Loss allocation in the newsvendor problem with fairness concerns

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Abstract
This paper studies the loss allocation problem in a supplier-retailer supply chain when the retailer reveals fairness concern. A loss-sharing contract is provided to coordinate such a loss and some new interesting management insights are found.

Keywords: Loss allocation, Supply chain coordination, Fairness concern

Introduction
This paper studies the loss allocation problem for a supplier-retailer supply chain. In our paper both the supplier and the retailer are independent decision-makers seeking to maximize their individual profit under the assumption of information asymmetry. The supplier produces seasonal products and markets them through a retailer who faces a stochastic market. The supplier offers the retailer a loss sharing contract to achieve cooperation of information sharing. Along with the contract, the supplier provides the retailer a demand signal. Assuming the contract is acceptable, the retailer sets an order to the supplier based on their collaborative forecasting. Then the supplier produces exactly according to the retailer’s orders quantity. Also, we incorporate the fairness to the collaborative forecasting scenario in which we assume the retailer is fair-minded.

In such a collaborative setting whether or not considering the fairness, both the supplier and the retailer are benefiting from the cooperation platform. For the supplier, he knows exactly how many the retailer orders, therefore, no over-stocking loss for him. For the retailer, his forecasting ability may improves with the information from the supplier, in other words, the loss of demand uncertainty decreases for a lower forecasting deviation.

As we mentioned before that the supplier’s information should be considered by the retailer to forecast the market demand, therefore, the retailer’s order decision may be manipulated by the supplier. In specific, if the supplier is optimistic to the market demand, he can provide a positive signal to the retailer for a higher order, and otherwise if the
supplier is bearish to the market, she will provide a negative signal to induce the retailer to place fewer order. Obviously, such a setting is not inequitable for a retailer. There is a significant incidence of cases in which firms, like individuals, are motivated by concerns of fairness in channel relationships (Kahneman et al. 1986). For an inequity-averse retailer, his decision may be influenced by the supplier’s inequitable actions, like inequitable loss allocation and which may sacrifice the retailer’s profit. In the final part of this paper, we investigate the effects of information sharing and inequity aversion to the agents’ decision making and some interesting results are shown in the later.

**Literature Review**

Literature related to this paper focuses mainly on the supply chain coordination and the fairness concerns. As the issue of supply chain coordination has been well studied in the literature and we refer readers to Cachon (2003) for a review. Thus, we pay our attention to the literature of fairness concern in this part.

Research in fairness concern in the past ten years has drawn more and more researchers’ attention. Fehr and Schmidt (1999) propose a distributive fairness model between players by incorporating people’s aversion to inequity in their utility function in which the agent’s utility is negatively related to the difference from the monetary payoff of others. They find that players dislike unequal outcomes compared to the payoffs of their opponents, and are willing to sacrifice a monetary amount to reach an outcome that they believe to be fairer through the theoretical and experimental research methods. Since we usually assume that supply chain players only care about their profit in traditional supply chain models, Cui et al. (2007) extend the concept of distributive fairness to the application of supply chain by considering a fair-minded agent cares about other agents’ profit simultaneously and first investigate how fairness may affect channel coordination. They find that a manufacturer in a dyadic channel can use a simple wholesale price above her marginal cost to maximize both channel profit and channel utility when the retailer has strong concerns of fairness or when both the retailer and manufacturer are fairness-concerned. Then Pavlov and Katok (2009), Caliskan-Demirag et al. (2010) extend Cui’s study respectively. Specially, they suggest that the channel can also be coordinated with linear pricing contract when demand functions are non-linear or when there is information asymmetry between channel members. Cui and Mallucci (2010) embed the fairness utility function of channel members in a quantal response model, in which firms may make mistaken decisions due to bounded rationality. They study the consequences of fairness on channel members’ decision-making in situations involving choices on both investments and prices. They analytically show that firms are more willing to invest in a channel in order to affect the formation of the equitable payoff, when firms are fair-minded and make pricing decisions to assure their pecuniary payoffs reflect their respective contributions to the channel. Recently, Ho and Su (2009) develop peer-induced fairness to explain the phenomenon that people may look to their peers as a reference to evaluate their own payoffs and suggest that peer-induced fairness can induce a monopoly firm to offer uniform prices. Then Chen and Cui (2013) incorporate
consumers’ concerns of peer-induced price fairness into a model of price competition and show that a uniform price for branded variants may emerge in equilibrium. They obtain that uniform pricing induced by consumers’ concerns of fairness can actually help mitigate price competition and hence increase firms’ profits if the demand of the product category is expandable. An individual firm may not have an incentive to unilaterally mitigate consumers’ concerns of price fairness to its own branded variants, which suggests the long-run sustainability of the uniform pricing strategy. In addition to the research of the model aspects, there are many empirical studies.

