Investment decision for supply chain resilience based on Evolutionary Game theory

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Abstract
We study the factors and improvement methods of supply chain resilience, then analyze the investment decisions between supply chain members based on the Evolutionary Game theory. We deduce that the government should use punitive and subsidy methods to control supply chain members’ behaviors and enhance the resilience of supply chain.

Key Words: supply chain resilience, supply chain disruption, Evolutionary Game theory

Introduction
Recently, supply chain disruption has been studied by experts and scholars in every vocation. Due to supply chain disruptions, operating cost rising and brand influence declining often occur. When some unexpected disasters events just like 911 happen, immeasurable economic loss and casualties will occur. In 2000, the Philips chip factory in US fired, causing the procurement of Ericsson interrupted, resulting in a direct loss of $400 million, and its impact force Ericsson's market share dropped from 12% to 9%. It is increasingly important to improve the supply chain resilience to reduce disruption losses.

In this paper, we first study the factors and improvement methods of supply chain resilience based on previous studies by other scholars. In order to improve supply chain resilience, companies need to invest in many aspects and only all members of supply chain invest, the resilience of the whole supply chain can be enhanced. Then we build the model of investment decision for resilience to analyze investment decision between suppliers and manufacturers based on the evolutionary game. After that, we use regulatory mechanisms to prompt all members of supply chain to invest for supply chain resilience.

For factors of supply chain resilience, Petti et al. (2010) considered that the factors of resilience can be classified as capability and vulnerability factors. If capabilities increase and vulnerabilities decrease, supply chain resilience will be improved. For the improvement methods of resilience, Lee et al. (2005) learned from total quality management, found that companies can achieve a high level of safety through proper management measures and process reengineering. Christopher and Peck proposed a few important principles which can help companies to enhance resilience. Sheffi (2001) found that there are 3 ways to enhance...
supply chain resilience: increase redundancy, enhance agility, and change corporate culture.

Evolutionary game theory combines game theory and dynamic evolution, studies the trends and stability of the fraction of members in the game. Recent years, many scholars use evolutionary game to study supply chain problems. Xiao et al. (2006) studied supply chain disruption and coordination based on evolutionary game. Zhu et al. (2007) studied the evolutionary game between core companies and governments in green supply chain. Because supply chain members have bounded rationality, they can not necessarily make the optimal decisions. So it is more practical to use evolutionary game to analyze this topic.

The factors and improvement methods of Supply chain resilience

As mentioned above, after getting elicitation from Petti’s study, we consider that firms should consider the vulnerability and capability factors of supply chain, and build the more balanced resilience. In our study, via researching previous studies of other scholars, we propose the specific methods to enhance resilience in three aspects. The classification is internal to the firm, external to the firm and virtual supply chain, shown as Table 1. It should be noted that, the virtual supply chain refers to information systems and information transmission system on the Internet which the actual supply chain relies on.

