Model and algorithm for the simultaneous pickup and delivery vehicle routing problem with split loads

Li Kunpeng, Li Hanmei

(School of management, Huazhong University of Science and Technology, Wuhan 430074, China)

1021986216@qq.com

Abstract
This paper establishes a model for a large scaled vehicle routing problem with simultaneous pickup and delivery, the objective is to minimize total transportation cost. The initial feasible solution is obtained via a heuristic transportation efficiency based algorithm and improved by a local search algorithm based on variable neighborhood search.

Key words: Split Loads, Simultaneous Pickup and Delivery， Variable Neighborhood Search

INTRODUCTION

Vehicle routing problem is a key link in the process of logistics distribution. High transportation cost and low efficiency are common issues in the industry with multiple production plants. Take a tobacco manufacturer in China as an example, the company has eight production plants and 54 different warehouses that are located in different regions. Since there is no central warehouse, all the production materials have to be stored distributed in these warehouses. Each month, the Logistics Center needs to allocate the materials needed by each production plant according to the production plan, and the required materials are carried to the destination by trucks of the plant. Since all trucks are back empty, considerable waste is caused during material transportation. In order to increase the utilization of trucks in travel and reduce the total travel cost, we will research on the vehicle routing problem derives from materials allocation and transportation process of this company.

The allocation and transportation process is conducted in a joint distribution network which covers all plants (vehicle depots) and warehouses. According to the real situation of the company, we find that the feature of the vehicle routing problem can be concluded as: large scale, vehicles have to travel among 62 points to carry more than 50 different materials, the most important is that the inventory of all materials are limited. Split delivery, each warehouse has both pickup and delivery requirements, all demands are not required to be satisfied only by one time, but we have to decide the demand and supplying pairing within the network. Thus this problem can be defined as unpaired vehicle routing problem with multiple materials to be delivery and pickup split. It is obvious that this problem combines the characters of the Split Delivery Vehicle Routing Problem (SDVRP) and the Vehicle Routing Problem with Simultaneous Delivery and Pickup (VRPSDP).

Section 2 reviews literature related to the problem described. Section 3 presents a detailed description of the problem and a mathematical formulation. Section 4 shows in detail the solution algorithm, Section 5 describes the comparison between the computing results and the practical cost of the company in six months. Finally, section 6 presents the conclusions.

LITERATURE REVIEW

Since vehicle routing problem was introduced by Dantzing and Ramser in 1959, it has been a hot topic in the field of operation research and optimization scheduling. SDVRP and VRPSDP are two important branches of VRP.

SDVRP is proposed by Dror et al. in 1989, and proved that the travel distance and number of vehicles can be reduced in case of split delivery. A lot of research has been devoted in solving SDVRP with heuristics algorithm and exact algorithm. See the local search algorithm proposed by Dror et al. (1989), the tabu search in Archetti et al. (2006), and the genetic algorithm and the heuristic method based on construction in Wilk et al. (2012a, 2012b). Though, exact algorithm is limited to solve small scale problems, there are a lot of achievements, see the two phase algorithm with valid inequality in Jin et al. (2007), column generation in Moreno et al. (2010), and the branch and cut in Archetti et al. (2014).

In allusion to VRPSDP, it was first proposed by Min et al. in 1989. The problem derives from books delivery and pickup between a central library and 22 local libraries. Most of the research efforts aimed at solving the VRPSDP have focused on the heuristic algorithm. Jin et al. (2009) present a particle swarm algorithm. Zachariadis et al. (2009) combine tabu search algorithm with local search. Gajpalet al. (2009) and Tasanet al. (2012) use ant algorithm and genetic algorithm respectively. Subramanian et al. (2010) investigate a parallel algorithm consist of variable neighborhood search and iterated local search. Curz et al. (2012) adopt genetic algorithm, variable neighborhood search, variable neighborhood decrease, tabu search and path relinking all together in solving VRPSPD, Goksalet al. (2013) adapt a hybrid heuristic algorithm based on particle swarm optimization， with the help of variable neighborhood decrease the algorithm enhanced the solution of iteration, in addition, the simulated annealing helps to maintain the diversity of particle swarm.