Although some literature on supply chain coordination has involved the fairness concerns, there is still no study which takes the fairness into consideration of the loss allocation. As people prefer to pay more attention to the unfair in the allocation of loss than profit, a little inequity might cause very strong sense of aversion. So it is necessary to study the fairness effects on a loss allocation of a supply chain. Therefore, we develop such a loss sharing contract to induce the supply chain members to share their demand information to make the supply chain a higher profit by considering their fairness concern.

Model Formulation

This paper considers a single period production and inventory system with the assumption that the information is asymmetric and the market demand is stochastic. We investigate both cases with non-collaborative forecasting and collaborative forecasting. In the non-collaborative forecasting scenario, the decision procedure is given in Fig. 1. The supplier first decides her production and wholesale price based on her forecasting of the market demand, and then the retailer decides the order quantity based on the supplier’s wholesale price and her forecasting of the market demand. They bear their own loss of overstocking independently. In collaborative forecasting scenario, the decision procedure is given in Fig.2. Both the supplier and retailer make decisions based on their collaborative forecasting to the market demand and bear the loss of overstocking together. In previous studies of newsvendor model only the retailer bears the loss of overstocking, while in our paper both of them need to bear the loss. Thus the loss allocation scheme is the main problem for their collaborative forecasting.

Based on the above description, the non-collaborative forecasting scenario will be analyzed as the benchmark to examine the benefit of the collaborative forecasting scenario. In the final section of this paper, we will extend our paper by considering the retailer’s loss aversion.

Model Analysis

In this section, we analyze the allocation of overstocking loss in the non-collaborative forecasting scenario, collaborative forecasting without fairness scenario and collaborative forecasting with fair-minded retailer scenario, respectively. Furthermore, the scenario one will be analyzed firstly as the benchmark.
Benchmark: Non-Collaborative Forecasting

In this case, the supply chain agents make their decisions in a decentralized way. For the retailer, he makes his decisions of order quantity according to his forecasting of the market demand. We assume that the cumulative distribution function (CDF) of the market demand forecasted by the retailer is \( F(x|\xi_r) \) and the probability density function (PDF) is \( f(x|\xi_r) \), where \( x \) is the market demand forecasted by the retailer and the \( \xi_r \) is the information he based on. For the supplier, he decides his production and wholesale price also based on the market information he collected. We assume that the CDF of the market demand forecasted by the supplier is \( F(y|\xi_s) \) and the PDF is \( f(y|\xi_s) \), where \( y \) is the marked demand forecasted by the supplier and the \( \xi_s \) is her information.

As the supplier and the retailer are non-collaborative, i.e. the forecast information is asymmetric, thus \( \xi_r \) is not equal to \( \xi_s \).

Based on the above forecasting information of \( F(x|\xi_r) \), the retailer can forecast \( q_r \) as his optimal order quantity and his expected profit function forecasted by himself is

\[
E(\Pi_r) = -wq_r + \int_{-\infty}^{x} psf(x|\xi_r)dx + \int_0^{x} sq_r-x)f(x|\xi_r)dx + \int_{x}^{\infty} pq_r f(x|\xi_r)dx
\]

\[
= (p-w)q_r - \int_0^{x} (p-s)(q_r-x)f(x|\xi_r)dx
\]

\[
= (p-w)q_r - \bar{L}_r
\]

where \( \bar{L}_r=(p-s)(q_r-x)^* \), \( \bar{L}_r(q_r)=E[L_r(q_r)] \) is the expected losses generated by the deviation of the retailer’s order quantity from the market demand. Then the retailer can calculate his optimal order quantity as

\[
q_r^* = F^{-1}(\frac{p-w}{p-s}|\xi_r)
\]

(1)

Since the supplier would not product based on the “make-to-order” policy, but plans her production for her own interest, then her optimal production quantity is to maximize the following expected profit function with respect to the production quantity. Similarly, the supplier can forecast \( q_s \) as her production decision according to the above forecasting information of \( F(x|\xi_s) \). The supplier’s expected profit function forecasted is

\[
E(\Pi_s) = -cq_s + \int_{-\infty}^{y} wsf(y|\xi_s)dy + \int_0^{\infty} s(q_p-y)f(y|\xi_s)dy + \int_{y}^{\infty} wq_p f(y|\xi_s)dy
\]

\[
= (w-c)q_s - \int_0^{\infty} (w-s)(q_p-y)f(y|\xi_s)dy
\]

\[
= (w-c)q_s - \bar{L}_s
\]

(2)

\[
(3)
\]
where \( L_s = (w - s)(q_p - y)^+ \), \( \bar{L}_s(q_p) = E[L_s(q_p)] \) is the expected losses of the supplier. Furthermore, the supplier can calculate her optimal product quantity

\[
q_p^* = \frac{w - c}{w - s} \text{F}^{-1}(s \xi)
\]

After the real retailer’s order quantity and market demand are realized, we can easily find that the \( q_p^* \) may not equal to the \( q_r^* \) and the \( q_r^* \) may not equal to the real market demand, too.