<table>
<thead>
<tr>
<th>Classification</th>
<th>The improvement methods</th>
<th>Meanings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal to the firm</td>
<td>Appropriate redundancy</td>
<td>Build the safety stock of raw material and final product; Keep extra production ability and workers etc.</td>
</tr>
<tr>
<td></td>
<td>Supply chain flexibility</td>
<td>Redesign and standardize product and procedures; Reduce the types of component; Make flexible contract etc.</td>
</tr>
<tr>
<td></td>
<td>Supply chain agility</td>
<td>Reduce reaction time and total time of materials moving on the supply chain; Control of inventories and production schedules.</td>
</tr>
<tr>
<td></td>
<td>Visibility of the firm</td>
<td>Business intelligence; Information gathering; Technology upgrades and other means.</td>
</tr>
<tr>
<td></td>
<td>Dispersibility of the firm</td>
<td>Decentralize decision-making, production capabilities, personnel and other critical resources.</td>
</tr>
<tr>
<td></td>
<td>Risk analysis and prevention</td>
<td>Establish some relevant departments; Develop disruption reaction mechanisms; Reserve relevant resources etc.</td>
</tr>
<tr>
<td></td>
<td>Disruption response measures</td>
<td>Establish emergency response departments; Develop interrupt response mechanism; Deploy stuff and resources effectively etc.</td>
</tr>
<tr>
<td></td>
<td>Risk management culture</td>
<td>Risk assessment and continuous communication among employees; Knowledge sharing and learning.</td>
</tr>
<tr>
<td>External to the firm</td>
<td>Design resilience in supply chain</td>
<td>Design resilience in bottle neck and critical path; Balance the cost, efficiency and risk based on company strategy etc.</td>
</tr>
<tr>
<td></td>
<td>Collaboration between firms</td>
<td>Exchange information with suppliers and customers; Collaborative planning, forecasting and replenishment etc.</td>
</tr>
<tr>
<td></td>
<td>Fitness to the environment</td>
<td>Fitness to the external environment such as natural, political, economic, legal and cultural environment.</td>
</tr>
<tr>
<td>Virtual supply chain</td>
<td>Design resilience in supply chain</td>
<td>Design resilience in bottle neck and critical nodes of the software system; Reserve interfaces to expand functions and</td>
</tr>
<tr>
<td>software system</td>
<td>coverage; Consider firm’s strategies when designing etc.</td>
<td></td>
</tr>
<tr>
<td>----------------</td>
<td>-----------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>Redundancy of supply chain system</td>
<td>Keep redundancy of hardware and software; Ensure appropriate speed and bandwidth of network; Deploy relatively staff to administrate the virtual supply chain system etc.</td>
<td></td>
</tr>
<tr>
<td>Protect virtual supply chain against risks</td>
<td>Purchase anti-virus software to prevent computer viruses; Increase investment to prevent network disruptions, and information distortion in transmission etc.</td>
<td></td>
</tr>
</tbody>
</table>

**Model Description**

In order to enhance supply chain resilience, firms should invest in many aspects such as increasing redundancy, improving flexibility and agility of supply chain and so on, all of which require capital investment. This paper differs from prior papers, study supply chain members’ investment decision for supply chain resilience based on evolutionary game (Webull 1998), and propose some mechanisms to control members’ behavior.

**Assumptions**

In this paper, we study the supply chain constituted by suppliers and manufacturers (denoted by S and M). In our model, an individual of supplier population (for short individual S) randomly plays a one-shot game with a matched individual of manufacturer population (for short individual M) every time. Every individual has two kinds of investment strategies for resilience: invest (for short I) or not invest (for short N). By invest for resilience, firms can reduce supply chain disruption losses, and improve revenues of them. Assumptions are:

1. If both individuals don’t invest for resilience, their revenues when disruptions occur are \((1-a_i)R_i\), \((1-b_i)R_m\); \(R_i(R_i>0)\) and \(R_m(R_m>0)\) are respectively normal revenue of individual S and individual M when disruptions do not occur. \(a_i(a_i>0)\) and \(b_i(b_i>0)\) are respectively revenue loss rate of individual S and individual M when disruptions occur.

2. If both individuals invest and improve the resilience, their profitability can be improved, then the disruption losses of them can be reduced. Their revenues are \(R_s-a_iR_s+a_2R_s-C_s\) and \(R_m-b_iR_m+b_2R_m-C_m\), respectively; \(a_2R_s(a_2R_s>0)\) and \(b_2R_m(b_2R_m>0)\) are compensation value of resilience for individual S and individual M after they invest, respectively. \(a_2(a_2>0)\) and \(b_2(b_2>0)\) are compensation rate of individual S and individual M, respectively. \(C_s(C_s>0)\) and \(C_m(C_m>0)\) are investment cost of individual S and individual M, respectively.

3. If individual S invests and individual M does not invest, the revenue of individual S is \(R_s-a_iR_s+a_2R_s-C_s\) when disruption occurs. Because individual M can benefit from individual S’ invest, for example manufacturers’ procurement can be guaranteed in quality, and more prices can be selected when purchasing, the cost of manufacturers can be reduced. Due to individual M’ free-riding, the revenue of individual M when disruption occurs is \(T_m(T_m>R_m-b_iR_m)\).