Vehicle Routing Problem with Split Delivery and Split Pickup is the combination of VRPSDP and SDVRP. Each customer has both delivery demand and pickup demand, and the demands are allowed to be satisfied by different vehicles. In allusion to the situation which the pickup and delivery requirements are paired beforehand, Mitra et al. (2008) construct a mixed integer linear programming model of this problem, and propose two heuristic algorithms. Nowak et al. (2008) named their research as Pickup and Delivery with Split Loads (PDPSL), the relationship of pickup and delivery is signified by the edges in the transportation network, and two terminal points of each edge represent the start and the end of pickup and delivery. Nowak et al. (2009) research the influence on partial shipment under the situation of multiple vehicles, the influence factors are the size and type of materials, the number of loading and unloading points. Sahin et al. (2013) solve the VRPSDSP with multiple vehicles by the combining tabu search algorithm with simulated annealing algorithm, the computational results indicate that the transport mileage can be reduced effectively when split delivery is allowed. In allusion to the situation that the supply point for each delivery point is not known beforehand, Chen et al. (2014) study an unpaired VRPPD with multiple commodities, they adopt a variable neighborhood search, the initial solution of which is obtained by a heuristic Transportation Efficiency Based Algorithm (TEBA), and six different neighborhood structure are implemented to improve the situation. Comparison with the exact solution solved by CPLEX concluded that the VNS can obtain the optimal solution of the problem with six materials need to be transported among ten customers in a reasonable computing time.

PROBLEM DESCRIPTION AND FORMULATION

Problem Description

The problem could be stated as follows: there is a set of homogeneous vehicles, $K=\{1,2,\dots|K|\}$. Each vehicle subjects to one of the production plants (vehicle depots) $O=\{1,2\dots n\}$, they pick up and delivery different kinds of materials in a graph with a set of warehouses together with vehicle depots $N=\{1,2\dots m\}$ ($O\subset N$) and a set of arcs $A=\{(i,j):\ i,j\in N,\ i\neq j\}$. Each arc (i,j) has a non-negative length and cost c_{ij} being equivalent to the distance between vertices connected by the arc and satisfying the triangle inequality. Each warehouse could have both pickup and delivery demand, they could in need of several types of materials and supply different types of materials at the same time. Let $R=\{0,1,2\dots r\}$ representing a set of materials to be transported in the graph, each material $r\in R$ is characterized by a set of supply warehouses $S=\{S_0,S_1,S_{2,\dots,}S_r\}$ and a set of demand warehouses $D=\{D_0,D_1,D_{2,\dots,}D_r\}$. As a consequence, each vehicle needs to know the type of materials and the quantities of which to be carried from one specific warehouse to another specific warehouse. It is not necessary to satisfy the demand of a warehouse at one time, but each warehouse could be visit by the same vehicle once at most.

In this problem, each vehicle departs from the vehicle depot they belong to, and back to the same depot after visiting several warehouses with a series of loading and unloading operations. The objective is to minimize the trip total cost used to satisfy all delivery requests while meeting loading constraints and time limit.

Mathematical Formulation

Sets

N: Warehouse nodes or vehicle depots indexed in i or j

O: Vehicle depots indexed in i or j

S: Supply warehouses indexed in i or j

D: Demand warehouses indexed in i or j

K: Available vehicles indexed in k

A: Arcs indexed in (i, j), with $i, j \in N$

R: Materials indexed in r

Parameters

P: Unit of cost

c_{ij} : Distance between vertices in NQ: Loading capacity of vehicles

T: Maximum time vehicles are allowed to travel

V: Travel speed of vehicles

d_{ir} : Demand quantity of material r in warehouse i

s_{jr} : Inventory of material r in warehouse j

y_{ij} : 1 represents nodes i , j are subject to the same plant; 0 otherwise

z_{ik} : 1 represents vehicle k belongs to depot i

Decision variables

x_{ij}^{k} : 1 if the arc (i, j) is in vehicle k's route; 0 otherwise

q_{ir}^{k} : Quantity of material r that is loaded or unloaded on vehicle k as it departs from warehouse i

L_{ijr}^{k} : Quantity of material r that is carried on vehicle as it travels from warehouse i to warehouse j

Mathematical model

$$MinP\sum_{k\in K}\sum_{r\in R}\sum_{i\in N}\sum_{j\in N, j\neq i} c_{ij}L_{ijr}^{k} \tag{1}$$

s.t.

