Single-machine scheduling with upper bounded actual processing times and maintenance times

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Abstract
We consider the scheduling problem of setting the upper bounds for the actual processing times and maintenance times under assumption that the actual processing times of a job is a position-dependent power function. The maintenance duration is a position-dependent exponential function. Through building a corresponding jobs scheduling model, we found the scheduling problem can be transformed as a classic assignment problem to solve and obtained jointly the optimal frequency to perform maintenances and the optimal job sequence to minimize the total cost, which is a linear function of makespan and total tardiness.

Through applying polynomial time algorithm to solve the scheduling problem we studied, we proved that the jobs scheduling model is computable and the computational complexity of the scheduling problem is $O(n^4)$.

Keywords: Position-dependent, Maintenance activities, Total cost

Introduction

In recent years, scheduling problems with the aging effect have attracted increasing attention. In case of the aging effect, the actual processing times of a job will be longer when it is scheduled later in a sequence. Browne and Yechiali (1990) initiated research on scheduling problem with the deteriorating effect, where the actual processing times of a job is a linear non-decreasing start-time dependent function. A time-dependent aging model was proposed by Rudek (2012), where actual time required to perform a job is a function of the sum of the normal processing time of jobs already processed. For extensive surveys related to time-dependent processing time, the reader can refer to the papers (Cheng et al. 2012, Hsu et al. 2011, Huang and Wang 2011, Wang 2009). Hsu et al. (2011) studied single-machine scheduling and due date assignment problems with position-dependent processing time. They showed that the problems are polynomial time solvable. Mosheiov (2012) investigated the scheduling problem with general, non-decreasing, job-dependent and position-dependent deterioration function under the setting of parallel identical machines to minimize total load. Rustogi and Strusevich (2012) presented polynomial-time algorithms for single machine problems with generalized positional deterioration effects under machine maintenance. They assumed that the decisions should be taken regarding possible sequences of jobs and on the number of maintenance activities to be included into a schedule to minimize the overall makespan. More recent surveys have considered position-dependent processing time could be seen in papers (Lai and Lee 2010, Wang and Guo 2010, Yang and Yang 2010).

Many papers have been conducted to address the scheduling problems with job completion time due window. A job has to be stored in inventory when it is finished earlier than its due-date, which results in an earliness penalty, while a job will get a tardiness penalty when it is finished later than its due-date because it violates contractual obligation with the customer. For extensive surveys related to scheduling problems with the job completion time due window, the reader can refer to the papers (Baker and Scudder 1990, Cheng and Gupta 1989, Liman 1988). In this paper, we set the upper bound for the actual processing time of each job, and the actual processing time of the jobs is required to restrict in a given interval otherwise tardiness penalty should be paid. Which is motivated by some real manufacturing processes. For example, in porcelain manufacturing process, the actual processing time of the porcelain can not exceed a given upper bound otherwise the porcelain obtained may have quality flaws.

On the other hand, it is reasonable and necessary to perform maintenance in manufacturing processes, because it can help improve the production efficiency. Some scheduling problems with deteriorating effect and machine maintenance have been studied. A single-machine scheduling problem with a cyclic process of aging effect and maintenance activities was addressed by Kuo and Yang (2008). For the problems, they provided polynomial algorithms to minimize the makespan. Zhao and Tang (2010) extended the model of Kuo and Yang (2008). The position-dependent aging effect they considered is described by a general exponential function. Chen (2009) studied a single-machine scheduling problem with periodic maintenance activities and non-resumable jobs to minimize the number of tardy jobs. Yang and Yang (2010) considered a single-machine scheduling with a position-dependent aging effect under variable maintenance activities to minimize the makespan of all jobs. It is necessary to maintenance the machine, but the maintenance should be completed within a time interval otherwise it will affect the machine efficiency (Kubzin and Strusevich 2006, Lee and Chen 2000). Thus, in this paper, we set the upper bound for the time of each maintenance, and if the maintenance time exceeds the upper bound, the tardiness penalty of maintenance should also be paid.

