Locating electric vehicles charging station based on cooperative coverage

Jun Yang, Lian Wu
School of Management, Huazhong University of Science & Technology, Wuhan 430074, China
wlandyliu@163.com

Abstract
The study follows a concept of “cooperative cover” and vehicle-refueling logics, using a mixed integer programming to formulate the recharging stations location model to meet user's demand with minimum cost. We will apply it to a reasonable program and analysis the difference of the new model compared with traditional ones.

Key words: Electric vehicle; Cooperative cover; Mixed integer programming

INSTRUCTION

Today, air pollution is becoming more and more serious, and the increment of the CO₂ changed the greenhouse effect of the world worse. About 65% of CO₂ in the atmosphere come from the use of energy, with 21% from the burning of fossil fuels in the transportation industry. As the energy and environment situation is increasingly serious today, many countries and regions attach importance to electric vehicles because of its advantages of energy saving and clean. Over the past 10 years, with the development of powered battery technology, electric vehicles have been developed to a certain scale in Europe, America, Japan and other developed countries. China has also proposed a development plan to reach 5 million electric vehicles (including hybrid vehicles, pure electric vehicles, fuel cell vehicles, etc.). So, to speed up the construction of urban charging stations and the scientific layout planning has a very important significance for the future development of electric vehicles.

Location theory has attracted significant research effort since the beginning of 1960s. Since then, they have put forward a lot of locating problems of different types and a variety of solutions which can be applied to practice. In 1964, C.A.Rogers first proposed the concept of covering location. Then, C.Revelle and R.L.Church were the first to put forward “The maximal covering location problem”(MCLP).MCLP addresses the issue of locating a certain number of facilities in order to maximize the number of demand points that can be covered. In a MCLP model, a
demand point is assumed to be covered completely if located within the critical distance of the facility and not covered at all outside of the critical distance. The maximal cover location problem has proved to be one of the most useful facility location models from both theoretical and practical points of view. And the problem usually can be solved by linear programming when the optimal solution is integer and by branch and bound. But the MCLP model also has its limitations. Since the optimal solution to a MCLP is likely sensitive to the critical distance ("coverage radius") R, determining a critical distance value when the coverage does not change in a crisp way from "fully covered" to "not-covered" at a specific distance may lead to erroneous results. On this basis, O.Berman proposed a new concept that is "The gradual covering decay location problem" (GMCLP) in 2002. In this model, two coverage radius l and u (l≤u) are defined for each demand point. If the closest facility is located within distance l of the demand point, then the demand point is considered to be fully covered. If the closest facility is further than u away, none of the demand is covered. When the distance to the closest facility is d, with l<d<u, the demand point is considered to be partially covered, with the level of coverage given by the "coverage decay function" f(d)∈[0,1].

Location models with cover objectives are one of the “classical” types of facility location models. Coverage models above all have one thing in common that whether or not a demand point is covered is determined by a single (the closest) facility. We thus refer to the standard model as the individual coverage model. Then the concept of cooperative coverage was put forward in 2009. In a cooperative-covering family of location model, the coverage objective could be viewed to result from the following mechanism: each facility emits a “signal” which decays over the distance. A demand point is covered if the signal it receives is sufficiently strong; if its aggregate signal exceeds a given threshold. Thus facilities cooperate to provide coverage, as opposed to classical coverage location model where coverage is only provided by the closest facility. It is shown that this cooperative assumption is appropriate in a variety of applications. In comparison, “cooperative coverage” is more reasonable than “individual coverage”.

There are two main types of coverage models: one is to cover the demand of user with minimum cost; another one is to maximally satisfy the needs of users under the condition of certain cost.

The above two cases can be described as the following two models under the assumption of cooperative coverage.

The first case:

\[
\begin{align*}
\text{Minimize} & \quad \sum_{i\in N} c_i x_i \\
\text{s.t.} & \quad \sum_{j\in N} f(d_i(x_j))x_j \geq TH_i \quad \forall i \in N \\
& \quad \sum_{i\in N} p_i H_i \geq \alpha P \
\end{align*}
\]  

(1)

(2)

(3)
\[ H_i \in \{0,1\} \quad \forall i \in N \]  
\[ x_i \in \{0,1\} \quad \forall i \in N \]  