$$\sum_{j\in S} x_{ij}^{k} y_{ij} z_{ik} \leq 1 \qquad \forall i \in O, k \in K \tag{2}$$

$$\sum_{j\in D} x_{ji}^{k} z_{ik} \leq 1 \qquad \forall i \in O, k \in K \tag{3}$$

$$L_{ojr}^{k} = L_{jor}^{k} = 0 \qquad \forall o \in O, j \in W, k \in K, r \in R \tag{4}$$

$$\mathrm{x}_{\mathrm{o_i o_j}}^{\mathrm{k}} = 0 \qquad \forall \mathrm{o_i}, \mathrm{o_j} \in \mathrm{O}(\mathrm{i} \neq \mathrm{j}), \mathrm{k} \in \mathrm{K} \tag{5}$$

$$\sum_{\mathrm{i}\in \mathrm{O}} \mathrm{z_{ik}} = 1 \qquad \forall \mathrm{k} \in \mathrm{K} \tag{6}$$

$$\sum_{i\in O} y_{ij} = 1 \qquad \forall j \in W \qquad (7)$$

$$\sum_{i\in W} x_{ij}^{k} \le 1 \qquad \forall k \in K, \forall j \in W(i \ne j) \qquad (8)$$

$$\sum_{k\in K} q_{ir}^{k} = d_{ir} \qquad \forall i \in D, r \in R \qquad (9)$$

$$\sum_{k\in K} q_{jr}^{k} \le s_{jr} \qquad \forall j \in S, r \in R \qquad (10)$$

$$\sum_{r\in R} L_{ijr}^{k} \le Q x_{ij}^{k} \qquad \forall i, j \in N(i \ne j), k \in K \qquad (11)$$

$$L_{ijr}^{k} - x_{ij}^{k} q_{jr}^{k} = x_{ij}^{k} L_{jpr}^{k} \sum_{p\in N, p\ne j} x_{jp}^{k} \qquad \forall i \in N, j \in D_r (i \ne j), k \in K, r \in R \qquad (12)$$

$$L_{ijr}^{k} + x_{ij}^{k} q_{jr}^{k} = x_{ij}^{k} L_{jpr}^{k} \sum_{p\in N, p\ne j} x_{jp}^{k} \qquad \forall i \in N, j \in S_r (i \ne j), k \in K, r \in R \qquad (13)$$

$$\sum_{k\in K}\sum_{i\in S} q_{ir}^{k} = \sum_{k\in K}\sum_{j\in D} q_{jr}^{k} \qquad \forall r \in R \qquad (14)$$

$$\sum_{i\in N}\sum_{j\in N, j\ne i} x_{ij}^{k} c_{ij} / V \le T \qquad \forall\ k \in K \qquad (15)$$

$$L_{ijr}^{k} \ge 0 \qquad \forall i, j \in N(i \ne j), k \in K, r \in R \qquad (16)$$

$$q_{ir}^{k} \ge 0 \qquad \forall i \in N, k \in K, r \in R \qquad (17)$$

Equation (1) represents the objective function of the model, which intends to minimize the total cost of the transportation. Equation (2)—(4) determine that each vehicle must depart from and return to the same depot after visiting several warehouses, besides the vehicle and the first warehouse visited must belong to the same plant together. Equation (5) ensures that no truck could travel from one depot to another depot directly. Equation (6) and (7) define that each truck only belongs to one vehicle depot and each warehouse only belongs to one plant. Equation (8) represents that each warehouse could be visit by the same vehicle once at most. Equation (9) represents that each warehouse is allowed to be visited by more than one vehicle, and the demand should be satisfied in total. Equation (10) guarantees that the quantity of each material loaded on the vehicle should be less than its inventory in the warehouse. Equation (11) refers to the loading constraint for each vehicle, it also indicates that if $x_{ij}^{k} = 0$, then $L_{ijr}^{k} = 0$. Equation (12)—(14) represent flow conservation equations for each vehicle. Equation (15) guarantees that travel duration for each vehicle cannot be more than the allowed working duration. Equation (16) and (17) represent the nature of model variables.

THE VARIABLE NEIGHBORHOOD SEARCH

Since the variable neighborhood search was proposed by Mladenovic ´and Hansen in 1997, it has been widely used to solve the large scale vehicle routing problem. In this paper we try to solve the MUPVRPSDP with a VNS-based local search algorithm. Firstly, VNS starts from an initial solution S, which is obtained by TEBA heuristic. Then, six different types of neighborhoods are implemented to obtain the local optimal solution S'. If S' is better than S, it replaces S and the search continues with the first neighborhood $\beta = 1$. If S' is worse, S is not replaced and the next neighborhood is used in the subsequent iteration $\beta = rand()\%\beta_{max} + 1$. Whenever $\beta = \beta_{max}$ is attained, the search continues with the first neighborhood. This is repeated until the iteration reaches 1000 times or there is no better solution more than 500 times. The following is the procedure of the VNS algorithm.