However, to the best of our knowledge, research on scheduling simultaneously with upper bounded actual processing times and upper bounded maintenance times under aging effect considerations has rarely been studied. Motivated by these points, this paper investigates a scheduling problem with upper bounded actual processing times and upper bounded maintenance times under aging effect. If the actual processing times exceeds the upper bound, tardiness penalties of jobs should be paid, and if the maintenance times exceeds the corresponding upper bound, which will affect the machine efficiency and tardiness penalty of maintenance should also be paid. We assume that the machine may be subject to several maintenance activities during the scheduling horizon and the duration of each maintenance is a variable function. The objective is to minimize the total cost, which is assumed to conclude production fee and total tardiness cost, through finding jointly the optimal maintenance frequency, the optimal maintenance position, and the optimal job sequences. We show that the studied problem in the scheduling problem remains polynomially solvable.

The remainder part of this paper is structured as follows. We formally introduce the notation and terminology used throughout the rest of this paper in the next section. In section 3, we propose the main results of this paper. In section 4, the paper concludes with a summary of the results, and suggests directions for future.

Notations and problem formulation

Assume that there are n independent jobs $J = [J_1, J_2, \ldots J_n]$, which are all available for processing at time zero. The machine can handle one job at a time. In the manufacturing process, the job preemption is not allowed. To improve the production efficiency, maintenance activities may be performed on the machine. During maintenance the machine is stopped, and the maintenance will revert the initial state of the machine. We assume that the actual processing time of a job will be longer when it is scheduled later in a sequence due to the aging effect of the machine. And the maintenance duration is a function of the maintenance position of the machine. The jobs will be processed from a group consecutively. Thus, the schedule can be denoted as $\sigma = [G_1, M_1, G_2, M_2, \ldots, G_k, M_k, G_{k+1}], 0 \le k \le (n-1)$, where G_i , $1 \le i \le (k+1)$, denotes the i th group and M_i , $1 \le i \le k$, denotes the i th maintenance. $C_{[i,r]}$ is the completion time of the job scheduled in r th position of the i th group. The following a positional deterioration model of the actual processing time of job J_j is discussed. The actual processing time of job J_j, if scheduled in position r of group G_i, is given by:

$$p_{[i,j]}^{r} = p_{[i,j]} r^{a_{[i,j]}}, \text{ for } i = 1,2,\ldots,k+1, j, r = 1,2,\ldots,n_i, \quad (1)$$

where $p_{[i,j]}$ denotes the normal processing time of job J_j, and $a_{[i,j]}$ denotes the aging factor of job J_j. The number of jobs of group G_i is denoted as n_i.

In this study, we examine a model of the maintenance duration which concerns the position-dependent aging effect. If the maintenance is the i th maintenance in the sequence, its actual maintenance duration is defined by

$$m_i = t_0 b^{(i-1)}, \text{ for } i = 1,2,\ldots,k, \quad (2)$$

where $t_0 > 0$ denotes the basic maintenance time and $b > 1$ is the aging factor of maintenance. If the maintenance is arranged later in the sequence due to the aging effect, the actual maintenance duration is longer in this model.

Observing from $Eq.(1)$, we find no matter what the group is, the actual processing time of the job J_j is only dependent on its position in a group. For the simplicity, we reformulate $Eq.(1)$ as follows:

$$p_j^r = p_j r^{a_j}, \text{ for } j = 1,2,\ldots,n,\ r = 1,2,\ldots,n_i, i = 1,2,\ldots,k+1, \quad (3)$$

where p_j and $a_j > 0$ are the normal processing time and the aging factor of job J_j, respectively.

Let $p_j b_0$ denote the upper bound of the processing time of job J_j, where $b_0 > 1$ is a constant number. The tardiness of job J_j is denoted as T_j, i.e., $T_j = \max\{0, p_j^r - p_j b_0\}$. Then it can be obtained that the total tardiness of all jobs is $\sum_{j=1}^{n} T_j$. Let the $t_0 u$ denote the upper bound of the maintenance time, where $u > 1$ is a constant number. The tardiness of the i th maintenance is denoted as T_i', i.e., $T_i' = \max\{0, t_0 b^{(i-1)} - t_0 u\}$. Then the total tardiness of all maintenances is denoted by $\sum_{i=1}^{k} T_i'$. Let C_{max} denote the makespan, i.e., $C_{max} = \max\{C_j \mid j = 1,2,\ldots,n\}$. The formulation of the objective is given as follows.