The second case:  

\[
\text{Minimize} \sum_{i \in N} c_i x_i 
\]

s.t.
\[
\sum_{j \in N} f(d_i(x_j)) x_j \geq TH_i \quad \forall i \in N 
\]
\[
\sum_{i \in N} c_i x_i \leq B \quad \forall i \in N 
\]
\[ H_i \in \{0,1\} \quad \forall i \in N \]  
\[ x_i \in \{0,1\} \quad \forall i \in N \]  

Let \( N \) be the demand space, \( T \) be the threshold signal strength required to achieve coverage, and \( X \) be the location space. We use \( f(d) \) to represent the strength of the signal at a distance \( d \) from the facility, where \( f(d) \) is non-negative, and non-increasing in \( d \). Thus a demand point \( i \in N \) receives a signal of coverage \( f(d) \) from the facility \( j \). The overall signal of coverage received by \( i \in N \) from the location space \( X \) is represented by constraint (2) and (5). And \( c_i \) represents cost of locating a station at point \( i \); \( x_i \) means whether locating at \( i \) or not; \( H_i \) means whether point \( i \) is located or not; \( p_i \) means the population at node \( i \); means the proportion of covered users.

**MODEL FORMULATION**

In this paper, we study the location problem of electric vehicle charging station. We will apply the concept of cooperative coverage to the research of location of electric vehicle charging stations, thus making a combination of cooperative coverage and charging logics to establish a new model for locating charging stations. We have already introduced the concept of cooperative coverage in detail before, now we will introduce the concept of refueling logics before the formulation of the model.

**Vehicle refueling logics**

Before the introduction of the model, we first introduce a concept that how the vehicle get charged, and that is refueling logics before mentioned above. This concept has been described
Wang Ying-Wei (2009) in detail. Vehicle range is the key factor for determining the locations and the number of refueling stations for completing a journey, especially a long-distance one. Following is a simple example to show the characteristics of vehicle refueling. Fig.1 shows the shortest path of 220km between O and D, with three nodes. Assuming that a vehicle’s maximum range is 100km and it is full charged at the original. We must establish a charging station at node 1, because it takes 50km of electricity from O to node 1 and only 50km of electricity left; however there are 70km between node 1 and 2. So if the vehicle can not be charged at node 1, it will not travel to node 2. Assuming there is charging station at node 1, the vehicle get full charged at node 1. Then there is no need to build a charging station at node 2, as the vehicle has enough power to complete the journey left from node 1 to 3. However, it is out of energy at node 3, so a charging station must be established to arrive at the destination.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.jpg}
\caption{Fig.1.}
\end{figure}

From this simple example, the refueling logics of the vehicle can be obtained. Assuming a vehicle is at a path (i-j-k), and then the vehicle refueling logics can be described in the following statements.

(A) If a vehicle m at node i can arrive at node j, it means that its remaining fuel must fully cover the distance between node i and j. If not, the vehicle must be refueled at node i, and thus a station must be located at this point. This can be formulated as \( b_{im} + r_{im} \geq d_{ij} \), where \( b_{im} \) is the remaining amount of fuel at node i for a vehicle m, \( r_{im} \) is the refueling amount of fuel at node i for a vehicle m, and \( d_{ij} \) is the distance between node i and j.

(B) The remaining fuel at node j is calculated by the remaining fuel plus the refuel at the prior node i, and minus the fuel consumption for the distance travelled between them. This can be described as \( b_{jm} = (b_{im} + r_{im}) - d_{ij} \).

(C) If a vehicle must be refueled at node i, the fuel being refueled can not be greater than the amount of fuel consumed at this point. That is, the refueling amount of fuel(\( r_{im} \)) is equal to or less than the refueling capacity(\( \beta \)) minus the remaining amount of fuel(\( b_{im} \)), which can be described as \( r_{im} \leq \beta - b_{im} \). Actually, \( b_{im} \) and \( r_{im} \) are no less than zero(\( b_{im} \geq 0, r_{im} \geq 0 \)).

Model introduction

We have introduced two main types model of location, one is to minimally the cost, another is to maximally the demand of users. In this paper, the two cases are combined to establish a double-objective function model. In the object function of the model, a weight coefficient \( w \) is set up, and when \( w \) takes the limit values 0 or 1, the model is simplified as a model of a single-objective function model, that is, only the cost or the user’s demand are
considered. The new model is formulated as:

Sets

- $N$: set of all nodes
- $M$: set of vehicles travelling along paths

Indices

- $i,j$: nodes or locations
- $m$: electric vehicle

Parameters

- $c_i$: the cost of locating a station at node $i$
- $p_i$: the population at node $i$
- $\beta$: refueling capacity
- $l$: electricity consumption per unit distance
- $w$: weight coefficient (totaling 1)
- $d_{ij}$: distance between nodes $i$ and $j$
- $T$: threshold value
- $k$: a large positive integer

