An LAV Methodology For Forecast Combination In Services Forecasting

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Abstract:

A number of previous studies have shown that a combination of forecasts typically outperforms any component forecast. Service managers may wish to use forecast combination to improve forecast accuracy in predicting retail sales. In this study, revenue data from an actual service company is used to generate and test a least absolute value (LAV) regression model for forecast combination. The LAV forecast, developed by the authors, is determined by minimizing weighted deviations from the component forecasts. The accuracy of this approach is compared to the accuracy of some traditional methods.

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Introduction

Reliable forecasting methodologies play a crucial role in effective service management. Since services typically cannot be "stockpiled" in anticipation of future demand, service managers usually attempt to schedule sufficient capacity to meet future demand requirements. To do this, accurate forecasts of future demand are necessary. Often, service companies have access to more than one forecast of future demand. A variety of forecasting methodologies such as simple moving average, exponential smoothing and regression analysis can be applied in many service contexts. In addition, expert opinion can be utilized to develop purely judgmental (or qualitative) forecasts. Since service managers are faced with so many forecasting techniques, an effective methodology for combining multiple forecasts into a single forecast would be useful.

For at least thirty years researchers have argued that a combination of forecasts tends to be more accurate than any of the individual forecasts comprising the combination. (See, for instance, de Menzes, Bunn and Taylor, 2000) Forecast accuracy tends to improve even if only two forecasts are combined. (Russell and Adam, 1987) In this study, revenue data from an actual service company is used to generate and test a model for forecast combination, which is based on least absolute value (LAV) regression. The performance of this approach is then compared to the accuracy of a standard method for forecast combination.

Forecast Combination

Forecast combination is not a new idea. During the past thirty years a number of researchers have proposed a wide variety of forecast combination techniques. In this section only a short summary of some well-established approaches to combining forecasts will be presented. A survey of non-traditional combination models can be found in Bunn's (1996) review paper .

One of the best known methods of forecast combination is the simple average approach. With this method, a combined forecast for period t (denoted by F_t) is generated by taking the arithmetic mean of two or more component forecasts for period t. Thus,

$$F_t = \Sigma f_i / n \qquad (1)$$

where f_i = an individual forecast and n = the number of individual forecasts to be combined. The simple average approach is easy to use and has performed well empirically. (Clemen, 1989) The accuracy of the combined forecast produced by the arithmetic mean reflects the accuracy of the component forecasts. (Gupta and Wilton, 1987) Thus, a disadvantage of this approach is that inaccurate individual forecasts reduce the accuracy of the combined forecast. Another disadvantage of the simple average is that each individual forecast receives the same weight in the combination - even if individual models differ significantly in accuracy and structure. Gupta et al. (1987, p.357) have observed that the simple average "treats the forecasts as though they are interchangeable: i.e., indistinguishable from one another." They argue that component models, which exhibit greater accuracy, should be given greater weights than those with lower accuracy; furthermore, the assigned weights should be "intuitively meaningful." (Gupta et al., 1987)

Like the simple average, the "outperformance" technique proposed by Bunn (1975) is easy to use; it also avoids the "exchangeability" assumption inherent in an equal weighting approach. The "outperformance" method is based on a weighted average in which each weight is the proportion of time the corresponding forecast performed the best in the past. Thus, an advantage of the outperformance method is that it utilizes individual forecast weights, which have intuitive meaning for the decision-maker. An additional advantage of this method is that it performs well when data are sparse. Finally, this method is well suited to situations in which expert judgment influences the choice of the weights. (de Menzes et al., 2000) Despite its advantages, the outperformance model does not utilize all available information about the component forecasts. Specifically, “information regarding the relative performances within the set is discarded.” (Gupta et al., p.359) Thus, relative performance among the outperformed models is ignored.