4. Similarly, if individual M invests and individual S does not invest, the revenue of individual M is \(R_m-b_iR_m+b_2R_m-C_m\). Due to individual S’ free-riding, the revenue of it is \(T_s(T_s>R_s-a_iR_s)\).

For the convenience of demonstration, we denote \(R_i^a\) and \(R_m^a\) as individual S’ and individual M’ revenue when all of them don’t invest, in other words, replace \((1-a_i)R_i\) with
$R_s$ and $(1-b_1)R_m$ with $R_s^m$, respectively; denote $R_s^b$ and $R_m^b$ as individual $S'$ and individual $M'$ revenue when all of them invest, in other words, replace $(1-a_1 + a_2)R_s - C_s$ with $R_s^b$ and $(1-b_1 + b_2)R_m - C_m$ with $R_m^b$, respectively. According to the above assumptions, we establish the payoff matrix of individual $S$ and individual $M$, which is shown in Table 2.

<table>
<thead>
<tr>
<th>S</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>I</strong></td>
<td>$I$</td>
</tr>
<tr>
<td><strong>N</strong></td>
<td>$T$, $(1-b_1 + b_2)R_m - C_m$</td>
</tr>
</tbody>
</table>

Replicator dynamic system

According to the payoff matrix, we can get the replicator dynamic system of supplier population (for short population $S$) and manufacturer population (for short population $M$). When the game begins, we let the fraction of individuals $S$ using strategy $I$ be $p$, so the fraction of individuals $S$ using strategy $N$ is $1-p$; Similarly we let the fraction of individuals $M$ using strategy $I$ be $q$, so the fraction of individuals $M$ using strategy $N$ is $1-q$.

The revenue functions when individual $S$ chooses strategy $I$, strategy $N$, and the average revenue function of population $S$ respectively are:

\[ U_1 = q[(1-a_1 + a_2)R_s - C_s] + (1-q)[(1-a_1 + a_2)R_s - C_s] \]  
\[ U_2 = qT + (1-q)(R_s - a_1R_s) \]  
\[ \bar{U} = pU_1 + (1-p)U_2 \]  

Similarly, the revenue functions when individual $M$ chooses strategy $I$, strategy $N$, and the average revenue function of population $M$ respectively are:

\[ V_1 = p[(1-b_1 + b_2)R_m - C_m] + (1-p)[(1-b_1 + b_2)R_m - C_m] \]  
\[ V_2 = pT_m + (1-p)(R_m - b_1R_m) \]  
\[ \bar{V} = qV_1 + (1-q)V_2 \]  

Because individuals have bounded rationality, after a period of evolution they will re-choose strategies based on the revenues they get, the individuals who have less revenues will change their strategies. Then the fraction $p$ and $q$ will change over time. According to Malthusian dynamic system (Friedman. 1991), we can obtain that the replicator dynamic system for population $S$ and population $M$ (denoted by system1) is

\[ \]
\[
\begin{align*}
\frac{dp}{dt} &= p(U_i - U) = p(1 - p)[q(1 - a_i)R_i + a_2R_i - C_i - qT_i] \\
\frac{dq}{dt} &= q(V_i - V) = q(1 - q)[p(1 - b_i)R_m + b_2R_m - C_m - pT_m]
\end{align*}
\]  \hspace{1cm} (7)

**Proposition 1.** For the system given by Eq. (7), we have the following results:

1. \((0, 0), (0, 1), (1, 0)\) and \((1, 1)\) are its equilibria.