Decision variables

- $H_i$: 1, if demand at point $i$ is covered, and 0, otherwise
- $x_i$: 1, if a station is located at node $i$, and 0, otherwise
- $b_{im}$: amount of fuel remaining at node $i$ for an EV $m$ on a path
- $r_{im}$: amount of fuel replenishing at node $i$ for an EV $m$ on a path
- $Y_{im}$: 1, if an EV $m$ is refueled at node $i$ on a path, and 0, otherwise

The location problem of cooperative coverage is formulated, as follows:

\[
\begin{align*}
\text{Minimize} & \quad w \sum_{i \in N} c_i x_i - (1-w) \sum_{i \in N} p_i H_i \\
\text{s.t.} & \quad \sum_{j \in N} f\left(d_i(x_j)\right)x_j \geq TH_i \quad \forall i \in N \\
& \quad b_{im} = (b_{jm} + r_{jm}) - d_{ij}l \quad \forall i \in N, \forall m \in M \\
& \quad r_{im} \leq \beta - b_{im} \quad \forall i \in N, \forall m \in M \\
& \quad r_{im} \leq Y_{im}\beta \quad \forall i \in N, \forall m \in M \\
& \quad \sum_{m \in M} Y_{im} \leq k x_i \quad \forall i \in N \\
& \quad H_i \in \{0,1\} \quad \forall i \in N 
\end{align*}
\]
Constraint (1) represents the objective function with weighted dual objectives of minimum cost and maximum population coverage. Constraint (2) represents that if a demand node \( i \) is covered, the overall signal of coverage received by a demand node \( i \) must be no less than the threshold value. To maximize the second objective (population coverage), \( H_i \) tends to be 1. If all \( H_i \) are equal to 1, that can ensure all the demand nodes are covered. However, the cost of locating stations will be large, this depends on the weight coefficient assigned to the objective. Constraint (3) and (5) represent the refueling logics problem mentioned above. Constraint (6) indicates that if a vehicle is charged at node \( i \), there must be located a station at the node. Constraint (7) and (10) defines the range of values of the variables.

**CASE STUDY**

In this part, we will apply the model to a specific case. To solve the problem, we will use CPLEX 12.6 to get the solution of the model with branch and bound method. Then this paper will further study the location model proposed, based on the analysis of solution of the case. We will take the Wuhan city which consists of 15 regions as an example, to illustrate the construction of charging station with the least cost to maximize the satisfaction of the demand.

As we know, Wuhan city can be divided into 15 areas, they are Jiangxia, Hongshan, Wuchang, Hanyang, Qiaokou, Jianghan, Qingshan, Huangpi, Xinzhou, Dongxihu, Caidian, Jiangan, Hannan, East Lake Development Zone and Wuhan Development Zone.

**Data setting**

In order to solve the case above, we selects 15 paths among the fifteen regions. They are shown in Table 1. The maximum range of electric vehicle is between 200km to 500km, it is set to 250km in the process of solving. At the beginning of each path, we assume the electric vehicle is full charged, so it is no need to locate a station at the beginning point. To simplify the process of solving, it is assumed that the cost of locating a station is the same at each point and only take the basis cost of establishment that is about 1125000 dollars. The number of users each node is shown in Table 2. Assumed that the electric vehicle is always travelling along the shortest route between two points to complete the entire journey from original to destination. The distance between any two sites is shown in Table 3. Weight coefficient \( w \) and the threshold value \( T \) are ranging from 0 to 1 depending on the actual situation. The function of cooperative coverage \( f(d) \) is assumed to be a linear function, and the independent variable is the distance between any two nodes and changes from 0 to \( D \), and the degree of coverage is reduced from 1 to 0 uniformly.
### Table 1 - Paths

