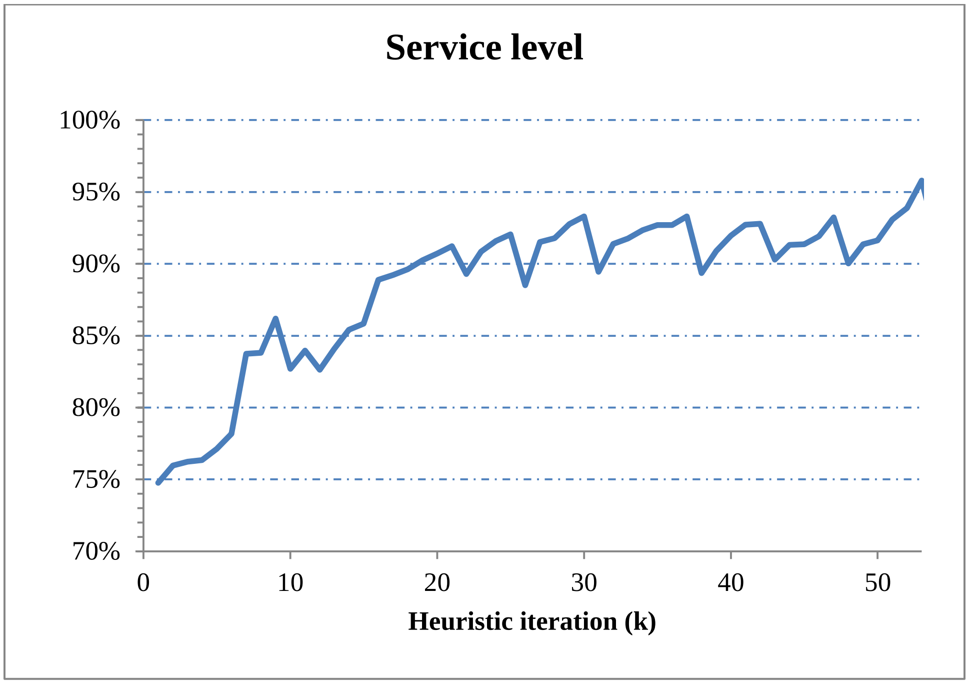

Figure 2. Evolution of the service level value

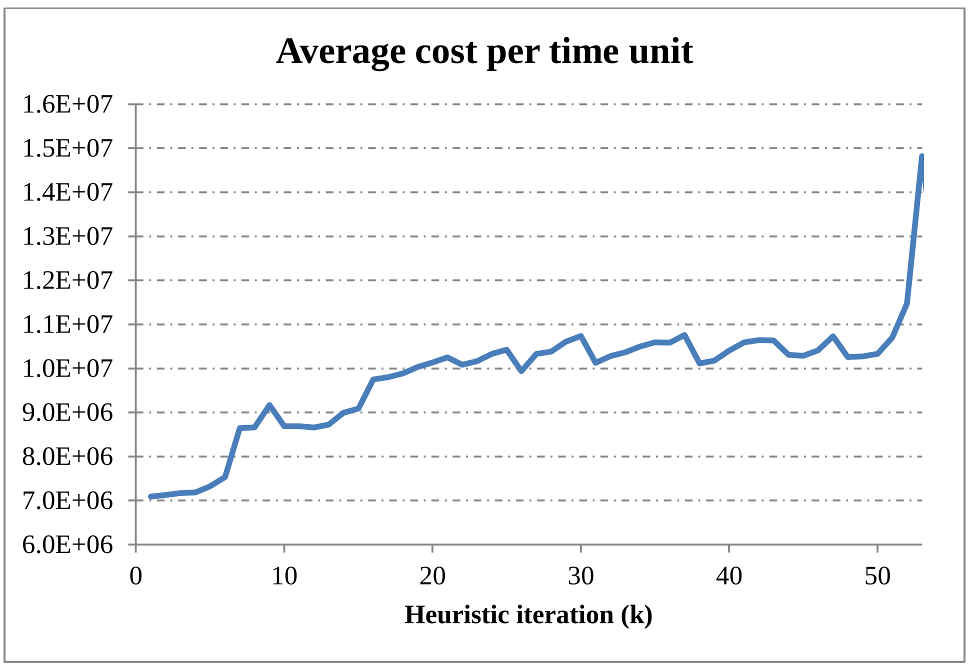

Figure 3. Evolution of the average cost per time unit

A comparison between the initial and final solutions is presented in Table 3, where N_0 and N_F represent the size vectors for the initial and final solutions respectively, while *%Gap* is the relative increment in size observed by each buffer, computed with respect to the initial size.

Table 3. Initial and final solutions

Buffer *i*	N_0	N_F	*%Gap*
1	200	220	10,00%
2	180	220	22,22%
3	250	270	8,00%
4	170	190	11,76%
5	200	220	10,00%
6	240	260	8,33%
7	160	200	25,00%
8	210	280	33,33%
9	150	190	26,67%
10	150	190	26,67%
11	220	250	13,64%
12	220	240	9,09%
13	140	190	35,71%
14	140	190	35,71%
15	230	260	13,04%
16	200	210	5,00%
17	550	550	0,00%

Conclusion

This paper deals with the problem of allocating intermediate buffer storage capacities in a production system subject to random failures and process times. Different unitary holding costs were considered for each buffer. The objective of this research is to minimize the average cost per time unit while maintaining the average service level above a certain pre-specified value. A greedy heuristic was proposed to dynamically increase the size of one buffer at a time until the objective service level value is reached. In order to estimate the average cost per time unit and service level, a discrete-event simulation model is proposed. To assess the performance of the proposed approach, a test instance from the literature was used. The proposed heuristic effectively solved the test instance within a reasonable amount of time. Further computational experiments need to be conducted with several test instances of different size and complexity in order to better understand the performance of the heuristic under these different circumstances.

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