Little’s Law, a fundamental tool to define supply chain metrics

Sangdo (Sam) Choi
Harry F. Byrd, Jr. School of Business, Shenandoah University, Winchester, VA 22601, USA
knowhow.schoi@gmail.com

Abstract
We review Little's Law to define inventory turnover and other asset-turnovers. We relate Little's Law with EOQ model and turnovers. We suggest a new definition of cash-to-cash cycle for manufacturers, because the current one is for retailers. We analyze several industries using an earns-turns matrix based on Little's Law.

Keyword: Little’s Law · Inventory Turnover · Earns-Turns Matrix

IS INVENTORY AN ASSET OR LIABILITY?

Supply chain professionals regard inventory as liability, whereas it appears as an asset on a financial statement, balance sheet. Inventory investments are substantial in a balance sheet, 10 ~ 30% of total investments for manufacturers, 25 ~ 40% for retailers. As a new technology or product penetrates in the market, the value of inventory decreases. Invested funds in inventories are tied up until sales occur. Physical spaces are required to keep inventories and information systems are used to keep track of inventory records. Funds tied up with inventories are hidden cash or reserves that, if managed efficiently, can be utilized to finance the current and future expansion of the business. The cash used to finance the account receivable and inventories should be freed as soon as possible and in amount as much as possible, so that the available cash can be invested in other more profitable avenues and thereby gain and hold a competitive edge over other players in supply chains (Attari and Raza 2012).

Inventory can be measured in four different ways for operations management. First, supply chain professionals use physical unit to manage inventory. Stock keeping unit is typically used and depends upon shape or material type. Secondly, accounting professionals use dollar amounts to measure and valuate inventory. These two metrics are static measure without respect to company size, or revenue amount. Combining revenue or sales quantity with inventory, not only supply chain professionals but also accounting professionals are able to measure how fast inventory moves in the market and how long average inventory supply to the market using a dynamic measure. Inventory turnover (ITR) and days-of-supply (DOS) are the other two inventory-related performance metrics. ITR is defined as the number of times that an inventory cycles during the fiscal year (Blackstone Jr. 2013) as the following:

\[ ITR = \frac{\text{Cost of Goods Sold}}{\text{Average Inventory}} \]  

(1)
Typically, beginning and ending balances of the fiscal year are used to compute the average inventory. Cost of goods sold (CGS) reflects the cumulative sales amount during the same fiscal year that the average inventory is taken into account. The cumulative sales amount during the fiscal year (i.e., flow rate) is valued at the cost, not the price at which revenue is valued. DOS (i.e., flow time) is the reciprocal value of ITR.

Figure 1 depicts all supply chain transactions. If a retailer buys a product from a supplier, then product inventory account is debited and increases in value and account payable (AP) is credited and increases in value. The retailer has a liability to the supplier via AP. Later, the retailer sells the product to a customer. Inventory is credited and decreases in value; CGS is debited and increases in value. Inventory and CGS accounts are corresponding each other and should be posted at the same time when a sales transaction takes place. At the same time, account receivable (AR) is debited and increases in value; Revenue is credited and increases in value. AR and revenue are also corresponding each other. If the customer pays account payables (AR) for cash, Cash is debited and increases in value; AR is credited and decreases in value. If the retailer pays AP for cash, Cash is credited and decreases in value; AP is debited and decreases in value. All balance sheet accounts increase and decrease depending upon nature of transaction during the fiscal period, whereas all income statement accounts set to zero at the begin of the fiscal period and increases monotonously.

We relate inventory-related measures to Little’s Law in next sections. The objectives of this paper are to (1) clearly explain analogy of Little’s Law to the relationship between ITR and DOS, (2) apply Little’s Law to economic order quantity (EOQ) model, (3) review different account payable turnovers (APT) and suggest corporate APT and supply chain APT, (4) suggest new Ears-Turns matrix with ITR and profitability, and (5) analyze Dow-Jones indexed companies and their competitors with the earns-turns matrix.

This paper is organized as follows. Next, we explain how Little’s Law defines inventory turnover and other asset turnovers. Then, we address the difference and similarity between Little’s Law and EOQ model. Later, we introduce account payable turnover and cash conversion cycle (CCC). We provide an earns-turns matrix to analyze supply chain firms and industries and shows industry analyses using the earns-turns matrix. Lastly, we conclude our discussion.