In the traditional newsvendor problem that the demand information is symmetrical, and the supplier’s production is exactly equal to the retailer’s order which means the supplier does not need to forecast, thus, there is no uncertain to the supplier’s profit. In this paper, the supplier’s production is based on his own forecasting of the market demand, i.e. the supplier’s decision variables isn’t determined by the retailer. Thus, the supplier’s profit is uncertain unless his forecasting of the information is totally accurate.

**Collaborative Forecasting without fairness concerns**

In this scenario, the supplier and the retailer collaborate to forecast the information of the market demand and they bear the loss of overstocking together. We let \( F(x|\xi, \zeta_s) \) and \( f(x|\xi, \zeta_s) \) be their collaborative forecasting of the CDF and PDF of the market demand, where \( \zeta_s \) is the information provided by the supplier to the retailer. Comparing with the non-collaborative forecasting scenario in benchmark, the demand information of forecasting becomes symmetrical. For the supplier, she can produce exactly what the retailer needs to get a certain profit without bearing the loss of forecasting bias. For the retailer, his profit may be improved because the supplier shares part of the loss of overstocking and her forecasting becomes more accurate with the information \( \zeta_s \). Hence, it may be valuable for the supplier and the retailer to achieve a collaboration of collaborative forecasting.

![Figure 2-Decision tree diagram in collaborative forecasting scenario.](image)

In the non-collaborative forecasting scenario, the supplier and the retailer make their own decision in a decentralized way, agents generally take recourse in coordination contracts to improve the performance of the supply chain. In this part, we provide a loss sharing contract to improve the supply chain performance.

In a loss sharing contract, we assume the total loss of overstocking is \( L_s \) and the loss shared by the retailer and the supplier are \( \gamma L_s \) and \( (1-\gamma) L_s \), respectively. Thus, we can get the expected profit functions of the supplier, retailer and supply chain.
In Eq. (5), \( w^* \) and \( q^*_r \) are the wholesale price provided by the supplier and order quantity from the retailer under the collaborative forecasting scenario, and \( L_e=(p-w^*)q^*_r-d^* \) and \( \bar{L}_e=E(L_e) =\int_{0}^{\infty}(p-s)(q-x)f(x|\xi, \zeta_x)dx \).

Using simple algebraic, \( E(\Pi'_r) \) is a concave function with respect to \( q^*_r \); therefore, we acquire the optimal order quantity \( q^*_r \) by solving the following equation:

\[
\frac{\partial E(\Pi'_r)}{\partial q^*_r} = p-w^*-\gamma(p-s)F(q^*_r) = 0 .
\]

From Eq. (6), the optimal order quantity of the retailer and the optimal order quantity of the supply chain are

\[
q^*_r = F^{-1}(\frac{p-w^*}{\gamma(p-s)}|\xi, \zeta_x), \quad q^*_c = F^{-1}(\frac{p-c}{p-s}|\xi, \zeta_x)
\]

Let \( q^*_c=q^*_w \), we have \( w^* = p-\gamma(p-c) \). In the situation when the supplier set a wholesale price \( w^* \), the retailer’s order quantity enables the supply chain to achieve the best profit performance. If both the retailer and the supplier achieve a better profit performance with the loss sharing contract comparing to that in the decentralized way, the loss sharing contract coordinates the supply chain.

Hence, while both the supplier and the retailer are willing to cooperate, the following conditions must be also satisfied

\[
\begin{align*}
\Pi'_r &\geq \Pi_r, \\
\Pi'_c &\geq \Pi_c, \\
\end{align*}
\]

**Theorem 1.** When the supply chain achieves coordination under the loss sharing contract, the supplier’s and the retailer’s profits are both proportional to the supply chain’s profit that \( \Pi'_r = (1-\gamma)\Pi'_c \) and \( \Pi'_c = \gamma \Pi'_c \), respectively. Furthermore, the following conditions should be satisfied that

\[
\begin{align*}
&\begin{align*}
&\max\left[0, \frac{(w^*-c)q^*_r-(w^*-c)q^*_c-L_e}{L_e} \right] \leq \gamma \leq \min\left\{1, \frac{(p-w^*)q^*_c-(p-w^*)q^*_r+L_e}{L_e} \right\} .
\end{align*}
\end{align*}
\]

