Two level apparel supply chain joint contract with buy back and pricing flexible

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Abstract

The supply chain contract is an effective way to coordinate activities among partners in the supply chain. The buy back contract can coordinate supply chain under fixed price conditions. However, the single buy back contract doesn't work perfectly under changing price condition. Considering the changing price characteristics of apparel supply chain, a joint contract which combines buy back and pricing flexible contracts is modeled in order to perfectly coordinate apparel supply chain in this paper.

Keywords: Apparel supply chain coordination, Joint contract, Pricing flexible contract

Introduction

Apparel market faces with varying retail price and customer demand. Apparel supply chain which has the seasonal apparel products has the characteristics of flexibility and uncertainty. Apparel supply chain consists of the retailer, apparel manufacture, and fabric supplier. The retailer sells the apparel to the consumer. The manufacturer is the apparel producer. The fabric supplier provides the required fabrics to the manufacturer. They may have their own production facility or may be a distributor of fabrics to the apparel manufacturers. Coordination between partners in apparel supply chain, which could be coordination between fabric supplier and apparel manufacturer, or apparel manufacturer and retailer, needs the sophisticated approach to improve entire supply chain management performance.

The supply chain contracts are effective way to coordinate activities among partners in the supply chain. Considering different benefits assignment and product demand changing conditions, various supply chain contracts have been proposed for supply chain coordination, such as, revenue sharing contracts, buy-back contracts, quantity discounts contracts, ,and pricing flexible contracts, etc. Cachon (Cachon and Lariviere 2005) study the revenue-sharing contracts in a general supply chain model with revenues determined by each retailer's purchase quantity and price under the deterministic demand. According to Pasternack (Pasternack 1985), buy back contracts can coordinate the channel under two returns policies: full return for partial credit and partial return for full credit. Under the first police, buy-back contracts are independent of the retailer's demand distribution. Tsay (Tsay 1999) study the quantity flexibility contract. Liu and Zhao (Li et al. 2008, Zhao and Wang 2002) study the different combined contracts for supply chain coordination.

In this paper, we will focus on the research of joint contract with buy back and pricing flexible for two level apparel supply chain coordination which is apparel manufacturer (apparel supplier) and retailer. At the beginning, the general concentrated two level apparel supply chain coordination model will be discussed. Then, the single buy back contract is applied to coordinate the apparel supply chain under fixed retail price condition. Furthermore, considering the price changing feature of apparel product, the join contract with buy back and pricing flexible is proposed to coordinate the apparel supply chain and share revenue among supply chain partners.

The two level general apparel supply chain model
Consider a two level apparel supply chain that consists of an apparel manufacturer (supplier) and a retailer as showed in Fig.1. And given the apparel's wholesales price decision, the retailer places a single order with the manufacturer before the selling period; the retailer makes no further order before and during the selling period.
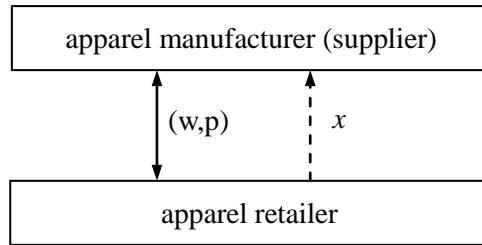

Figure 1- two level apparel supply chain

In general case which contract doesn't applied in the supply chain, supplier and retailer make the management decision respectively to maximize their own profits. The supplier set the wholesale price w based on the production cost c. Then, the retailer make the order decision with the optimal order quantity Q. Supposed consumer demand x is stochastic, but independent and stationary across periods. We assume there are ordering costs and shortage penalty costs, but no inventory holding, and the delivery cost is negligible. Each unsold item has residual value s. And, $s < c < w < p$
If the order quantity is more than market demand, $Q \geq x$, then the expected profit of retailer is,

$$E_1 Z_R(Q) = \int_0^Q [(p - w)x - (w - s)(Q - x)] f(x) dx \tag{1}$$

And, if the order quantity is less than market demand, $Q \leq x$, then the expected profit of retailer is,

$$E_1 Z_R(Q) = \int_Q^\infty (p - w) \cdot Q \cdot f(x) dx \tag{2}$$

Combining equation (1) and (2) the total expected profit of retailer is as follows,

$$E_1 Z_R(Q) = \int_0^Q [(p-w)x - (w-s)(Q-x)]f(x)dx + \int_Q^\infty (p-w) \cdot Q \cdot f(x)dx$$

$$= -wQ + (p-s)\int_0^Q xf(x)dx + s\int_0^Q Qf(x)dx + p[\int_0^\infty Qf(x)dx - \int_0^Q Qf(x)dx] \qquad (3)$$

$$= (p-w)Q + (p-s)\int_0^Q xf(x)dx - (p-s)\int_0^Q Qf(x)dx$$

In order to get the optimal order quantity Q under maximized expected profit, derivative equation (3) as follow：

$$\frac{dE_1 Z_R(Q)}{dQ} = (p-w) + (p-s)Qf(Q) - (p-s)[\int_0^Q f(x)dx + Qf(Q)]$$

$$= (p-w) - (p-s)\int_0^Q f(x)dx \qquad (4)$$

Let, $\dfrac{dE_1 Z_R(Q)}{dQ} = 0$, then from equation (4) we can get $\int_0^Q f(x)dx = \dfrac{p-w}{p-s}$

Let, $\int_0^Q f(x)dx = F(Q)$, then $F(Q) = \dfrac{p-w}{p-s}$,

The optimal order quantity Q is：

$$Q^* = F^{-1}(\frac{p-w}{p-s}) \qquad (5)$$

Because $\dfrac{d^2 E_1 Z_R(Q)}{dQ^2} = -(p-s)f(Q) < 0$, retailer can obtain maximization expected profit

when order quantity is Q^*.