The optimal method for forecast combination proposed by Bates and Granger (1969) represents another common weighted average approach. In this model, linear weights are constructed to minimize the error variance of the forecast combination, assuming that individual forecasts are unbiased. Granger and Ramanthan (1984) have demonstrated that the optimal method is equivalent to a least squares regression model which omits the intercept and constrains the weights to sum to one. Granger et al. (1984) have also noted that the optimal method may fail to produce an unbiased forecast when component forecasts are biased. In addition, de Menzes at al. (2000, p.192) have observed that this method requires the covariance matrix *S* of forecast errors to be properly estimated and that “in practice, *S* is often not stationary, in which case it is estimated on the basis of a short history of forecasts and thus becomes an *adaptive* approach to combining forecasts.” (de Menzes et al., 2000, p. 192)

Ordinary least squares (OLS) regression models constitute another frequently used methodology for forecast combination. In this approach, component forecasts serve as the independent variables while the observed value for the forecasted variable is the dependent variable. (Coulson and Robbins, 1993) Thus, if two component forecasts are used in the combination, the model has the form:

$$y_t = b_o + b_1 f_{1t} + b_2 f_{2t} + e_t \quad (2)$$

where y_t = the actual value of the forecasted variable for period t
f_{it} = the forecast for period t generated by component forecast model i
b_o = the constant term
b_i = the regression coefficient for component forecast i
e_t = the error term for period t

The regression coefficients and constant term are found which minimizes the sum of squares of the error terms.

De Menzes et al (2000, p.192) report that this regression approach is superior to the optimal model in that “an unbiased combined forecast is produced regardless of whether the constituent forecasts are biased.” However, Narula and Korhonen (1998, p.71) have argued that, despite its popularity, the OLS model may not be the best regression approach in some instances. They have observed that the OLS model “implicitly assumes that the loss function is proportional to the square of the errors” and that it “is known that in many situations the quadratic loss function is inappropriate.” Furthermore, the results of OLS forecasts are frequently reported in terms of relative percentage error, which is based on the absolute value of the ratio of the error term to the observed value. Given this practice, Narula et al. (1998, p.71) conclude that “it is more appropriate to consider a loss function proportional to the absolute value of the errors rather than the square of the errors.” They recommend that least absolute value (LAV) regression be considered as an alternative to OLS regression. In (2) above LAV regression finds coefficients and constant term which minimizes the sum of the absolute values of the error terms.

The LAV model has long been viewed as an alternative to OLS regression; however, in contrast with OLS regression, there are no formulas for estimating the slope and intercept of the LAV regression line. Several algorithms exist for calculating these estimates (Birkes and Dodge, 1993); in particular, the linear (goal) programming approach has received much attention.

(Hanna, 1992) Regression lines estimated by ordinary least squares are “more severely affected by outliers or extreme data points” than those estimated with the LAV model; furthermore, the LAV method may provide better estimates of regression coefficients than the OLS model when normality assumptions are not met. (Dielman and Pfaffenberger, 1998, p. 734)

A final approach to forecast combination relies on the use of expert judgment. In their study, Flores and White (1989) found the accuracy of subjective forecast combinations equaled and at times surpassed the accuracy of such traditional mathematical methods as the simple average and the optimal method. They also suggested that at most four component forecasts be included in the combination process.

Case Study

Competition within the cellular phone industry has increased dramatically during the past several years. This has forced providers of cellular phone services to intensify their efforts in attracting new customers while maintaining current customers. The cellular phone company, which provided the research context for this case study, faced such a challenge. Located in the southeast United States, this company once had only one major competitor in its market; however, in recent years, the number of competitors had increased to eight. In the past, the company characterized demand for its services as “limitless”; now the company is uncertain how demand will grow in the future. Given this uncertainty, the company is particularly interested in improving its forecasts of total revenue, which is the sum of phone access and phone usage revenues.

Working with recent company data on total revenue, the authors developed combined forecast models for this situation and then examined the accuracy of these combination models. Two years of monthly total revenue figures comprised the data set. The research effort proceeded in the following stages:

Stage 1: Data Preparation

The raw data was smoothed to adjust for the fact that the number of days per month varied. An index was developed for each month to “deseasonalize” the data. Plots of the deseasonalized data indicated the presence of a trend in total revenue. We denote the adjusted revenue data by r(t).

Stage 2: Generation of the Component Forecast Models

The first 12 monthly total revenue values (i.e., Year 1 data) were used to develop two individual forecast models. The first model, REG(t), was a simple linear regression model, which estimated a trend line for the 12 values. The second model, SEST(t), was an exponentially smoothed forecast with trend adjustment. In the development of this model, a search was done to determine the values of the smoothing constants (alpha and beta) which ensured the greatest accuracy.