In the manufacturing process, the length of the working time determines the production fee. The tardiness penalties are assumed to be linear relationship with the total tardiness of all jobs and all maintenances, respectively. Thus, in the case of setting the upper bounds for the processing times of jobs and maintenance times of machine simultaneously, we define the total cost as follows:

$$TC = \alpha C_{max} + \beta \sum_{j=1}^{n} T_j + \gamma \sum_{i=1}^{k} T_i', \quad (4)$$

where α, β and γ are the unit production fee, the unit tardiness cost of the job and the unit tardiness cost of maintenance, respectively. α, β and γ should be positive numbers, i.e., $\alpha > 0, \beta > 0, \gamma > 0$. The objective of this study is to minimize the total cost, we have to explore jointly the optimal maintenance frequency, the optimal maintenance positions, and the optimal job sequences to minimize the total cost.

Total cost minimization

The problems studied in this section can be denoted by $1\left|p_j^r = p_j r^{a_j}, M = k, m_i = t_0 b^{(i-1)}\right|TC$, where M *and* k denote the maintenance and the maintenance frequency, respectively. We consider setting the upper bounds for the processing times of jobs and maintenance times of machine simultaneously. If the processing times of jobs exceeds the upper bound, the tardiness penalties should be paid, and if the maintenance times also exceeds the corresponding upper bound, tardiness penalty of maintenance should also be paid. The objective of the problem $1\left|p_j^r = p_j r^{a_j}, M = k, m_i = t_0 b^{(i-1)}\right|TC$ is

$$TC = \alpha C_{max} + \beta \sum_{j=1}^{n} T_j + \gamma \sum_{i=1}^{k} T_i'. \quad (5)$$

A *group balance principle* is presented by Kou and Yang (2008). In the next, we will prove that the *group balance principle* remains valid for the problem $1\left|p_j^r = p_j r^{a_j}, M = k, m_i = t_0 b^{(i-1)}\right|TC$. Assume that there are n independent jobs to be assigned. If the machine is maintained k times in a schedule, then the jobs are divided into $k+1$ groups. Applying the *group balance principle* ensures that the numbers of jobs in the groups are as close as possible.

Group balance principle: Assume that the machine is maintained k times in a schedule and the jobs are divided into $k+1$ groups. The number of the jobs in every group is $\lceil n/(k+1)\rceil - 1$ *or* $\lceil n/(k+1)\rceil$, *i.e.,* $\lceil n/(k+1)\rceil - 1 \le n_i \le \lceil n/(k+1)\rceil$.

Lemma 1. For the problem $1|p_j^r = p_j r^{a_j}, M = k, m_i = t_0 b^{(i-1)}|TC$, there exists an opti-mal schedule such that the number of jobs in groups satisfies the *group balance principle*.

Proof. Assume that an optimal schedule σ which consists of n independent jobs and k maintenance activities, does not satisfy the *group balance principle*. The maintenance and group sequence σ can be described as $\sigma = [G_1, M_1, G_2, M_2, \dots, G_k, M_k, G_{k+1}]$. Then somewhere in σ there must exist at least two groups G_i and G_j, in which the difference in the number is greater than one. We assume that $n_i > n_j$, then $n_i - n_j > 1$, where n_i and n_j denote the number of jobs in the G_i and G_j, respectively. Let π_1, π_2 and π_3 denote the partial schedules of the σ, then $\sigma = [\pi_1, G_i, M_i, \pi_2, G_j, M_j, \pi_3]$, where $G_i = [J_{[i,1]}, J_{[i,2]}, \dots, J_{[i,n_i-1]}, J_{[i,n_i]}]$ and $G_j = [J_{[j,1]}, J_{[j,2]}, \dots, J_{[j,n_j-1]}, J_{[j,n_j]}]$, respectively.