**LITTLE’S LAW DEFINES INVENTORY TURNOVER AND OTHER ASSET-TURNOVERS**
Little’s Law applies to supply chain and defines the relationship among average inventory level, average flow rate (e.g., CGS), and average flow time (e.g., DOS). Originally, Little’s Law deals with queueing systems, consisting of discrete objects (e.g., customers, patients, job requests), each of which arrives at a rate (e.g., arrival rate) to the system, joins one or more queues, and receives service at a rate (e.g., service rate) and exits from the system at a rate (e.g., leaving rate) (Little 1961, 2011). Little’s Law relates the average time spent in the system of objects and the average number of objects in the system via the average rate of arrival to the system. Law 1 establishes the formal statement of Little’s Law.

**Law 1 (Little 1961)** Let \( W \) be the average time spent in the system of objects; \( L \), the average number of objects in the system; \( \lambda \), the average rate of arrival to the system, respectively. Then \( L = \lambda W \) holds.

Little’s Law holds under steady state conditions (Little 1961) as well as practical situations (Stidham Jr. 1974, Whitt 1991, Wolff 2011). Little’s Law applies to individual queues and networks and subnetworks. Little’s Law has become widely used in not only Queueing (Stidham Jr. 1974, Whitt 1991), but also operations management (Hopp and Spearman 2000, Cachon and Terwiesch 2004) and computer architecture (Lazowska et al 1984, Gunther 2010). Queuing theory takes into account the arrival rate that is endogenous, whereas operations management typically takes into account the leaving rate that is determined by exogenously. The arrival rate, which is endogenous, is a primary input to queuing theory, whereas the leaving rate, determined by exogenously, is a primary input to the operations management. Manufacturers target output first. The operations manager decides the arrival to the system to achieve the output target (Little and Graves 2008). Queuing theory and operations management generally use different notation for Little’s Law parameters (Hopp and Spearman 2000, Cachon and Terwiesch 2004, George 2002). Hopp and Spearman (2000) offer alternative words and expanded definitions for manufacturing firms: Throughput (TH) is the average output of a production process per unit time, work in process (WIP) as the inventory between the start and the end points of a product routing, and cycle time (CT) as the average time between the beginning and the end of the routing. Corollary 2 establishes the expanded Little’s Law for a production process.

**Corollary 2 (Hopp and Spearman 2000)**

\[
TH = \frac{WIP}{CT}
\]  

(2)

where \( TH \) is the average output per unit time at a product routing, \( WIP \) is the inventory between the start and the end points of the routing, and \( CT \) is the average time between the beginning and the end of the routing.

George (2002) has alternative descriptors, calling \( TH \) the velocity of products through the plant in items per hour, \( WIP \) the average number of items in process in the plant, and \( CT \) the process lead time, i.e., the average hours each item spends in the plant. George (2002) uses Little’s Law to provide a methodology for improving organizational effectiveness. Corollary 3 establishes Little’s Law for process innovation.
Corollary 3 *(George 2002)*

\[
\text{Process lead time} = \frac{\text{Average number of Things in process}}{\text{completions per hour}}
\]

Corollary 4 establishes Little’s Law for supply chain firms. Cachon and Terwiesch (2004) introduced the following equation:

**Corollary 4 *(Cachon and Terwiesch 2004)*

\[
I = RT
\]

where \(I\) is average inventory, equivalent to \(L\); \(R\) is average flow rate, equivalent to \(\lambda\); \(T\) is average flow time, equivalent to \(W\), respectively.

Hence, Little’s Law is expressed in different forms such as equations (2), (3), or (4). In other words, \(L: \lambda: W = W IP: CT = I: R: T.\) (1) is the retailer version of Little’s Law, because transactions between the retailer and the supplier are taken into account. Manufacturer’s version of Little’s Law involves three types of inventory in the manufacturer: raw material (RM), work-in-process (WIP), and final goods inventory (FGI) as shown Figure 2. Take into accounts the following value-added activities of the manufacturer cross supply chain: purchasing, production, final assembly, and sales. Figure 2 depicts changes of the related accounts cross supply chain. We skip explanation about purchasing and sales transactions, because they are the same as retailer’s transactions in Figure 1.