Stage 3: Generation of Alternative Combined Forecasts

Each model developed in Stage 2 was used to generate a series of monthly revenue forecasts for Year 1. These forecasts were then used as input to 2 alternative models for forecast combination. The first combined model was the frequently used multiple regression model described by (2). The second combined model was an LAV regression model, denoted by WLAV, which was developed by the authors. For month $t = 2,3,...,12$ in Year 1, the model formulates a weighted absolute deviation from the WLAV forecast to each of the pair of values obtained from the two component forecasts. Linear (goal) programming was used to find the slope and intercept for the regression equation, which minimized the sum of the weighted absolute deviations. The model formulation is given in Exhibit 1.

For each month $t = 2,3,...,12$, weights: $w1(t), w2(t),$ were computed for the deviations from the WLAV regression line to each of the two component forecasts: $f1(t) = SEST(t), f2(t) = REG(t).$ Determination of these weights reflect two basic ideas. (1) The component forecast, which is closest to the actual revenue, should have the largest deviation weight. Hence, the weights reflect the accuracy of the component forecasts. (2) In determining the WLAV regression equation for the current time period, the weights associated with earlier time periods should be reduced. Hence, deviations from component forecasts for earlier time periods will be less important when establishing the regression line. The following formulas were used in computing weights $wi(t).$ For each month $t = 2,3,...,12$ and each component forecast $fi(t),$ compute:

(3)

1. Fraction Absolute Error: $ei(t) = | r(t) - fi(t) | / r(t).$ (4)
2. Fraction Accurate: $ai(t) = 1 - ei(t).$ (5)
3. Time Period Weight: $w(t) = b^{12-t}$; $0 < b \leq 1.$
   (In our example $b = 0.9.$) (6)
4. Absolute Deviation Weight: $wi(t) = w(t) \bullet ai(t)$

The model uses time (months) as the independent variable. In particular, our model differs from traditional applications of LAV regression to forecast combination in that the individual forecasts were not used as predictor variables.

Stage 4: Model Comparisons in Year 1

Year 1 reseasonalized monthly forecasts generated by the OLS and WLAV methods are given in Table 1 and are illustrated in Figure 1. Year 1 results revealed that the OLS combination forecast produced a smaller MAD (582562.4803) than the WLAV method MAD (892207.9364).

Stage 5: Generation of Forecasts for Year 2

The OLS and WLAV models found in Stage 3 were used to predict monthly total revenue over a 12 month forecast horizon (Year 2). The 12-month forecasts for Year 2 were then compared to the actual revenues that ultimately materialized over Year 2 (see Table 1 and Figure 1). The WLAV method clearly outperformed the OLS approach with the MAD for the OLS model (3149967.079) nearly three times the MAD for the WLAV model (1063622.78).

Discussion

Results indicate that while the OLS method appeared more accurate than the WLAV approach initially, the WLAV method actually provided much better predictions of total revenue for the forecast horizon (Year 2). Additional research is needed to determine if the WLAV approach will continue to outperform the OLS method for other variables of interest to this company. Additional research is also needed to ascertain how well the WLAV approach compares to other common methods for forecast combination.

Since this study represents only an initial investigation of the use of combination methods for services forecasting, it is impossible to conclude that the WLAV approach is a superior methodology. However, it should be noted that the WLAV model possesses some advantages that the OLS model does not. Unlike the OLS model, the WLAV model can be written as a linear program and sensitivity analysis can be used to determine the effect of an outlier or mis-specified point on model parameters. Sensitivity analysis can also indicate whether a data point can be dropped from the WLAV analysis entirely without affecting the WLAV line. This may prove useful when the forecaster must consider how to deal with a mis-specified point.

In summary, this study offers only preliminary results on the performance of WLAV regression and OLS regression as combination methodologies in service forecasting. The initial findings presented in this paper do suggest, however, that the WLAV model developed by the authors has potential as a forecasting tool in the service industries.

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Exhibit 1

Weighted LAV Regression Model

Let WLAV(t) denote the weighted LAV forecast for month(t). The form of the WLAV Regression Equation is given:

WLAV(t)=$m \bullet t+b$, where m and b are the slope and constant term of the regression line.

The linear programming formulation is given:

Let o(t) and u(t) denote the deviations (over or under) of WLAV(t) from SEST(t) in month(t), t=2,…,12.
Let a(t) and b(t) denote the deviations (over or under) of WLAV(t) from REG(t) in month(t), t=2,…,12.
Let w1(t) and w2(t) denote the weights assigned to the absolute deviations of WLAV(t) from SEST(t) and REG(t), respectively, t = 2, … , 12.

<u>Objective</u>: minimize $\sum w1(t) \bullet (o(t)+u(t)) + \sum w2(t) \bullet (a(t)+b(t))$,
t = 2, …, 12.