<table>
<thead>
<tr>
<th>Number of node</th>
<th>Node i</th>
<th>Path</th>
<th>Nodes of the path</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Jiangxia</td>
<td>1</td>
<td>1–8–10–11–13–1</td>
</tr>
<tr>
<td>2</td>
<td>Hongshan</td>
<td>2</td>
<td>2–9–11–2</td>
</tr>
<tr>
<td>3</td>
<td>Wuchang</td>
<td>3</td>
<td>3–13–6–3</td>
</tr>
<tr>
<td>4</td>
<td>Hanyang</td>
<td>4</td>
<td>4–8–6–5–4</td>
</tr>
<tr>
<td>5</td>
<td>Qiaokou</td>
<td>5</td>
<td>5–9–7–1–5</td>
</tr>
<tr>
<td>6</td>
<td>Jianghan</td>
<td>6</td>
<td>6–13–10–8–6</td>
</tr>
<tr>
<td>7</td>
<td>Qingshan</td>
<td>7</td>
<td>7–9–8–2–1–14–7</td>
</tr>
<tr>
<td>8</td>
<td>Huangpi</td>
<td>8</td>
<td>8–13–15–6–1–8</td>
</tr>
<tr>
<td>9</td>
<td>Xinzhou</td>
<td>9</td>
<td>9–4–15–10–8–9</td>
</tr>
<tr>
<td>10</td>
<td>Dongxihu</td>
<td>10</td>
<td>10–13–15–12–10</td>
</tr>
<tr>
<td>11</td>
<td>Caidian</td>
<td>11</td>
<td>11–10–8–9–11</td>
</tr>
<tr>
<td>12</td>
<td>Jiangan</td>
<td>12</td>
<td>12–1–14–7–9–12</td>
</tr>
<tr>
<td>13</td>
<td>Hannan</td>
<td>13</td>
<td>13–15–12–1–9–8–13</td>
</tr>
<tr>
<td>14</td>
<td>East Lake</td>
<td>14</td>
<td>14–7–9–8–1–14</td>
</tr>
<tr>
<td>15</td>
<td>Wuhan Region</td>
<td>15</td>
<td>15–4–5–6–1–8–10–15</td>
</tr>
</tbody>
</table>

### Table 2 - Populations

<table>
<thead>
<tr>
<th>Number of node</th>
<th>Node i</th>
<th>Population</th>
<th>Number of vehicles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Jiangxia</td>
<td>644835</td>
<td>161209</td>
</tr>
<tr>
<td>2</td>
<td>Hongshan</td>
<td>1049434</td>
<td>262359</td>
</tr>
<tr>
<td>3</td>
<td>Wuchang</td>
<td>1266768</td>
<td>316692</td>
</tr>
<tr>
<td>4</td>
<td>Hanyang</td>
<td>584077</td>
<td>146020</td>
</tr>
<tr>
<td>5</td>
<td>Qiaokou</td>
<td>828644</td>
<td>207161</td>
</tr>
<tr>
<td>6</td>
<td>Jianghan</td>
<td>683492</td>
<td>170873</td>
</tr>
<tr>
<td>7</td>
<td>Qingshan</td>
<td>485375</td>
<td>121344</td>
</tr>
<tr>
<td>8</td>
<td>Huangpi</td>
<td>874938</td>
<td>218735</td>
</tr>
<tr>
<td>9</td>
<td>Xinzhou</td>
<td>848760</td>
<td>212190</td>
</tr>
<tr>
<td>10</td>
<td>Dongxihu</td>
<td>451880</td>
<td>112970</td>
</tr>
</tbody>
</table>
Solutions

As in the model, T\(w\), and the value of D and range is not unique, we first just consider one situation to solve the problem. And then we will give sensitivity analysis, when they take different values. When the four parameters above are 0.12, 0.8, 70 and 250, we can get the solution that the number of charging station is three, respectively are node 6(Jianghan), node 8(Huangpi), and node 12(Jiangan). There are eleven nodes covered, node 2(Hongshan), node 3(Wuchang), node 4(Hanyang), node 5(Qiaokou), node 6(Jianghan), node 7(Qingshan), node 8(Huangpi), node 10(Dongxihu), node 12(Jiangan), node 14(East Lake), node 15(Wuhan Region). The rate of demand node covered is 73.33\%(11/15), and percentage of user’s coverage is 79.28%.

We can take some demand nodes to verify the function of cooperative coverage, the distance between node 2, node 6, node 8 and node 12 are 10, 43 and 33. According to the coverage function, we can calculate the solution to prove that the solution is correct, and that is \((70-10)/70+(70-13)/70+(70-33)/70>T\). In the same way, we know that the distance between node 4 and 2, 6, 8 are 12, 45 and 13, obviously it satisfies the inequality above. Now we will consider the following two extreme cases, at a weight of 0, the model is simplified as a maximum coverage model based on cooperative cover. The number of refueling stations sited is four(6,7,8,15), and all of the fifteen nodes are covered, which can achieve a ratio of 100\%, the whole users are covered. In contrast, at a weight of 1, the model is just a model of cost optimization, with the minimum cost to meet the needs. The number of charging stations is three (6,9,13), number of nodes covered is ten(2,3,4,5,6,7,9,12,13,14). Ratio of nodes covered is 66.67\%, rate of user’s coverage is 73.41\%. This shows that when the value of w range from 0 to 1 gradually, the results is also constantly changing.