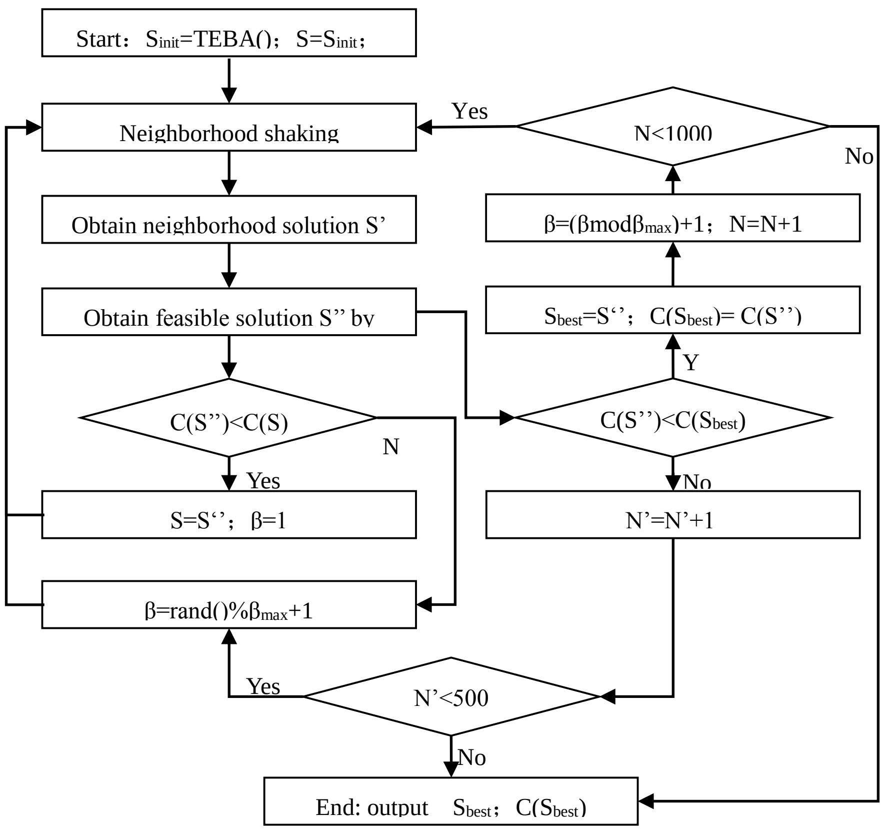

Figure1-the procedure of the VNS algorithm

Construct Initial sub-route based on TEBA

The company's unit of transportation cost account is ton kilometer, as for the same freight volume, the shorter the vehicle travels, the lower the corresponding cost. Popken (2006) proposes that increasing efficient utilization of the vehicle's capacity by controlling order circuit is more useful to solve practical sized pickup and delivery problems. Chen (2014) proposes a heuristic Transportation Efficiency Based Algorithm according to this idea, the core of TEBA is to find a solution with increased value of δ, which is the ratio of the transportation volume and the transportation distance. The greatest difference between this research and Chen (2014) is the limited inventory of all materials. Thus, before we introduce TEBA we need to fix a supply and demand pairing for each warehouse to ensure that the materials in highest demand is supplied by the nearest warehouse. The following is the description of the fixing operation.

Step 1: order the warehouses by the total demand of all materials, so we can get $A = \{D_1,\ D_2 \ldots D_n\}$, which is an array of warehouses. Then, ordering the materials in each warehouse by the demand amount to get $B(D_{i)} = \{r_1,\ r_2 \ldots r_m\}$, which is an array of materials in each demanding warehouse.

Step 2: pair supply warehouse for each demand warehouse of A in sequence. First, we have to know the feasible supply warehouses of D_i, they are selected by the material r with maximum requirements in D_i, and the inventory of r in the supply warehouse should not be zero. Then, the distance to D_i should be the shortest.

Step 3: determine the quantity of material distributed according to the storage amount, the demand quantity and capacity of vehicle.

Step 4: find whether there is pairing for another materials, if it is, repeat step 3 until no material could be allocated. Then updating the storage amount, the demand quantity, and the rest of capacity.

Step 5: fix the pairings in step 4, which can be regarded as a dummy warehouse, and the inventory and the demand of which are determined by the updated storage amount, demand quantity, and rest of capacity all together.

Then let us construct the initial solution by TEBA.

Step 1: suppose that the objective of the initial solution is a large number, set $C_{TEBA} = 1000000$.