The expected profit of the retailer determined by the retailer's optimal order quantity Q^* as follow:

$$E_1 Z_S(Q^*) = (w-c)Q^* \qquad (6)$$

The total expected profit of entire supply chain is the sum of expected profit of the supplier and retailer as follows：

$$E_1 Z(Q^*) = E_1 Z_R(Q^*) + E_1 Z_S(Q^*) \qquad (7)$$

So, the supplier's profit depends on the retailer's order quantity in the general decentralized supply chain case in which the supply chain doesn't coordinated by contract between partners. In the concentrated supply chain case, the retailer and supplier simultaneously decide and operate. So the expected profit of supply chain and retailer's order quantity can be optimized in concentrated case. Although this concentrated supply chain model is less applied in the real supply chain operations, the academics and industry take it as perfect coordination

standard of supply chain system.

Under concentrated controlled supply chain, the expected sale quantity $\Phi(x)$ can be given as;

$$\phi(x) = \int_0^Q xf(x)dx + \int_Q^\infty Qf(x)dx$$
$$= [1 - F(Q)]Q + \int_0^Q xf(x)dx$$
(8)

And, the expected profit of entire supply chain system can be calculated as;

$$EZ_1(Q) = (p-c)Q - (p-s)\int_0^Q F(x)dx$$
（9）

So, $\dfrac{p-c}{p-s} = F(Q)$, And optimal order quantity on concentrated controlled supply chain is as follows：

$$Q_1^0 = F^{-1}(\frac{p-c}{p-s})$$
（10）

Compared the equation (5) and (10), we can find that $Q^* < Q^0$, because $p > w > c > s$. So, the retailer's order quantity Q^* under decentralized supply chain model is always less than the order quantity Q^0 under concentrated supply chain model. This means that the general decentralized model cannot coordinate the apparel supply chain.

Two level apparel supply chain coordination with joint contract
Buy back contract would coordinate the price-setting newsvendor if the supplier could commit to adjust the buy back and wholesale price in response to any price chosen by the retailer. According to Pasternak [2], under buy back contract the manufacturer charges a wholesale price *w* per unit and pays buy back price *b* per unit the unsold product to the retailer after the sale season finishes. The retailer still collects the *s* salvage revenue per unit. The retailer determines the order quantity based on the buy back price to optimize its profits. Supposed the marginal cost of supplies is *c*, then $s \leq b$，$c \leq w \leq p$

Under decentralized supply chain with buy back contract, the retailer's expected profit is as follows:

$$E_2Z_R(Q) = \int_0^Q [(p-w)x - (w-b)(Q-x)]f(x)dx + \int_Q^\infty (p-w) \cdot Q \cdot f(x)dx$$
$$= (p-w)Q - (p-b)\int_0^Q F(x)dx$$
（11）

The order quantity Q** under retailer's maximized expected profit in this case can be calculated as follows:

$$Q^{**} = F^{-1} \left(\frac{p-w}{p-b} \right)$$
(12)

And, the expected profit of supplier is：

$$E_2 Z_S (Q^{**}) = (w-c)Q^{**} - b\int_0^Q F(x)dx$$
(13)

The expected profit of entire supply chain with buy back contract is:

$$E_2 Z(Q^{**}) = E_2 Z_R (Q^{**}) + E_2 Z_S (Q^{**})$$
(14)

Obviously, in order to make total profit of this decentralized supply chain with buy back contract equals to the expected profit of concentrated supply chain, that is $E_2 Z(Q^{**}) = EZ(Q)$, it is necessary to let $Q^0 = Q^{**}$ and then：

$$F^{-1}(\frac{p-c}{p-s}) = F^{-1}(\frac{p-w}{p-b})$$
（15）

Because of the rigid monotony of $F^{-1}( )$, equation (15) can be changed into：

$$\frac{p-c}{p-s} = \frac{p-w}{p-b}$$
（16）

Then we get： $b = \dfrac{s(p-w) + p(w-c)}{p-c}$
(17)