2. If \(C_i < a_2R_i < T_i - (1 - a_i)R_i + C_i, C_m < b_2R_m < T_m - (1 - b_i)R_m + C_m\), \((p_0, q_0)\) is also an equilibrium of system 1.

   \[p_0 = \frac{b_2R_m - C_m}{T_m - (1 - b_i)R_m}, q_0 = \frac{a_2R_i - C_i}{T_i - (1 - a_i)R_i}.\]

**Proof.** For system 1, when \(dp/dt = 0, dq/dt = 0\), obviously we get that \((0,0), (0,1), (1,0)\) and \((1,1)\) are its equilibria. If \(a_2R_i\) and \(b_2R_m\) satisfy \(C_i < a_2R_i < T_i - (1 - a_i)R_i + C_i, C_m < b_2R_m < T_m - (1 - b_i)R_m + C_m\), we can easily know that \(0 < p_0 < 1, 0 < q_0 < 1\), thus \((p_0, q_0)\) is also an equilibrium.

*Evolutionarily stable strategies*

The equilibriums of system 1 which we have got are not necessarily ESS. So we should use Jacobian method (Hofbauer et al. 1998) to judge them. The Jacobian matrix of system 1 is

\[
J = \begin{bmatrix}
q(1 - a_i + a_2)R_i - C_i - qT_i & q(1 - q)(1 - b_i)R_m - T_m & (1 - 2p) & (1 - 2q) \\
q(1 - q)(1 - a_i)R_i - T_i & q(1 - q)(1 - b_i)R_m - T_m & p(1 - a_i)R_i - T_i & p(1 - b_i)R_m - C_m - pT_m
\end{bmatrix}
\]  \hspace{1cm} (8)

The local stability of equilibriums is determined by both determinant and trace. The determinant of the Jacobian matrix is

\[
detJ = (1 - 2p)[q(1 - a_i + a_2)R_i - C_i - qT_i](1 - 2q)[p(1 - b_i + b_2)R_m - C_m - pT_m] - p(1 - p)[(1 - a_i)R_i - T_i]q(1 - q)[(1 - b_i)R_m - T_m]  \hspace{1cm} (9)
\]

The trace of the Jacobian matrix is

\[
trJ = [q(1 - a_i + a_2)R_i - C_i - qT_i](1 - 2p) + [p(1 - b_i + b_2)R_m - C_m - pT_m](1 - 2q)  \hspace{1cm} (10)
\]

Furthermore, we can derive the following.

**Proposition 2.**

1. If \(0 < a_2R_i < C_i, 0 < b_2R_m < C_m\), the equilibrium \((0, 0)\) is the ESS of system 1.

2. If \(C_i < b_2R_m < T_m - (1 - b_i)R_m + C_m, 0 < a_2R_i < C_i\), \((0, 1)\) is the ESS of system 1.

3. If \(C_i < a_2R_i < T_i - (1 - a_i)R_i + C_i, 0 < b_2R_m < C_m\), \((1, 0)\) is the ESS of system 1.

4. If \(C_i < a_2R_i < T_i - (1 - a_i)R_i + C_i, C_m < b_2R_m < T_m - (1 - b_i)R_m + C_m\), \((p_0, q_0)\) exists, \((0, 1)\) and \((1, 0)\) are the ESS of system 1.

5. If \(a_2R_i > T_i - (1 - a_i)R_i + C_i, b_2R_m > T_m - (1 - b_i)R_m + C_m\), \((1, 1)\) is the ESS.

**Proof.** According to the Jacobian method, we know that if \(trJ < 0, detJ > 0\), the equilibrium of the replicator system is ESS. We can get the values of \(trJ\) and \(det\) on the equilibriums of system 1, then judge the stability of equilibriums and get the ESS. The
The analysis of ESS in every case is shown in Table 3.