Step 2: generate the initial sub-route: selecting one dummy warehouse i and another warehouse j out of the dummy warehouse set. Connect i and j with the vehicle depot o.

Step 3: consider the warehouses not in the sub-route, insert each warehouse into each possible position in the sub-route, and determine the maximal pickup and delivery for each possible supply and demand pairing from the first warehouse to its successors and from the first material type subject to the capacity constraint.

Step 4: if another warehouse could be inserted into the current sub-route, repeat step3; if not, take the sub-route as one route of the current solution S and update the storage amount, the demand quantity and the warehouse set excluding the warehouses without both pickup and delivery request. Then, execute step 5. If each material demand is satisfied then turn to step 6.

Step 5: select two warehouses, and one is from the virtual warehouse set, connect them with the vehicle depot to generate a new initial sub-route, obtain the corresponding value of δ, and select the sub-route with the largest value of δ. Turn to step 3.

Step 6: obtain the feasible solution of the problem. If $C_{TEBA} > C(S)$, then $C_{TEBA} = C(S)$, and S is the initial solution.

Neighborhood Structure

Since the supply and demand (S-D) pairing is a decision variable, the change of visiting sequence in every route will affect the supply and demand pairing, which further changes the visiting sequence for the other routes. Similar to Chen (2014) here we adopt six different neighborhood structures: ①delete partial S-D pairings; ②delete partial S-D pairings of one customer in one route; ③delete all S-D pairings of some material types in one route; ④delete all S-D pairings of some type in some routes; ⑤delete all S-D pairings of one customer in some routes; ⑥delete all S-D pairings of one customer with appearance than the least number.

Local Search

If we apply the above neighborhood structures to the initial solution, we will get several different neighborhood solutions as well as a set of new demands. In order to obtain a complete and local optimal solution, the general local search heuristic in the VNS algorithm for pickup and delivery problem in korsvik et al. (2011) is adapted for this problem.

Step 1: since new demands come from those deleted S-D pairings, before we try to satisfy them, we should first check out whether there is any empty route, if yes, delete the route and update the number of routes.

Step 2: randomly select an unsatisfied demand d_{ir} (the demand of material r in warehouse i), and examine each possible insertion point in each route. Then try to satisfy the demand by three ways in step 3 to 5 successively subject to the capacity constraint.

Step 3: if there is a warehouse j could satisfy d_{ir} totally by vehicle k , then insert $\boldsymbol{i}$ after j in the route; if more than one warehouse could meet the demand, choose the position with least additional cost in which i will be inserted. If the above situation is not existence, turn to step 4.

Step 4: if there are several supply warehouses in one route, and the total can satisfy the demand, then insert i after the last supply warehouse. And the supply quantity of each warehouse follows the principle "the closer the more". If more than one route could meet the demand, choose the one with least additional cost in which i will be inserted.

Step 5: establish a new route with least additional cost.

Step 6: remove the demand d_{ir} from demand set, update the sets of demand and supply, as well as the rest vehicle capacity. If the demand set is not empty, turn to step 2; if all the demand has been satisfied, turn to step 7.

Step 7: stop calculation and return the vehicle routes and the total cost.

COMPUTATIONAL RESULTS

In order to test the effectiveness of TEBA and the local search, we utilize the practical allocating data of the tobacco industry in the first half year of 2012. Table 3 shows the comparison of the practical transportation cost, the initial solution calculated by TEBA and the best solution obtained by VNS. GAP_1 represents the difference between the practical cost and the initial solution, and the average savings in six months is 20.219%. GAP_2 represents the difference between the initial solution and the best solution, and the average savings in six months is 3.811%. We can summarize from table 3 that the initial solution obtained by the approach of TEBA is much better than the practical cost, and the result could be improved again when a local search is introduced into the VNS.

Table 3-comparison of results

	Jan.	**Feb.**	**Mar.**	**Apr.**	**May**	**June**	**Average**
C(practical)	651214	371345	575237	641238	382576	574451	532677
$C(S_{init})$	483325	257725	484433	493484	309808	533629	427067
GAP_1	25.78%	30.59%	15.78%	23.04%	19.02%	7.106%	20.219%
$C(S_{best})$	480413	241365	463205	476866	299870	507288	411501
GAP_2	0.602%	6.374%	4.382%	3.367%	3.207%	4.936%	3.811%

CONCLUSIONS

In this paper, we develop a comprehensive mathematical model for the material allocation process of a large tobacco industry enterprise in China. The process can be described as a large scaled vehicle routing problem with multiple materials to be picked up and delivered in an unpaired network. The goal is to minimize the total transportation cost. Compared with the practical allocation operation of the company, the greedy heuristic TEBA considers both vehicle load and travel cost at the same time, which guides the vehicle to carry sufficient load in low cost transportation leg quickly. And savings of the total cost validates the effectiveness of both TEBA and VNS. Concerning future research, time window of each warehouse will be introduced into this allocation problem as a constraint condition since time requirement is also an important factor that impacts the decision of the company.