The retail price is fixed in above supply chain coordination with single buy back contract. When retail price is fixed, single buy back contract can coordinate the apparel supply chain. However, the apparel retail price is changing depending on the sales cycle in apparel market. And also, the apparel retailers change the retail price in order to maximize the revenue through optimal decision on order quantity and retail price. Thus, the joint contract with buy back and pricing flexible is applied to coordinate apparel supply chain which retail price is dynamically changing. The two level apparel supply chain with joint contract of buy back and pricing flexible is showed in Figure 2.
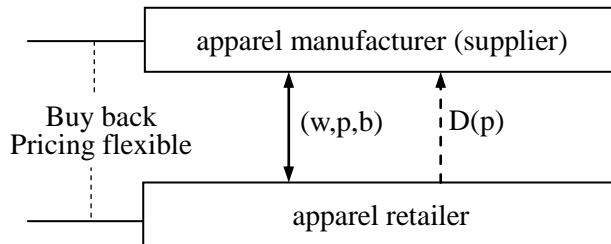

Figure 2- two level apparel supply chain with joint contract of buy back and pricing flexible

Supposed that the retail price is represented with P_0 for initial value and P for adjusted values. The market demand $D(p)$ varies related to changing retail price p. The cost of suppliers is c and buy back price is b. w is wholesale price and w_0 is basic wholesale price. w is the linear function of p.

$$\Delta w = \alpha \Delta p ; \quad b = w - \delta \tag{18}$$

α is a constant and $0<\alpha<1$. δ is the difference between wholesale price w and buy back price b. And,

$$\Delta p = p_0 - p ; \Delta w = w_0 - w; \tag{19}$$

let $\quad w^0 = \alpha p_0 + (1-\alpha)c \tag{20}$

$$\delta = (1-\alpha)(c-s) \tag{21}$$

The expected profit of retailers in this case is as follows,

$$E_3 Z_R(Q, p) = (p - w)Q - (p - b)E[Q - D(p)]^+ \tag{22}$$

And, introduce equation (18-21) to equation (22), we can get,

$$
\begin{aligned}
E_3 Z_R(Q, p) &= (p - w)Q - (p - b)E[Q - D(p)]^+ \\
&= [p - w_0 + \alpha(p_0 - p)]Q - [p - w_0 - \alpha(p - p_0) + (1-\alpha)(c-s)]E[Q - D(p)]^+ \\
&= (1-\alpha)(p-c)Q - [(1-\alpha)(p-c) + (1-\alpha)(c-s)]E[Q - D(p)]^+ \\
&= (1-\alpha)((p-c)Q - (p-s)E[Q - D(p)]^+)
\end{aligned}
\tag{23}
$$

The expected profit of supplier can be calculated as,

$$E_3 Z_S(Q, p) = (w - c)Q - (b - s)E[Q - D(p)]^+ \tag{24}$$

And, introduce equation (18-21) to equation (24), we can get

$$E_3 Z_s(Q, p) = \alpha((p-c)Q - (p-s)E[Q - D(p)]^+) \tag{25}$$

So, the expected profit of entire supply chain system can be calculated as,

$$E_3 Z(Q, p) = E_3 Z_R(Q, p) + E_3 Z_S(Q, p) = (p-c)Q - (p-s)E[Q - D(p)]^+ \tag{26}$$

Introduce equation（20） and equation (21）into equation（18）and equation（19），we can obtain w and b as following：

$$w = c + \alpha(p - c); \quad b = s + \alpha(p - s) \tag{27}$$

Compared equation (26) of supply chain expected profit with buy back and pricing flexible joint contract and equation (2) of general concentrated supply chain expected profit, we can find that buy back and pricing flexible joint contract can perfectly coordinate the supply chain for any given initial retail price p_0 when wholesales price varies with the retail price ($0 < \alpha < 1$).

If the wholesale price is not only related to retail price and also related to order quantity, the w and b to coordinate supply chain is as follow:

$$w = \alpha p + (1 - \alpha)c - \alpha(p_0 - p)E[Q - D(p)]^+; \quad b = s + \alpha(p - s) \tag{28}$$

We can find that the wholesale price w in equation (28) which considers the order quantity Q is lower than that in equation (27) without considering order quantity when retail price is decreasing ($p < p_0$). The wholesale price w decreases when order quantity Q increases. It means that suppliers willing to decrease their wholesale price to compensate for the retailers' loss caused by decreased price when retailers order more products. When retail price increased ($p > p_0$), the wholesale price w in equation (28) which considers the order quantity Q is higher than that in equation (27) without considering order quantity. It means that the retailers profit will increase and the wholesale price will increase too if retail price increases. And, the wholesale price w will increase when order quantity Q increases. In this case, the suppliers' profit will increase when retailers' profit increases. So, the buy back and pricing flexible joint contract can improve partners profit partitions and perfectly coordinate the supply chain.

Conclusion

The supply chain contract is an effective way to coordinate activities among partners in the supply chain. In this paper, we study the apparel supply chain coordination. The general apparel supply chain modeling shows that the general decentralized supply chain without contract can't be coordinated well. The apparel supply chain with single buy back contract can be coordinated under fixed retail price condition. However, the retail price is not fixed because of seasonal demand feature of apparel product. So, the joint contract of buy back and pricing flexible is modeled in this paper to coordinate apparel supply chain under changing retail price condition. The study shown that this joint contract can perfectly coordinate the apparel supply chain and flexibly partition the profit between the supplier and the retailer when the rate α of wholesale price of retail price is meet the condition of $0 < \alpha < 1$. In the further research, the formulation based on the apparel supply chain case will be given.

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