The manufacturing firm transforms RM into WIP incurring direct material (DM), direct labor (DL), and manufacturing overhead (OH) costs. If RM is taken for production, DM cost increase in value. The manufacturing firm transforms WIP into FGI incurring cost of goods manufactured (CGM). In a similar fashion, AP, RM, WIP, FGI, AR, and cash accounts change over transactions because they are balance sheet accounts. DM, CGM, CGS, and Revenue increase in value as transactions take place. As CGS is the flow rate of FGI, so CGM is the flow rate of WIP and DM is the flow rate of RM. Little’s Law assume that the arrival rate to the system is equal to the departure rate from the system, which is again the arrival rate to the next system. However, \(DM < CGM < CGS\) because supply chain activities are value-added. In practice, the manufacturer cannot synchronize the consumption rate of RM, the production rate of WIP, and the sales speed of FGI all the time. To take into account the inventory turnover of RM (WIP or
FGI), DM (CGM or CGS) should be used respectively. Corollary 5 establishes Little’s Law for respective ITRs in a manufacturing firm.

**Corollary 5**

\[
ITR_{RM} = \frac{DM}{\text{Average } RM} \quad (5)
\]

\[
ITR_{WIP} = \frac{CGM}{\text{Average } WIP} \quad (6)
\]

\[
ITR_{FGI} = \frac{CGS}{\text{Average } FGI} \quad (7)
\]

where \( ITR_{RM} \) is the RM inventory turnover, \( ITR_{WIP} \) is the WIP inventory turnover, and \( ITR_{FGI} \) is the FGI inventory turnover.

However, most public company on NYSE, Nasdaq, or else do not provide investors with DM or CGM information. Corollary 6 shows that DOS with total inventory is a lower bound of the real inventory days, shown as the followings:

\[
ITR = \frac{CGS}{\text{Average } FGI + \text{Average } WIP + \text{Average } RM} \quad (8)
\]

\[
DOS = \frac{\text{Average } FGI + \text{Average } WIP + \text{Average } RM}{CGS} \quad (9)
\]

**Corollary 6** 365 \times \frac{\text{Average } FGI + \text{Average } WIP + \text{Average } RM}{CGS} is a lower bound of the real inventory days, the sum of RM inventory days, WIP inventory days, and FGI inventory days.

**Proof** Because \( DM < CGM < CGS \), we have

\[
\frac{\text{Average } FGI + \text{Average } WIP + \text{Average } RM}{CGS} < \frac{\text{Average } RM}{DM} + \frac{\text{Average } WIP}{CGM} + \frac{\text{Average } FGI}{CGS} \quad (10)
\]

Little’s Law applies to AR turnover with \( I = \text{average AR}, R = (\text{net}) \text{ sales revenue}, \) and \( T = \text{AR days, the reciprocal value of ART} \). The meaning of ART is how fast the firm transforms AR into cash. Corollary 7 establishes the definition of ART and Corollary 8 asset turnover, respectively.

**Corollary 7**

\[
ART = \frac{\text{Net Revenue}}{\text{Average AR}} \quad (11)
\]

**Corollary 8**

\[
\text{Asset Turnover} = \frac{\text{Net Revenue}}{\text{Average Total Asset}} \quad (12)
\]

where the average total asset is the average value of beginning and ending balance of the total
LITTLE’S LAW AND ECONOMIC ORDER QUANTITY MODEL

We relate Little’s Law to economic order quantity (EOQ) model. Suppose that the firm has an annual demand, \( D \), and would like to decide the optimal order quantity, \( Q \). Each cycle length \( T \) is defined as \( Q/D \), which is equivalent to \( I=RT \) because of \( I: R: T=Q: D: T \). The flow rate, \( R \) is equal to the annual demand, \( D \). The difference is that \( R \) is measured at cost and \( D \) is quantity, a physical measure. The average inventory, \( I \) is equal to the order quantity, \( Q \). Little’s Law involves the average inventory because there are lots transactions related to inventory during the fiscal period. EOQ involves the order quantity, \( Q \), not average inventory \( Q/2 \), because EOQ has only one incoming transaction and many sales (outgoing) transactions during each cycle. The flow time of Little’s Law, \( T \) is equal to the cycle length of EOQ. Corollary 9 shows that the inventory turnover of Little’s Law is equivalent to the number of cycles of EOQ.

**Corollary 9** ITR under EOQ makes use of order quantity \( Q \), not average inventory \( Q/2 \).

\[
ITR = \frac{1}{R} = \frac{D}{Q}
\]  

(13)

Another difference between EOQ and Little’s Law is that EOQ model allows zero inventory, which is rare in practice. Hopp and Spearman (2000) show that the \( WIP \) never drops to zero in manufacturing systems and that there might be an explicit control rule that maintains some target level or range of \( WIP \).

