Batch Sizes in Push Manufacturing
Operations Planning, Scheduling and Control Track

Abstract: Isn't Push manufacturing dead and is the best batch size? Many manufacturing companies still use a Push system and they do so with batches of more than 1! They need to produce with little risk of late completion. To reflect this need, a new measurement, MUST (1%) (Minimum Upper Supply Time) was introduced. This is the throughput time that is only exceeded on 1% of occasions. The paper discusses the results of a series of simulation trials and the relationships which emerge between production batch size, plant utilization, average throughput time, WIP and MUST (1%).

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Introduction
The last three decades have seen a major swing away from ‘push’ manufacturing and towards a Just in Time approach, and yet there are still a number of manufacturers who persist in pushing products through their factories. In many cases they are doing so with less inventory than previously. The questions then arise: Are they wrong to persist with ‘push’ systems and, if they do, what stock levels should they use?

This paper seeks to move a little closer towards answering the first of these questions by exploring the answer to the second. In the course of the research a series of simulation models were constructed to explore the effects of changing batch size. These looked at the relationship between batch size, levels of WIP (Work in Progress or Work in Process in US terminology), lead times and utilisation, for simple multi-operation production facilities. Though average lead time gives some measure of responsiveness to customers’ demands, it is of limited use. What really matters to a producer is the lead time which can be offered to a customer and achieved, with a reasonable level of certainty. For this reason, a new metric was introduced, MUST(1%), the maximum upper supply time which is only exceeded on 1% of occasions.

The Study in Context
One of the earliest publications on Batch Sizes in Manufacturing was the hugely influential paper by Harris (1913). It was quickly realised by most authors that Harris’ square root formula strictly applied only to economic purchasing where there was uniform demand. It is wholly inapplicable to multi-stage manufacturing systems, since it ignores the effects of batch size changes on the level of WIP, and hence on the lead time. This affects the cost of working capital. There are also competitive advantages in being able to offer shorter lead times to a customer.

In a study of scheduling systems, Colley et al (1977) considered the effects of utilisation on WIP. They claimed that all scheduling systems tended to produce work on time with utilisations of less than 90% and all scheduling systems struggled with utilisation rates of over 92%. They described an approach for choosing an “optimum” capacity for balancing plant capital against WIP. The work seemed to treat the production batch size as a constant. However, more capacity enables smaller batches to be used with a far greater effect on WIP and lead times.

A review by Goyal and Gunasekaren (1990) of theoretical models of the relationships addressed in this paper shows a wealth of theoretical models in this area, many quite complex and mathematical. It also discusses a number of other simulation models in this area. Karmarkar (1993) similarly reviews the literature relating to lead times, order release and capacity loading.

In the course of a wide-ranging chapter, he reviewed the relationship between lead time and batch size for a number of theoretical queuing models.

Many papers on the choice of manufacturing batch sizes have since been published. These include Agawal et al (2000) (using the LETSA algorithm) and Hung and Chien (2000) (comparing the use of tabu search, simulated annealing and genetic algorithms). Each adds to the tools that can be used to examine the effects of changing batch size. With such a complex array of tools, it is appropriate to re-examine the relationships which underlie their application.

**The Methodology**

Since the mathematical analysis of even quite simple multi-stage manufacturing processes has proved so difficult, it was decided to build a series of simulation models using the SIMUL8 simulation package. For each such model, the simulation was re-run with a series of production batch sizes and using 20 trials for each batch size. In all cases the traditional relationships, that the production batch size is the same at each operation and that transfer batches are the same size as the production batches were adhered too. Eli Goldratt has been prominent among those demonstrating that such restrictions are often expensive mistakes (see for example Goldratt (1985)). However, for this initial study it has been retained for simplicity. Clearly, further development of this work could examine the situation when those links are broken.

See Figure 1

Figure 1 illustrates the simulation layout used in this paper. Orders are generated for the seven different products with an equal expected frequency for all seven. The symbol for the points where orders arrive in the model is shown in the key. Orders are accumulated until there are enough for a batch of production, at which point raw material is issued. They are then released as WIP. The issuing of raw material is not explicit in the diagram. It is only modelled by the choice of queues to be counted as WIP.

The model shows two machines at stage 1 (with production operations Start 1 run and Start 2 run) and 3 machines at stage 2 (with production operations Finish 1 run, Finish 2 run and Finish 3 run). The operations on either side of these production operations unpack a batch of components into its constituent parts and re-batch them for the next operation. The symbols for the machines on which production takes place are shown in the key to Figure 1. These machines are resources which are required for both the production operation and the operation to unpack a batch ready for production. This, with priority given to the production operation, ensures that no batch is unpacked before the previous batch has been completed. As soon as production of the last operation is completed, the product leaves the simulation at the despatch symbol (see the key).

Results from the simulation were transferred to Excel for further analysis and graphical presentation. For the graphs in this paper, I have shown the level of the three key variables, average WIP, average throughput time and MUST(1%) against the variation in the average machine utilisation.

The decision to show these variations against utilisation rather than against batch size reflect the fact that this appears to give a greater similarity in the patterns of values observed with different models. However, though the relationships have broadly similar shapes when expressed in terms of utilisation, there are variations. These are particularly marked between models with stable arrival rates and low variation in process times and those with highly variable order arrival rates and process times. When the set-up times, run times and production capacities are known, it is a simple task to calculate the average utilisation from the batch size.

The Results
In all cases examined, a batch size of 1 required so many set-ups that the facility was unable to produce at the required rate. With utilisations approaching 100% we know from queuing theory to expect large lead times and large average WIP. As the utilisations fall, somewhere in the region of 92-96% utilisation, both average WIP and average lead time reached minima in the facilities modelled in this study. Both then started to rise again.