As we mentioned before that the information of \( \xi \) is provided by the supplier for the retailer to forecast the market demand. Thus, providing \( \xi \) can be a kind of strategic behavior of the supplier. In specific, when the supplier is optimistic to the market demand she can provide positive information, \( \xi^*_s \), to the retailer to induce him order more quantity, and if the supplier is pessimistic to the market demand, she can show negative signal, \( \xi^*_n \), to the retailer to decrease the order quantity because of the risk of overstocking. Based on prospect theory (Kahneman et al.1979) that the negative information shows larger
influence to the retailer than the positive information, we have $\xi^+ > \xi^+$. Thus, the CDF under the collaborative forecasting scenario with positive or negative information are $F(x|\xi^+ \cup \xi^+_n)$ and $F(x|\xi^- \cup \xi^-_n)$, respectively. In our paper, Demand is stochastically increasing in the demand signal, i.e., $F(x|\xi_n) < F(x|\xi)$ for all $\xi_n > \xi_j$ (Donohue, 2000).

**Lemma 1.** The retailer orders more products from the supplier when he receives an optimistic signal of market demand than that with a negative signal.

Comparing with the non-collaborative forecasting scenario, the supply chain can be coordinated by the loss sharing contract both with the positive information (with a lower order quantity) or negative information (with a higher order quantity) provided by the supplier. We can calculate the supply chain optimal order quantity with the supplier’s positive information or negative information that

$$q^+_c = F^{-1}(\frac{p-c}{p-s}|\xi^+ \cup \xi^+_n)$$
$$\text{and } q^-_c = F^{-1}(\frac{p-c}{p-s}|\xi^+ \cup \xi^-_n). \tag{10}$$

**Theorem 2.** When the supply chain achieves coordination by the loss sharing contract, the supplier will provide positive information to the retailer to forecast market demand when $p > c + \Delta L / \Delta q$, otherwise, the negative information will be provided when $p \leq c + \Delta L / \Delta q$. Where

$$\Delta q = q^+_n - q^-_n > 0$$
$$\Delta L = (p-s)[\int_{q^+_n}^{q^-_n} (q^+_n - x) f(x|\xi^+_n + \xi^-_n) dx - \int_{q^+_n}^{q^-_n} (q^+_n - x) f(x|\xi^+_n + \xi^-_n) dx] > 0. \tag{11}$$

Theorem 2 shows that when the retail price of the product is high $(p > c + \Delta L / \Delta q)$, the supplier will show a positive signal to the retailer in order to fully satisfy the market demand. In specify, the optimal order quantity in Eq. (10) realizes that a higher retailer price induces the retailer orders fewer products from the supplier and at the same time reduces the risk of overstocking, which means the retailer is “timid” to the market demand. The optimistic information from the supplier can neutralize such conservative attitude of retailer and make the information become more objective. Contrary, a lower retail price $(p \leq c + \Delta L / \Delta q)$ makes retailer over confident to sells more products which increases the risk of overstocking. Fortunately, the more “calm” supplier can reduce this risk by showing a negative demand signal of the market to the retailer.

**Collaborative Forecasting with fair-minded retailer**

In this section, we investigate the situation where the retailer is fairness concern. As “people like to help who are helping them and to hurt those who are hurting them” (Rabin, 2001), the fairness concern retailer may show inequity aversion for taking more loss. We assume that the retailer’s equitable loss of demand uncertainty with collaborative forecasting is $\eta L$, where $0 \leq \eta \leq 1$ represents the maximal loss sharing rate of the retailer.

If the loss which the retailer shares under the collaborative forecasting scenario, $\gamma L$, is higher than the equitable loss, a disadvantageous inequality occurs, which will result in a disutility for the retailer in the amount of $\alpha$ per-unit difference in the two losses. The parameter $\alpha > 0$ captures the retailer’s inequity aversion to the disadvantageous inequality.
Algebraically, similar to the framework introduced by Cui et al. (2007), the retailer’s utility function with fairness concern is

$$U^r = \Pi^r - \alpha (\gamma - \eta)L_c.$$  \hfill (12)

where the $U^r$ and $\Pi^r$ mean the retailer’s utility under inequity aversion and the retailer’s profit under collaborative forecasting scenario. Such inequity aversion of loss sharing will raise the threshold of collaboration of forecasting, an inequity-averse retailer cannot collaborate with a much “greedy” partner. It is appropriate to note that our assumptions in this section are closer to the reality of business world and considering the fairness has much more substance than a simple mathematic representation (Cui, 2007). If $\gamma \leq \eta$, $U^r = \Pi^r$. Thus, the situation changes into the case without fairness concern. Our purpose in this section is to investigate the effects of fairness concern on the supply chain. When $\gamma > \eta$, the objective function of the retailer changes into

$$\max_{q^r} U^r = \Pi^r - \alpha (\gamma - \eta)L_c. \quad \text{s.t. } \gamma > \eta.$$  \hfill (13)