We move the last job of the group G_i to the last position of the group G_j, then the number of jobs in the group G_i becomes $n_i - 1$ and that of group G_j becomes $n_j + 1$, and we obtain a new schedule $\sigma' = [\pi_1, G_i', M_i, \pi_2, G_j', M_j, \pi_3]$,where $G_i' = [J_{[i,1]}, J_{[i,2]}, \dots, J_{[i,n_i-1]}]$ and $G_j' = [J_{[j,1]}, J_{[j,2]}, \dots, J_{[j,n_{j-1}]}, J_{[j,n_j]}, J_{[j,n_j+1]}]$, respectively. The moving of the job $J_{[i,n_i]}$ is illustrated by Figure 1. For simplicity, we let the job $J_{[i,n_i]}$ be the job J_j. Since the positions of the other jobs remain unchanged in schedule σ and σ′, the cost of the processing times and the tardiness of the jobs expect the job J_j remains unchanged. Let $TC(p_j)$ and $TC(p_j')$ denote the contribution of p_j to the total cost in the schedule σ and σ′, respectively. Since the maintenance times is only dependent on its position in the schedule, moving the last job of the group G_i to the last position of the group G_j can not change the value of the time of i th maintenance. Then in the schedules σ and σ′, $\gamma \sum_{i=1}^{k} T_i'$ remains unchanged.

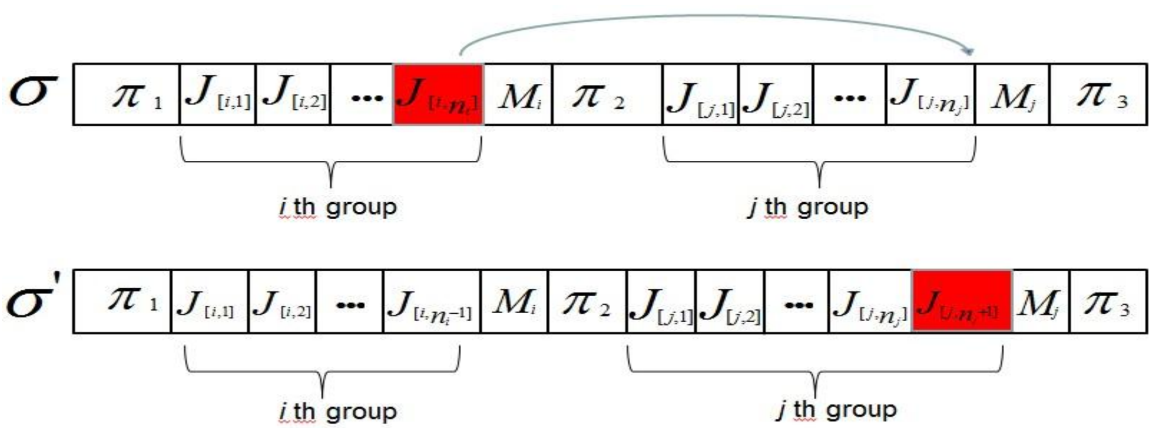

Figure 1 –The illustration of the moving of job $J_{[i,n_i]}$

In the schedule σ, the contribution of p_j to the total cost is

$$TC(p_j) = \alpha p_j n_i^{a_j} + \beta max\{0, p_j n_i^{a_j} - p_j b_0\}, \quad (6)$$

where a_j is the aging factor of the job J_j.

In the schedule σ', the contribution of p_j to the total cost is

$$TC'(p_j) = \alpha p_j (n_j + 1)^{a_j} + \beta max\{0, p_j(n_j + 1)^{a_j} - p_j b_0\}. \quad (7)$$

Combining equation (6) and (7), we get

$$TC(p_j) - TC'(p_j)$$
$$= \alpha p_j (n_i^{a_j} - (n_j + 1)^{a_j}) + \beta p_j (\max\{0, n_i^{a_j} - b_0\} - \max\{0, (n_j + 1)^{a_j} - b_0\}). \quad (8)$$

Since $\alpha > 0, \beta > 0, n_i - n_j > 1$, and $a_j > 0$, we can obtain that $TC(p_j) - TC'(p_j) > 0$. Hence, we can obtain that the total cost of the schedule σ' is less than that of the schedule σ, which contradicts the optimality of the schedule σ. *Lemma 1* is proved. □