Cachon and Terwiesch (2004) and Chopra and Meindl (2014) use the ending inventory or beginning inventory rather than average inventory to compute ITR. This approach is valid only if the beginning inventory and the ending inventory is the same or similar, which means by that the firm never grows in inventory size. Cachon and Terwiesch (2004), Anupindi et al (2011) use the average inventory (i.e., \( Q/2 \)) to compute ITR of EOQ model. The cycle length of EOQ, \( T \) is different from 2 the average flow time according to Cachon and Terwiesch (2004), Anupindi et al (2011).

Combining the inventory account from balance sheet and CGS from income statement, we graphically relate EOQ to Little’s Law. The upper graph in Figure 3 depicts changes of CGS and inventory over a fiscal year. An income statement account, CGS is set to zero when a new fiscal period starts and increases in value as each sales transaction takes place. A balance sheet account, inventory increases in value as the firm buys (or produces) goods to be sold and decreases in value as the firm sells a product to a customer. Solid lines are actual values and dashed lines are average values. The length to the point where two dashed lines meet is an order cycle length of EOQ, which is \( T \) in the below graph of Figure 3. The slope of CGS of the above graph is equal to the slope of demand rate of the below graph. The height of each order cycle of the below graph is equal to the average inventory of the above graph. The number of order cycles is ITR, i.e., how many times orders the firm makes during the fiscal period with the average inventory.
A business can be viewed as a process of obtaining cash to buy assets (i.e., financing activities), converting cash to assets (i.e., investing activities) and back to cash (i.e., operating activities). Supply chain professionals are more interested in operating activities such as buying RM, transforming RM into WIP and finally FGI, and selling FGI than financing activities or investing activities. The faster the firm operates, the more efficient its operations are. Operating cash cycle (OCC) is a measure of operational speed, length of time the firm’s cash is tied up working capital before money is finally returned when customers pay for the products sold or services rendered (Churchill and Mullins 2001). Cash-to-cash cycle (CCC) is the difference between cash inflow and cash outflow, measuring the time between cash outflows for resources and cash inflows from product sales (Akgun and Gurunlu 2010). Hager (1976) present the ideas of CCC initially and Gentry et al (1990) develop a weighted CCC. Generally, CCC is calculated by subtracting the debt turnover period from the sum of ITR and ART periods (Besley and Brigham 2007, Akgun and Gurunlu 2010, Attari and Raza 2012, Stewart 1995, Schilling 1996, Soenen 1993, Filbeck and Krueger 2005). CCC is a dynamic measure of liquidity management, because it combines not only balance sheet entries (e.g., AR, inventory, AP) but also income statement entries (e.g., CGS, net revenue) (Jose et al 1996, Akgun and Gurunlu 2010, Gitman and Sachdeva 1982).

Cash outflow is measured by days of payables outstanding, or AP days. AP makes the delivery of goods or services of services and their subsequent payment separated, which allows the firm to reduce the uncertainty of the payments (Garcia-Teruel and Martinez-Solano 2010). The higher the AP, the higher size, higher cost of financial debt, more growth in sales, more investment in current asset in general, and in AR and stock in particular (Garcia-Teruel and Martinez-Solano 2010). However, the calculation of AP days varies. Supply chain wide AP days involves the payment for the purchase of RM or a product, whereas corporate wide AP days takes into account AP, salaries, benefits, and other payments. Payment cycle across supply chain is calculated by AP turnover (APT). Akgun and Gurunlu (2010) define APT as the following:
\[ APT = \frac{CGS}{Average\ AP} \]  

(14)

where the average AP involves beginning and ending balances of the fiscal period. (14) involves CGS as the flow rate. However, CGS is the flow rate of sales, not payment or purchasing. (14) is valid for retail firms. The total payment to the suppliers should be taken into account as flow rate, which does not appear in any financial statement, but an internal cost accounting report.

If beginning inventory and ending inventory are the same, purchasing is equal to CGS. It is an ideal or a non-desirable to have a steady sales history over years. Equation (14) is invalid for manufacturing firms. A retailer does not transform a product, but buys and sells it. A manufacturer transforms RM to WIP; WIP to FGI, which is finally sold to a customer. The values of inventories across supply chain increases, i.e., \( RM < WIP < FGI \). Corollary 10 establishes Account Payable Turnover (APT) for a manufacturing firm.

**Corollary 10**

\[ APT = \frac{DM}{Average\ AP} \]  

(15)

where \( APT \) is account payable turnover, \( DM \) is direct material cost, and \( AP \) is account payable.