The MUST(1%) exhibits a similar profile to that of the average time in the system, but in each case examined, it hit its minimum at a slightly lower utilisation. It is this assured lead time that most manufacturers will wish to minimise. If the product were for direct sale to a customer, then the time used as the lead time would be the MUST time, or something close to it. If the average lead time were used, then output would be late on 50% of occasions. Similarly, if the product were used internally in an assembly or sub-assembly, then the required lead time would again be about the same as the MUST(1%) time. Products completed within the MUST(1%) time then need to wait for rest of the parts to be completed before assembly can start.

In each of the models, the graphs of average WIP against % utilisation are equally valid if each product in the simulation is taken to represent $n$ real products. The unit production times must then be the times to manufacture $n$ products. Each order arrival in the model must similarly represent the arrival of an order for $n$ products.

The number of products in WIP must be multiplied by $n$ to give the number of real products, but the relationship between WIP size and batch size will have the same shape with a minimum at the same utilisation. The relationship between batch size and lead time (both average lead time and MUST(1%)) will be the same for all values of $n$. 

Similarly the simulation works in time units, which need not represent minutes. Therefore, if all times (inter-arrival times, set up times and unit production times) are multiplied by a factor $t$, then the relationship between WIP and batch size will be unaffected. The average lead time and MUST(1%) times will have the same shape with minima at the same utilisations, but all times will be multiplied by this same factor $t$.

We can verify both of these generalisations in the model with the results shown in Figures 3 and 6 and with those shown in Figures 4 and 7. The latter model is obtained from the former by using values of $n$ and $t$, which are both 0.1. Notice the shapes of the two sets of graphs are identical, but the number of products in WIP and the lengths of time in the system have dropped to one tenth of their former value from the earlier to the later model.

**Conclusions**

The range of models that have so far been simulated is quite limited. Many more simulation runs of this sort are required, in order to get a better picture of the relationship between average utilisation, average WIP average throughput times, MUST(1%) and the other relevant parameters in batch manufacturing. When we have a clear understanding of the best that can be achieved with ‘push’ manufacturing and the batch sizes which produce that optimum performance, then we can better compare that approach with the alternatives. These would include kanban based pull production and OPT based systems. Such an analysis could also be used to provide the data for cost justification for a change in manufacturing method. Without a clear idea of the best that should be attainable with the current method, any such analysis is built on weak foundations.

For those companies that choose to retain a functional layout and to push product through it, then preliminary analysis suggest that the optimum batch size could be one that gives a utilisation of 94% - 96%. Until more complete mapping of the relationships involved, it is recommended that the particular characteristics of the company concerned should be entered into a simulation model, such as those discussed here. This would enable each company to determine its optimum batch sizes.

If a company wishes to minimise the lead time which it can be confident of achieving on 99% of occasions, then I recommend that they use the MUST (1%) measure, introduced above as the parameter they wish to minimise. The MUST (1%) time can then be used as the lead time for MRP or customer promises. In the models described in this paper, it is assumed that operations are performed in a sequence which takes the product through the factory with other products at the same stage. If the processes actually in use put products in competition with others at different stages of manufacturing, then careful choice of scheduling rules (e.g. Least Sack Scheduling) should enable shorter MUST (1%) times. This should be achieved by giving priority to products that are falling behind, particularly if they are nearing the end of their production process.

In the models simulated here, all products underwent the same sequence of operations, so there would be little scope for improving the MUST (1%) value by use of other scheduling rules, and none at all for the model with fixed processing times.
The level of utilisation which produces the shortest MUST (1%) varies between the models examined. I speculate that an important factor affecting this may be the degree of variation of the ratio of total demand to bottleneck capacity, over some standardised period of time. Let us define the standard deviation of this ratio as the Demand to Capacity Variation, abbreviated as the DCV. A suitable standardised time period may be the set-up time plus the run time for the batch size that makes the average demand exactly match the production capacity at the bottleneck operation.

The next stage of this research will be to examine a much wider range of models and to attempt to establish relationships between the size of the MUST (1%), the utilisation at which minimum MUST (1%) occurs, and the DCV.

**References**


Goldratt, M.E., “Devising a Coherent Production/Finance/Marketing Strategy Using the OPT Rules”, BPICS Control, April/May 1985


Figure 1 The Simulation Model Layout for Two Stage Manufacture
Figure 2: The Number of products in WIP compared with the average Utilisation, with Exponential Arrival of Orders, One at a time and Exponential Process Time
Total WIP against Utilisation

- Exponential Arrivals in batches of 10, Fixed Process Time & set-up time 50 time units x Number of machines at that Stage

Figure 3 The Number of products in WIP compared with the average Utilisation, with Exponential Arrival of Orders, in batches of 10 and Fixed Process Time
Figure 4 The Number of products in WIP compared with the average Utilisation, with Exponential Arrival of Orders, One at a time and Fixed Process Time
Total time in the system with Utilisation
- Exponential Arrivals in batches of 1, Exponential Process Time & set-up time 5 time units x Number of machines at that Stage

Figure 5: The Average Lead Time and MUST(1%) time, compared with the average Utilisation, with Exponential Arrival of Orders, One at a time and Exponential Process Time
Figure 6: The Average Lead Time and MUST(1%) time, compared with the average Utilisation, with Exponential Arrival of Orders, in batches of 10 and Fixed Process Time
Figure 7: The Average Lead Time and MUST(1%) time, compared with the average Utilisation, with Exponential Arrival of Orders, One at a time and Fixed Process Time