In the following, we show that the problem $1|p_j^r = p_j r^{a_j}, M = k, m_i = t_0 b^{(i-1)}|TC$ remains polynomially solvable and can be solved in $O(n^4)$ time. The total cost is given by

$$TC = \alpha C_{max} + \beta \sum_{j=1}^{n} T_j + \gamma \sum_{i=1}^{k} T_i'$$
$$= \alpha \left(\sum_{i=1}^{k+1} \sum_{r=1}^{n_i} p_{[i,r]} r^{a_{[i,r]}} + \sum_{i=1}^{k} t_0 b^{(i-1)} \right) + \beta \sum_{i=1}^{k+1} \sum_{r=1}^{n_i} \max\{0, p_{[i,r]} r^{a_{[i,r]}} - p_{[i,r]} b_0\}$$
$$+ \gamma \sum_{i=1}^{k} \max\{0, t_0 b^{(i-1)} - t_0 u\}$$
$$= \sum_{i=1}^{k+1} \sum_{r=1}^{n_i} (\alpha r^{a_{[i,r]}} + \beta max\{0, r^{a_{[i,r]}} - b_0\})\, p_{[i,r]} + \sum_{i=1}^{k} (\alpha b^{(i-1)} + \gamma \max\{0, b^{(i-1)}$$
$$- u\}) t_0. \quad (9)$$

Then, it can be seen whatever the group is, the contribution of a job to the total cost only depends on its position in a group, and for the given k , $\sum_{i=1}^{k} (\alpha b^{(i-1)} + \gamma \max\{0, b^{(i-1)} - u\}) t_0$ is a constant. We explore to find a polynomial to minimize the total cost. The problem $1|p_j^r = p_j r^{a_j}, M = k, m_i = t_0 b^{(i-1)}|TC$ can be reformulated as a standard assignment problem, which can be described as follows:

$$Minimize \sum_{j=1}^{n} \sum_{i=1}^{k+1} \sum_{r=1}^{n_i} w_{jir} x_{jir} + \sum_{i=1}^{k+1} (\alpha t_0 b^{(i-1)} + \gamma max\{0, t_0 b^{(i-1)} - t_0 u\}) \quad (10)$$

s.t.

$$\sum_{j=1}^{n} x_{jir} = 1, i = 1,2, \ldots, k + 1, r = 1,2, \ldots, n_i, \quad (11)$$

$$\sum_{i=1}^{k+1} \sum_{r=1}^{n_i} x_{jir} = 1, j = 1,2, \ldots, n, \quad (12)$$

$$x_{jir} = 0 \; or \; 1, j = 1,2,\ldots,n, i = 1,2,\ldots,k+1, r = 1,2,\ldots,n_i, \tag{13}$$

where $w_{jir} = (\alpha r^{a_j} + \beta \max\{0, r^{a_j} - b_0\}) p_j$. If job J_j is scheduled in the r th position in the group G_i, $x_{jir} = 1$ otherwise $x_{jir} = 0$. Constraint sets (11), (12) and (13) can ensure that each job is scheduled exactly once and each position is taken by one job. A special case should be noted as follows. In the case of $k = 0$, there is no maintenance in the schedule, and the objective of the assignment problem is not $\sum_{j=1}^{n}\sum_{i=1}^{k+1}\sum_{r=1}^{n_i} w_{jir}x_{jir} + \sum_{i=1}^{k+1}(\alpha t_0 b^{(i-1)} + \gamma max\{0, t_0 b^{(i-1)} - t_0 u\})$,but $\sum_{j=1}^{n}\sum_{i=1}^{k+1}\sum_{r=1}^{n_i} w_{jir}x_{jir}$.

It is known that the assignment problem can be optimally solved in $O(n^3)$ time by the classic Hungarian algorithm. In order to minimize the total cost, we propose a polynomial time algorithm to determine jointly the optimal k, and the optimal job sequence.

Algorithm 1.

Step 1. For each $k (k = 1,2,\ldots,n-1)$, solve the assignment problem (10)-(13) and let the corresponding objective value be $TC(k)$.

Step 2. Let $TC^*(k) = \min\{TC(k) | k = 1,2,\ldots,n-1\}$, and the corresponding schedule is the result schedule.

Theorem 1. The problem $1\left|p_j^r = p_j r^{a_j}, M = k, m_i = t_0 b^{(i-1)}\right|TC$ can be optimally solved by Algorithm 1 in $O(n^4)$ time.