**Lemma 2.** Under the collaborative forecasting scenario with an fair-minded retailer, when the retailer feels unfair for taking more loss, $\gamma > \eta$, his optimal order quantity is

$$q^r = \frac{p-w}{(p-s)(\gamma + \alpha(\gamma - \eta))} [\xi, \zeta].$$  \hfill (14)

where $q^r$ means the retailer optimal order quantity under inequity aversion. From Eq. (14) we can easily get $\frac{\partial q^r}{\partial \alpha} < 0$ and $\frac{\partial q^r}{\partial (\gamma - \eta)} < 0$, which means a more fairness concern retailer will order fewer from the supplier and a larger difference between his equitable loss and actual loss will also make him ordering fewer. It is easy to understand that if the retailer takes more loss from the supply chain, he will reduce his order quantity from the supplier to reduce the risk of loss, and the more loss he takes makes him more sensitive to the loss, therefore, his order further decreases.

**Lemma 3.** Comparing with the retailer’s order quantity without fairness concern, the inequity-averse retailer will increase his order quantity when he feels slightly inequity i.e., $\alpha < (1-\lambda)/(\gamma - \eta)$, and he will decrease the order quantity from the supplier when he feels heavily inequity, $\alpha \geq (1-\lambda)/(\gamma - \eta)$.

The lemma 4 shows something interesting that when the retailer feels slightly inequity for the loss taking, he keeps “silent” and orders more to make up his loss. Interestingly, the retailer pushes envelope when he feels heavily inequity to his loss sharing, he revenges the supplier by decreasing the order quantity which makes them internecine to their profits.

**Theorem 3.** The loss sharing contract can still coordinate the supply chain if the supplier reduces her wholesale price, i.e., $w^c = w - \alpha (\gamma - \eta)(p-c)$, to the inequity-averse retailer when $\gamma > \eta$ and the loss sharing rate satisfies
\[
\max \left\{ \frac{(w^*_w - c)q^{*w} - (w^* - c)q^*_r + L^r + L^w}{L^w} \right\} \leq \gamma \leq \min \left\{ \frac{(p - w^*_w)q^{*w} - (p - w^*)q^*_r + \eta L^w + L^r}{(1 + \alpha)L^w} \right\},
\]

where \( L^w = (p - s) \int_0^{q^{*w}} (q^{*w} - x)f(x|\xi, \zeta)dx \).

**Discussion and conclusions**

In this paper, we propose loss sharing contract in the case of collaborative forecasting in the newsvendor model. As in the traditional newsvendor model, numerous literatures share the common assumption that the retailer bear all the risks and loss caused by the uncertain demand. Utilizing the loss sharing contract, both the supplier and retailer collaboratively forecast the market demand then share the loss caused by the deviation and in some situation the supply chain can achieve coordination. Also, as fairness is an important norm that often motivates and regulates channel relationships, we incorporate fairness concerns to the supply chain.

Some meaningful results are found that: (i) the supplier’s and the retailer’s profits are both proportional to the supply chain’s profit when the supply chain achieves coordination in some situation under the loss sharing contract. Interestingly, the proportion of profit distribution is the same with the ratio of bearing the loss. (ii) since the supplier can offer positive or negative information to the retailer in the collaborative forecasting, the supplier can control the order quantity by deliberately providing egoistic signal. For example, when the retail price of the product is high, the supplier will show a positive signal to the retailer in order to fully satisfy the market demand. (iii) when the retailer is fair-minded, the loss sharing contract can still coordinate the supply chain only if the supplier reduces her wholesale price and the inequity-averse retailer will increase his order quantity when he feels slightly inequity while decrease the order quantity from the supplier when he feels heavily inequity.

It’s obvious that our research has deduced some useful insights, there are still several directions deserving future research. First, we only consider the retailer is fair minded. Since the supply chain we assumed consists of a supplier and a retailer, the model can be expanded to involve both fair-minded supplier and retailer. Second, it could be expanded a more complex models in which the supply chain is composed by one supplier and two retailers. As the two retailers are also related to fairness, which we called the peer-fairness, so it is interesting to study in such a background.

**References**