<table>
<thead>
<tr>
<th>Case</th>
<th>Equilibrium</th>
<th>$TrJ$</th>
<th>$DetJ$</th>
<th>Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$(0,0)$</td>
<td>$-$</td>
<td>$+$</td>
<td>ESS</td>
</tr>
<tr>
<td></td>
<td>$(0,1)$</td>
<td>$-$</td>
<td>$+$</td>
<td>Saddle point</td>
</tr>
<tr>
<td></td>
<td>$(1,0)$</td>
<td>$-$</td>
<td>$+$</td>
<td>Saddle point</td>
</tr>
<tr>
<td></td>
<td>$(1,1)$</td>
<td>$+$</td>
<td>$+$</td>
<td>Unstable</td>
</tr>
<tr>
<td>2</td>
<td>$(0,0)$</td>
<td>$-$</td>
<td>$+$</td>
<td>ESS</td>
</tr>
<tr>
<td></td>
<td>$(0,1)$</td>
<td>$-$</td>
<td>$+$</td>
<td>Saddle point</td>
</tr>
<tr>
<td></td>
<td>$(1,0)$</td>
<td>$-$</td>
<td>$+$</td>
<td>Saddle point</td>
</tr>
<tr>
<td></td>
<td>$(1,1)$</td>
<td>$+$</td>
<td>$+$</td>
<td>Unstable</td>
</tr>
<tr>
<td>3</td>
<td>$(0,0)$</td>
<td>$-$</td>
<td>$+$</td>
<td>Saddle point</td>
</tr>
<tr>
<td></td>
<td>$(0,1)$</td>
<td>$-$</td>
<td>$+$</td>
<td>Saddle point</td>
</tr>
<tr>
<td></td>
<td>$(1,0)$</td>
<td>$-$</td>
<td>$+$</td>
<td>ESS</td>
</tr>
<tr>
<td></td>
<td>$(1,1)$</td>
<td>$+$</td>
<td>$+$</td>
<td>Unstable</td>
</tr>
<tr>
<td>4</td>
<td>$(0,0)$</td>
<td>$+$</td>
<td>$+$</td>
<td>Unstable</td>
</tr>
<tr>
<td></td>
<td>$(0,1)$</td>
<td>$-$</td>
<td>$+$</td>
<td>ESS</td>
</tr>
<tr>
<td></td>
<td>$(1,0)$</td>
<td>$-$</td>
<td>$+$</td>
<td>ESS</td>
</tr>
<tr>
<td></td>
<td>$(1,1)$</td>
<td>$+$</td>
<td>$+$</td>
<td>Unstable</td>
</tr>
<tr>
<td>5</td>
<td>$(0,0)$</td>
<td>$+$</td>
<td>$+$</td>
<td>Unstable</td>
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<tr>
<td></td>
<td>$(0,1)$</td>
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<td>$+$</td>
<td>Saddle point</td>
</tr>
<tr>
<td></td>
<td>$(1,0)$</td>
<td>$-$</td>
<td>$+$</td>
<td>Saddle point</td>
</tr>
<tr>
<td></td>
<td>$(1,1)$</td>
<td>$+$</td>
<td>$+$</td>
<td>ESS</td>
</tr>
</tbody>
</table>

**Evolutionary analysis results**

Based on the analysis above, we get the evolutionary game process of population $S$ and population $M$ in five cases, shown in Figure 1, respectively. From the phase diagrams of dynamic evolution of system 1, we can obtain the following results.
(1) If \(0 < a_s R_s < C_s, 0 < b_s R_m < C_m,\) \(a_s R_s\) and \(b_s R_m\) are all less than the invest cost \(C_s, C_m\). So the revenue after individual \(S\) and individual \(M\) invest are less than the revenue when they don’t invest. See Fig. 1(a), (1,1) is unstable, (0,1) and (1,0) are saddle points, (0,0) is an ESS of system 1. In other words, \((N, N)\) is an ESS of system 1, population \(S\) and population \(M\) all don’t invest.

(2) If \(C_m < b_s R_m < T_m - (1-b_s)R_m + C_m, 0 < a_s R_s < C_s,\) compensation value \(a_s R_s\) is less than invest cost \(C_s\), so individual \(S\) won’t invest. \(b_s R_m\) is larger than invest cost \(C_m\), but \(R_m^b\) is less than free-riding revenue \(T_m\). However, individual \(S\) does not invest so that the free-riding behavior of individual \(M\) cannot be implemented, investing for resilience is optimal for individual \(M\). See Fig. 1(b), (1,1) is unstable, (0,0) and (1,0) are saddle points, (0,1) is an ESS of system 1. Therefore population \(S\) does not invest and population \(M\) invest.