Proof. For a fixed maintenance frequency k, we can obtain the optimal maintenance positions and the number of jobs in each group by Lemma 1. The problem $1\left|p_j^r = p_j r^{a_j}, M = k, m_i = t_0 b^{(i-1)}\right|TC$ can be optimally solved via the assignment problem (9)-(12) in $O(n^3)$ time. Note that k has n possible values. Then, $TC^*(k) = \min\{TC(k) | k = 1,2,\ldots,n-1\}$ is the optimal objective value for the considered problem. Therefore, to solve the problem $1\left|p_j^r = p_j r^{a_j}, M = k, m_i = t_0 b^{(i-1)}\right|TC$, the computational complexity is $O(n^4)$. □

Using the similar method of Theorem 1, the following corollary can be easily obtained.

Corollary 1. For the scheduling problem of only setting the upper bound for the actual processing times, it can be optimally solved in $O(n^4)$ time.

In what follows, we investigate a special case of the problem $1\left|p_j^r = p_j r^{a_j}, M = k, m_i = t_0 b^{(i-1)}\right|TC$, and explore to find a more efficient algorithm. We assume that $1\left|p_j^r = p_j r^{a}, M = k, m_i = t_0 b^{(i-1)}\right|TC$, where $a > 0$ is a constant number.

First, we give a lemma which is useful for the following results.

Lemma 2. If sequence $x_1, x_2, \ldots, x_n$ is ordered nondecreasingly and sequence $y_1, y_2, \ldots, y_n$ is ordered nonincreasingly, the sum $\sum_1^n x_i y_i$ of products of the corresponding elements is minimized (Hardy 2008).

Theorem 2. The problem $1\left|p_j^r = p_j r^{a}, M = k, m_i = t_0 b^{(i-1)}\right|TC$ can be optimally solved by scheduling the jobs in a non-increasing order of their normal processing time p_j and then arranging the jobs one by one into each group in turn. The time complexity of the problem is $O(nlogn)$.

Proof. For a given maintenance frequency $k = k_0$, let h be the remainder of n divided by $k_0 + 1$, i.e. $h = mod(n, k_0 + 1)$. If $h \neq 0$, without loss of generality, we assume that there are d jobs in each of the first h groups and $(d - 1)$ jobs in each of the other groups.

Let $w_{[i,r]} = \alpha r^a + \beta \max\{0, r^a - b_0\}$, where $w_{[i,r]}$ is the positional weight of corresponding job. Then the total cost is given as follows:

$$TC = \alpha C_{max} + \beta \sum_{j=1}^{n} T_j + \gamma \sum_{i=1}^{k_0} T_i'$$

$$= \sum_{i=1}^{h} \sum_{r=1}^{d} w_{[i,r]} p_{[i,r]} + \sum_{i=h+1}^{k_0+1} \sum_{r=1}^{d-1} w_{[i,r]} p_{[i,r]} + \sum_{i=1}^{k_0} (\alpha b^{(i-1)} + \gamma \max\{0, b^{(i-1)} - u\}) t_0. \quad (14)$$

Since α, t_0, b, γ *and* u are constant numbers, for the given k_0, $\sum_{i=1}^{k_0} (\alpha b^{(i-1)} + \gamma \max\{0, b^{(i-1)} - u\})\, t_0$ is a constant number. From $Eq.$ (13), it can be seen that

$\alpha + \beta\, max\{0,1 - b_0\} = w_{[1,1]} = w_{[2,1]} = \cdots = w_{[k_0+1,1]} < \alpha 2^a + \beta\, max\{0, 2^a - b_0\} = w_{[1,2]} = w_{[2,2]} = \cdots = w_{[k_0+1,2]} < \cdots < \alpha(d-1)^a + \beta\, max\{0, (d-1)^a - b_0\} = w_{[1,d-1]} = w_{[2,d-1]} = \cdots = w_{[k_0+1,d-1]} < \alpha d^a + \beta\, max\{0, d^a - b_0\} = w_{[1,d]} = w_{[2,d]} = \cdots = w_{[h,d]}$.

Hence, if

$p_{[1,1]} \geq p_{[2,1]} \geq \cdots \geq p_{[k_0+1,1]} \geq p_{[1,2]} \geq p_{[2,2]} \geq \cdots \geq p_{[k_0+1,2]} \geq \cdots \geq p_{[1,d-1]} \geq p_{[2,d-1]} \geq \cdots \geq p_{[k_0+1,d-1]} \geq p_{[1,d]} \geq p_{[2,d]} \geq \cdots p_{[h,d]}$.