We suggest that corporate wide payment cycle be the weighted average of AP days, Interest days, Employees’ payout days, and so on, including both primary and secondary activities. The weight is based on CGS and SGA. Corollary 11 establishes firm-wide payment cycle. Payments take place for all liabilities, especially current liabilities in the same fiscal year.

**Corollary 11**

\[ AP\ days = \frac{365 \times Current\ Liabilities}{CGS+SGA} \]  

(16)

**EARNs-TURNS MATRIX WITH INVENTORY TURNOVER AND VALUE CHAIN INCOME**

The firm mainly seeks for profit, while it should satisfy customers’ needs and wants and/or innovate products. Profitability measures have been regarded as the first to-go measure to analyze the firm’s performance, how efficient the firm leverages assets to generate earnings. Return on Asset (ROA) and profit margin are typical measures of profitability (Mottner and Smith 2009). The economic characteristics of each industry affect firm’s financial measures such as ROA, profit margin, and inventory turnover. Grocery stores have ROA of 4.03% and food industry ROA of 8.4% on average (Wahlen et al 2008). An earns-turns matrix explains what are industry characteristics and how a firm takes a strategic position in the matrix. ROA measure involves not only profitability measure, but also operational efficiency by decomposition as follows:

\[ ROA = \frac{Earning\ before\ interest\ and\ tax\ (EBIT)}{Average\ Asset} = \frac{EBIT}{Net\ Sales} \times \frac{Net\ Sales}{Average\ Asset} \]  

(17)

The elements of ROA are profit margin and asset turnover, to examine firms’ earning per sales dollar and efficiency in using assets to generate sales. Profit margin for ROA involves
EBIT or operating income, whereas profit margin of income statement is the ratio of gross profit to net sales. Gross profit is from the primary activities cross supply chain. Both primary and supportive activities cross supply chain contribute to generation of revenue. Hence, operating income or Earning before Interests and Tax (EBIT) is used. Operating income or EBIT does not involve supply chain costs properly. First, depreciation is non-cash expense. Other expenses and non-operating income do not involve operating activities but finance or investment activities. We prescribe value chain income (VCI) as the following:

\[ VCI = Revenue - CGS - SGA - R&D \]  

(18)

We analyze U.S. manufacturers and retailers using two dimensions of profit margin and asset turnover. To understand different industry characteristics, we analyze Dow Jones Indexed (DJI) firms with an earns-turns matrix. Figure 4 depicts the earns-turns analysis for DJI companies except banking or other financial institutions. Oracle (ORCL), McDonald (MCD), Apple (AAPL), Fedex (FDX), United Parcel Services (UPS), Home Depot (HD), Wal Mart (WMT), Best Buy (BBY), and Costco Wholesale (COST) are on the best ROA frontier. Technology-based industries including IT, Chemical, and Pharmaceutical firms are more profitable supply chain, whereas retailers are more lean supply chain pursuing high asset turnover. Technology-based firms generate high value chain margin with low asset turnover. Microsoft (MSFT), Google (GOOG), Apple, and IBM are IT-related firms. Chemical or pharmaceutical firms are similar to tech-firms. Pfizer (PFE), Johnson & Johnson (JNJ), Merck & Co (MRK), Proctor & Gamble (PG), 3 M (MMM) are examples of chemical or pharmaceutical firms. Retail firms operate efficiently with low value chain margin. Costco Wholesale, Best Buy, Wal-Mart, Target (TGT), Amazon (AMZN), and Home Depot are representative retailers. Traditional manufacturing firms or industrial goods suppliers have low value chain margin and operate less efficiently than other industries. Alcoa (AA), General Motor (GM), Advance Micro Device (AMD), Nucor (NUE), Caterpillar (CA), and Boeing (BA) are examples.

Figure 4 Earns-Turns matrix of DJI listed firms
CONCLUSION

We review Little’s Law in accounting perspective and verify asset turnovers such as ART and ITR. We analyze DJI listed firms using Earns-Turns matrixes. Little’s Law is a Newton’s Law in supply chain management to explain relationships among inventory, flow time, and turnover. Inventory plays an important role in defining turnover, days of supply, and cash-to-cash cycle. EOQ applies to define the relationship and sales (flow rate), order quantity (inventory), and cycle time (flow time). The earns-turns matrix with DJI listed firms show that each industry has its own positioning, either profitable supply chain (high profit margin and low ITR) or lean supply chain (high turnover and low profit margin).

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