(3) Similarly, if \(C_s < a_s R_s < T_s - (1-a_s)R_s + C_s, 0 < b_s R_m < C_m,\) \(b_s R_m\) is less than \(C_m\), so individual \(M\) won’t invest. \(a_s R_s\) is larger than \(C_s\), but \(R_s^b\) is less than \(T_s\). See Fig. 1(c), (1,1) is unstable, (0,0) and (0,1) are saddle points, (1,0) is an ESS of system 1. In other words, population \(M\) does not invest, population \(S\) invest.

(4) If \(C_s < a_s R_s < T_s - (1-a_s)R_s + C_s, C_m < b_s R_m < T_m - (1-b_s)R_m + C_m,\) \(a_s R_s\) and \(b_s R_m\) are larger than \(C_s\) and \(C_m\). Because \(T_s\) and \(T_m\) are larger than \(R_s^b\) and \(R_m^b\), population \(S\) and population \(M\) are more inclined to implement free-riding behavior. But if they all don’t invest, the free-riding behavior can’t be implemented. Therefore system 1 will eventually evolve to two ESS. See Fig. 1(d), (0,0) and (1,1) are unstable points, \((p_0, q_0)\) is a saddle point, (0,1) and (1,0) are ESS of system 1. In other words, population \(S\) does not invest and population \(M\) invest or population \(M\) does not invest and population \(S\) invest.

(5) If \(a_s R_s > T_s - (1-a_s)R_s + C_s, b_s R_m > T_m - (1-b_s)R_m + C_m,\) \(R_s^b\) and \(R_m^b\) are larger than \(T_s\) and \(T_m\), \(a_s R_s\) and \(b_s R_m\) are larger than \(C_s\) and \(C_m\). See Fig. 1(e), (0,0) is unstable, (0,1) and (1,0) are saddle points, (1,1) is the ESS. In other words, populations all invest.

**Evolutionary analysis under government control**

In order to improve the resilience of the whole supply chain and maximize the revenue, we should let (1, 1) be the unique ESS, in other words, populations all invest for resilience. Then we consider that the government should get involved in the management of supply chain, set up mechanisms to regulate the behaviors of populations.

**Evolutionary analysis under punitive mechanism**
If $C_s < a_2 R_s < T_s -(1-a_1) R_s + C_s, C_m < b_2 R_m < T_m -(1-b_1) R_m + C_m$, the ESS of system 1 are (0,1) and (1,0), the resilience of the whole supply chain cannot be fully improved, so the supply chain cannot achieve optimal operational efficiency. Therefore government should set up punitive mechanism to punish free-rider to prompt them to invest. For case 4 of system 1, assume punishment for free-rider is $P^*$, the payoff matrix is shown as Table 4.

**Table 4-Payoff matrix of individual S and individual M under punitive mechanism**

<table>
<thead>
<tr>
<th>S</th>
<th>M</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$(1-a_1+a_2) R_s - C_s , (1-b_1+b_2) R_m - C_m$</td>
<td>$(1-a_1+a_2) R_s - C_s , T_m - P^*$</td>
</tr>
<tr>
<td>N</td>
<td>$T_m - P^* , (1-b_1+b_2) R_m - C_m$</td>
<td>$(1-a_1) R_s , (1-b_1) R_m$</td>
</tr>
</tbody>
</table>

The replicator dynamic system under punitive mechanism (denoted by system 2) is

$$\begin{align*}
\frac{dp}{dt} &= p(1-p)[q(1-a_1) R_s + a_2 R_s - C_s - q(T_s - P^*)] \\
\frac{dq}{dt} &= q(1-q)[p(1-b_1) R_m + b_2 R_m - C_m - p(T_m - P^*)]
\end{align*}\tag{11}$$