Bibliography

Dantzig G B, Ramser J H. 1959. The truck dispatching problem. *Management science* 6(1): 80-91.

Dror M, Trudeau P. 1989. Savings by split delivery routing. *Transportation Science* 141-145.

Archetti C, Speranza M G, Hertz A. 2006. A tabu search algorithm for the split delivery vehicle routing problem. *Transportation Science* 40(1): 64-73.

Wilck IV J H, Cavalier T M. 2012. *A genetic algorithm for the split delivery vehicle routing problem*.

Jin M, Liu K, Bowden R O. 2007.A two-stage algorithm with valid inequalities for the split delivery vehicle routing problem. *International Journal of Production Economics* 105(1): 228-242.

Moreno L, De Arag ãO M P, Uchoa E. 2010.Improved lower bounds for the split delivery vehicle routing problem. *Operations Research Letters* 38(4): 302-306.

Archetti C, Bianchessi N, Speranza M G. 2014. Branch-and-cut algorithms for the split delivery vehicle routing problem. *European Journal of Operational Research* 238(3): 685-698.

Min H. 1989.The multiple vehicle routing problem with simultaneous delivery and pick-up points. *Transportation Research Part A: General* 23(5): 377-386.

Ai T J, Kachitvichyanukul V. 2009. A particle swarm optimization for the vehicle routing problem with simultaneous pickup and delivery. *Computers & Operations Research* 36(5): 1693-1702.

Zachariadis E E, Tarantilis C D, Kiranoudis C T. 2009.A hybrid metaheuristic algorithm for the vehicle routing problem with simultaneous delivery and pick-up service. *Expert Systems with applications* 36(2): 1070-1081.

Gajpal Y, Abad P. 2009. An ant colony system (ACS) for vehicle routing problem with simultaneous delivery and pickup. *Computers & Operations Research* 36(12): 3215-3223.

Tasan A S, Gen M. 2012. A genetic algorithm based approach to vehicle routing problem with simultaneous pick-up and deliveries. *Computers & Industrial Engineering* 62(3): 755-761.

Subramanian A, Drummond L M A, Bentes C, et al. 2010. A parallel heuristic for the vehicle routing problem with simultaneous pickup and delivery. *Computers & Operations Research* 37(11): 1899-1911.

Cruz R C, Silva T C B, Souza M J F, et al. 2012. GENVNS-TS-CL-PR: A heuristic approach for solving the vehicle routing problem with simultaneous pickup and delivery. *Electronic Notes in Discrete Mathematics* 39: 217-224.

Goksal F P, Karaoglan I, Altiparmak F. 2013. A hybrid discrete particle swarm optimization for vehicle routing problem with simultaneous pickup and delivery. *Computers & Industrial Engineering* 65(1): 39-53.

Mitra S. 2008. A parallel clustering technique for the vehicle routing problem with split deliveries and pickups. *Journal of the operational Research Society* 59(11): 1532-1546.

Nowak M, Ergun Ö, White III C C. 2008. Pickup and delivery with split loads. *Transportation Science* 42(1): 32-43.

Nowak M, Ergun O, White C C. 2009. An empirical study on the benefit of split loads with the pickup and delivery problem. *European Journal of Operational Research* 198(3): 734-740.

Şahin M, Çavuşlar G, Öncan T, et al. 2013. An efficient heuristic for the multi-vehicle one-to-one pickup and delivery problem with split loads. *Transportation Research Part C: Emerging Technologies* 27: 169-188.

Chen Q, Li K, Liu Z. 2014. Model and algorithm for an unpaired pickup and delivery vehicle routing problem with split loads. *Transportation Research Part E: Logistics and Transportation Review* 69: 218-235.

Mladenovic ́, N., Hansen, P., 1997. Variable neighborhood search. Comput. Oper. Res. 24 (11), 1097–1100.

Popken, D.A. 2006. *Controlling order circuity in pickup and delivery problems.* Transp. Res. Part E 42, 431–443.