Then, by Lemma 3, the total cost is the least one. Therefore, there exists an optimal schedule in which jobs are scheduled in non-increasing order of their normal processing time. Then, schedule the job in the first position of each group one by one. If the first position of each group is filled, then schedule the remaining job in the second position of each group one by one. If all the second positions are filled, fill the third position, and so on, until all jobs are scheduled. The time complexity of arranging the jobs in a non-increasing order of their normal processing time is $O(nlogn)$. The time complexity of assigning n jobs one by one to each group in turn in a non-increasing order of their normal processing time is $O(1)$. Thus, the problem $1|p_j^r = p_j r^a, M = k, m_i = t_0 b^{(i-1)}|TC$ can be optimally solved in $O(nlogn)$ time.

We demonstrate the results of the theorem 2 in the following example:

Example. Data: $n = 5, p_1 = 3, p_2 = 5, p_3 = 5, p_4 = 8, p_5 = 11, a = 0.2, \alpha = 2, \beta = 25, \gamma = 100, t_0 = 4, b = 1.1, u = 1.2, b_0 = 1.3$. Let $v = \sum_{i=1}^{k_0} (\alpha b^{(i-1)} + \gamma max\{0, b^{(i-1)} - u\}) t_0$. The value of the number of jobs in each group, positional weights, the optimal schedule, the v and the total cost is given in table 1.

Table 1 –The number of jobs in each group, positional weights, the optimal schedule, the value of v and the total cost

k_0	The number of jobs in each group	Positional weights	The optimal schedule	v	TC
0	$n_1 = 5$	$w_{[1,1]} = 2.00, w_{[1,2]} = 2.30, w_{[1,3]} = 2.49, w_{[1,4]} = 3.13, w_{[1,5]} = 4.75$	(11,8,5,5,3)	0	82.75
1	$n_1 = 3, n_2 = 2$	$w_{[1,1]} = 2.00, w_{[1,2]} = 2.30, w_{[1,3]} = 2.49, w_{[2,1]} = 2.00, w_{[2,2]} = 2.30$	(11,5,3,8,5)	2.00	70.47
2	$n_1 = 2, n_2 = 2, n_3 = 1$	$w_{[1,1]} = 2.00, w_{[1,2]} = 2.30, w_{[2,1]} = 2.00, w_{[2,2]} = 2.30, w_{[3,1]} = 2.00$	(11,5,8,3,5)	4.20	70.60
3	$n_1 = 2, n_2 = 1, n_3 = 1, n_4 = 1$	$w_{[1,1]} = 2.00, w_{[1,2]} = 2.30, w_{[2,1]} = 2.00, w_{[3,1]} = 2.00, w_{[4,1]} = 2.00$	(11,3,8,5,5)	17.88	82.78
4	$n_1 = 1, n_2 = 1, n_3 = 1, n_4 = 1, n_5 = 1$	$w_{[1,1]} = 2.00, w_{[2,1]} = 2.00, w_{[3,1]} = 2.00, w_{[4,1]} = 2.00, w_{[5,1]} = 2.00$	(11,8,5,5,3)	80.93	144.93

Observing from table 1, it can be seen that the case of $k_0 = 1$ is optimal. The jobs should be divided into 2 groups, where $n_1 = 3, n_2 = 2$. The optimal schedule is (11, 5, 3, 8, 5). Then TC^*= 70.47.

Conclusions

The paper investigated a single-machine scheduling problem with upper bounded actual processing times and upper bounded maintenance times under aging effect. The objective is to minimize the total cost that is a linear function of the makespan and tardiness penalties. We showed that the studied problem can be optimally solved in $O(n^4)$ time. Moreover, for a special case that the aging factor of the processing time is assumed as a constant, we showed that the total cost minimization problem with aging effect can be solved in $O(nlogn)$ time. Future research may focus on the scheduling problem with upper bounded actual position-dependent processing times and upper bounded maintenances times under aging effect in the context of multiple machines scheduling problems or job-shop scheduling problems.

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