**Proposition 3.** For the system 2 given by Eq. (11), we derive the following:
(1) $(0, 0), (0, 1), (1, 0)$ and $(1, 1)$ are its equilibriums.
(2) If inequation (12) satisfies, $(p^*_0, q^*_0)$ is also an equilibrium of system 2,

$$\begin{align*}
p^*_0 &= \frac{b_2 R_m - C_m}{T_m -(1-b_1) R_m - P^*} , \quad q^*_0 = \frac{a_2 R_s - C_s}{T_s -(1-a_1) R_s - P^*} .
\end{align*}$$

$$\begin{align*}
\max[T_s -(1-a_1+a_2) R_s + C_s , T_m -(1-b_1+b_2) R_m + C_m ] < P^* \\
< \min[T_s -(1-a_1) R_s , T_m -(1-b_1) R_m]
\end{align*}\tag{12}$$

**Proof.** For system 2, we let $dp / dt = 0, dq / dt = 0$, obviously, we get that $(0, 0), (0, 1), (1, 0)$ and $(1, 1)$ are its equilibriums. If inequation (12) satisfies, we can easily know that $0 < p^*_0 < 1, 0 < q^*_0 < 1$, then $(p^*_0, q^*_0)$ is also an equilibrium.

**Proposition 4.** The necessary and sufficient condition under which $(1, 1)$ is the unique ESS of system 2 is:

$$P^* > \max[T_s -(1-a_1+a_2) R_s + C_s, T_m -(1-b_1+b_2) R_m + C_m]\tag{13}$$

Obviously, inequation (13) is equivalent to $P^* > \max[T_s - R_s, T_m - R_m]$.

**Proof.** First we know that the necessary and sufficient condition is $trJ < 0, detJ > 0$. Easily, we get $-[(1-a_1+a_2) R_s - C_s -(T_s - P^*)] - [(1-b_1+b_2) R_m - C_m -(T_m - P^*)] < 0$, and $[(1-a_1+a_2) R_s - C_s -(T_s - P^*)][(1-b_1+b_2) R_m - C_m -(T_m - P^*)] > 0$, then we can get $(1-a_1+a_2) R_s - C_s -(T_s - P^*) > 0$ and $(1-b_1+b_2) R_m - C_m -(T_m - P^*) > 0$. Therefore we see
that \( P^* > \max[T_s - (1 - a_i + a_2)R_s + C_s, \ T_m - (1 - b_i + b_2)R_m + C_m]\). Secondly when inequation (13) satisfies, based on Table 5 we can obtain \((0, 0)\) is an unstable point of system 2, \((0, 1)\) and \((1, 0)\) are saddle points, \((1, 1)\) is the unique ESS of system 2.

### Table 5: Stability analysis of equilibriums for system 2

<table>
<thead>
<tr>
<th>Equilibrium</th>
<th>(trJ)</th>
<th>(detJ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>((0, 0))</td>
<td>((a_2R_s - C_s) + (b_2R_m - C_m))</td>
<td>((a_2R_s - C_s)(b_2R_m - C_m))</td>
</tr>
<tr>
<td>((0, 1))</td>
<td>([(1 - a_i + a_2)R_s - C_s - (T_s - P^*)] - (b_2R_m - C_m))</td>
<td>(-(b_2R_m - C_m)[(1 - a_i + a_2)R_s - C_s - (T_s - P^*)])</td>
</tr>
<tr>
<td>((1, 0))</td>
<td>([(1 - b_i + b_2)R_m - C_m - (T_s - P^*)] - (a_2R_s - C_s))</td>
<td>(-(a_2R_s - C_s)[(1 - b_i + b_2)R_m - C_m - (T_s - P^*)])</td>
</tr>
<tr>
<td>((1, 1))</td>
<td>[-[(1 - a_i + a_2)R_s - C_s - (T_s - P^<em>)] - ] ([(1 - b_i + b_2)R_m - C_m - (T_s - P^</em>)])</td>
<td>[(1 - a_i + a_2)R_s - C_s - (T_s - P^*)]</td>
</tr>
</tbody>
</table>

This illustrates that when \( P^* > \max[T_s - R_s, \ T_m - R_m]\), the punishment \( P^* \) is larger than the difference between free-rider’s revenue and the revenue when firms invest, the revenue when firms invest is larger than the revenue of free-riding, then supply chain members all tend to invest for resilience, the resilience of the whole supply chain can be improved.