Next we will analyze the effect of different value of maximun driving range(\(\beta\)) and threshold T on the results of solution. The range is taken 250 and 300, and T is 0.7 and 0.9. So we can get four combinations of \(\beta\) and T (\(\beta\), T), thus (250, 0.7). (250, 0.9), (300, 0.7), (300, 0.9), to solve the problem in the four different cases. It has analyzed the effect of the weight on the results. Therefore, we will take a fixed value of w which is range from 0 to 1, thus 0.2. Table 4 is the result.

From Table 4, it shows that when w is certain, the number of stations is 3, 3, 2, and 2, the number of covered site is 15, 11, 12, and 10, the ratio of coverage is 100\%,79.28\%, 83.49\%, 75.21\% in the four cases. It is obvious that the number of located stations depends mainly on \(\beta\), with the range increasing, the number of the located stations is reducing. However, the coverage rate of users demand mainly depends on the value of T. When \(\beta\) is certain, the smaller T is, the larger the coverage rate is.
Table 3

<table>
<thead>
<tr>
<th>Weight</th>
<th>Stations</th>
<th>Nodes</th>
<th>Nodes of covered</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
<td>6,7,8,15</td>
<td>1 to 15</td>
<td>100%</td>
</tr>
<tr>
<td>0.12</td>
<td>3</td>
<td>6,8,12</td>
<td>1,2,3,4,5,6,7,8,9,10,12,14,15</td>
<td>94.60%</td>
</tr>
<tr>
<td>0.2</td>
<td>3</td>
<td>6,8,12</td>
<td>1,2,3,4,5,6,7,8,9,10,12,14,15</td>
<td>94.60%</td>
</tr>
<tr>
<td>0.4</td>
<td>3</td>
<td>7,8,15</td>
<td>2,3,4,5,6,7,8,10,12,14,15</td>
<td>79.28%</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>6,9,13</td>
<td>2,3,4,5,6,7,9,12,13,14</td>
<td>73.41%</td>
</tr>
</tbody>
</table>

Table 4

<table>
<thead>
<tr>
<th>(β,T)</th>
<th>Stations</th>
<th>Nodes</th>
<th>Nodes of covered</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>(250,0.7)</td>
<td>3</td>
<td>6,8,12</td>
<td>1 to 15</td>
<td>100%</td>
</tr>
<tr>
<td>(250,0.9)</td>
<td>3</td>
<td>6,8,12</td>
<td>2,3,4,5,6,7,8,10,12,14,15</td>
<td>79.28%</td>
</tr>
<tr>
<td>(300,0.7)</td>
<td>2</td>
<td>6,8</td>
<td>2,3,4,5,6,7,8,10,11,12,14,15</td>
<td>83.49%</td>
</tr>
<tr>
<td>(300,0.9)</td>
<td>2</td>
<td>6,8</td>
<td>2,3,4,5,6,7,8,10,12,15</td>
<td>75.21%</td>
</tr>
</tbody>
</table>

CONCLUSIONS

The study proposed a new location model of the charging station based on the concept of cooperative coverage and vehicle refueling logics. That makes up for the limitation of the traditional cover problem to some extent. The model was successfully applied to a practical case, and the influence of the parameter variation on the results in the model is also analyzed. The results of the sensitivity analysis show that the greater the vehicle's range, the fewer the number of stations that need to be sited, and the weight and threshold are also factors that influence this number and the site’s locations.

The greatest contribution of the paper is to extend the scope of the concept of coverage location by adding the constraints of nodal demand coverage based on cooperative cover. However, there are still some shortcomings in the establishment and solution of the model. For example, it is assumed that the cost of locating a station at each site is the same to solve the case easily. Since the objective is the minimum locating cost and the maximal coverage, the station cost and vehicle’s range are the critical factors to the solution of the model. Therefore, how to decrease the station cost and increase the range of an EV are important for the future development of the study in electric vehicles. And we also define the covering function as a linear function to solve the problem, which it is not entirely reasonable. These are what we should improve and study in the future.

Bibliography