#### Evolutionary analysis under subsidy mechanism

If \(0 < a_2R_s < C_s\), \(0 < b_2R_m < C_m\), the ESS of system 1 is \((0,0)\), the resilience cannot be improved, and this will form a vicious circle. So the government should set up subsidy mechanism to complement the firms to prompt them to invest. For case 1 of system 1, assume subsidy for firms is \( S^* \), then we get the payoff matrix shown as Table 6.

### Table 6: Payoff matrix of individual \( S \) and individual \( M \) under subsidy mechanism

<table>
<thead>
<tr>
<th></th>
<th>( I )</th>
<th>( N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I )</td>
<td>((1-a_i+a_2)R_s-C_s+S^<em>, \ (1-b_i+b_2)R_m-C_m+S^</em>)</td>
<td>((1-a_i+a_2)R_s-C_s+S^*, \ T_m)</td>
</tr>
<tr>
<td>( N )</td>
<td>( T_s, \ (1-b_i+b_2)R_m-C_m+S^*)</td>
<td>((1-a_i)R_s, \ (1-b_i)R_m)</td>
</tr>
</tbody>
</table>

The replicator dynamic system under subsidy mechanism (denoted by system 3) is

\[
\begin{align*}
\frac{dp}{dt} &= p(1-p)[q(1-a_i)R_s + a_2R_s - C_s + S^* - qT_s] \\
\frac{dq}{dt} &= q(1-q)[p(1-b_i)R_m + b_2R_m - C_m + S^* - pT_m]
\end{align*}
\]

(14)

**Proposition 5.** For the system 3 given by Eq. (14), we derive the following:

1. \((0, 0), \ (0, 1), \ (1, 0)\) and \((1, 1)\) are its equilibriums.
2. If inequation (15) satisfies, \((p_0^2, q_0^2)\) is also an equilibrium of system 3.

\[
p_0^2 = \frac{b_2R_m - C_m + S^*}{T_m - (1-b_i)R_m}, \quad q_0^2 = \frac{a_2R_s - C_s + S^*}{T_s - (1-a_i)R_s}.
\]
\[
\max[(C_s - a_s R_s)(C_m - b_m R_m)] < S^* \\
\min[T_s - (1 + a_s - a_i)R_s + C_s, \ T_m - (1 + b_m - b_i)R_m + C_m]
\] (15)

**Proof.** Similar to Proposition 3.

**Proposition 6.** The necessary and sufficient condition under which (1, 1) is the unique ESS of system 3 is:

\[
S^* > \max[T_s - R_{s2}, \ T_m - R_{m2}]
\] (16)

**Proof.** Similar to Proposition 4.

This illustrates that when \(S^* > \max[T_s - R_{s2}, \ T_m - R_{m2}]\), the subsidy \(S^*\) is larger than the difference between free-rider’s revenue and the revenue when firms investing. Suppliers or manufacturers selecting the invest strategy will get more revenue than free-riding, then they all tend to invest. So when \(a_s R_s\) and \(b_m R_m\) are relatively small, government should use subsidy method to control the behaviors of firms to let them invest for resilience.

**Conclusion**

This paper first studies the factors and improvement method of supply chain resilience, then analyzes the investment decision of suppliers and manufacturers based on Evolutionary Game theory. We find that with the increase of \(a_s R_s\) and \(b_m R_m\), there appears different ESS such as (0, 0), (0, 1), (1, 0), (1, 1). In order to improve the resilience of the whole supply chain and maximize the revenues of supply chain members, we consider that government should use punitive and subsidy methods to prompt firms to invest. Thus the resilience can be improved and the disruption loss can be reduced.

There are several areas to extend this research. First we can study resilience improvement methods of multi-echelon supply chain. Secondly, we can consider coordination contracts of supply chain in the investment decision based on Evolutionary Game theory.

**Reference**