<u>Constraints</u>:

$(m \bullet t+b)+u(t)-o(t)=SEST(t)$ t=2,…,12.
$(m \bullet t+b)+b(t)-a(t)=REG(t)$ t=2,…,12.

$u(t), o(t), b(t), a(t) \geq 0$, all t.
m, b unrestricted in value.

Figure 1: MULT. REG. & WLAV. FORECASTS vs ACTUAL REVENUE

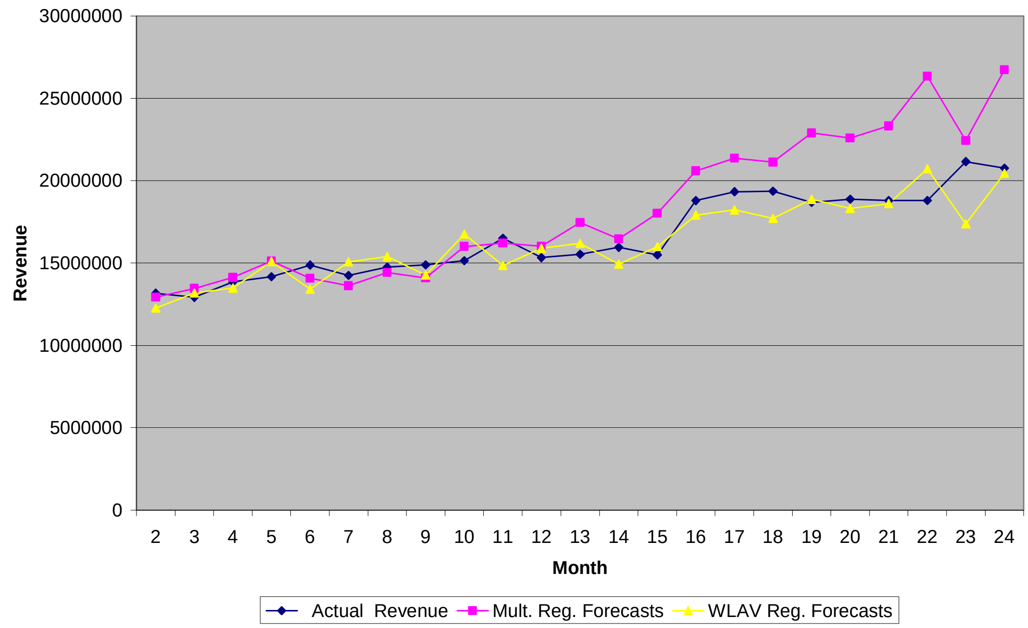

Table 1: MULTIPLE REGRESSION AND WLAV REGRESSION FORECASTS

Month	Actual Revenue	Mult. Reg. Forecasts	WLAV Reg. Forecasts		Mult. Reg. MAD (Months 2-12)	Mult. Reg. MAD (Months 13-24)	WLAV Reg MAD (Months 2-12)	WLAV Reg. MAD Months (13-24)
2	13166957	12925582.44	12262015		241374.2363		904941.68	
3	12912701	13452705.28	13176757.13		540004.5151		264056.365	
4	13858640	14122918.88	13478398.5		264278.5456		380241.83	
5	14167864	15129021.03	15092424.63		961156.9389		924560.535	
6	14871489	14064428.26	13411125		807061.1815		1460364.44	
7	14245411	13614506.28	15068242.75		630904.6019		822831.87	
8	14751254	14424827.56	15384248		326426.2268		632994.21	
9	14881695	14081742.96	14272957.5		799952.2903		608737.75	
10	15140529	16006107.51	16744270.25		865578.8517		1603741.59	
11	16497933	16210616.39	14847512.5		287316.684		1650420.57	
12	15330133	16014266.25	15891529.5		684133.2114		561396.46	
13	15532047	17460140.25	16193170.88			1928093.235		661123.855
14	15949844	16459665.14	14923877.75	**MAD=**	**582562.4803**	509820.8721	**892207.9364**	1025966.52
15	15496587	18023267.1	15996622.5			2526680.414		500035.81
16	18787710	20592627.99	17912290			1804917.5		875420.49
17	19328223	21359612.12	18228295.25			2031388.854		1099928.02
18	19356764	21120889.65	17701377.75			1764125.7		1655386.2
19	18677290	22893630.43	18860305.75			4216340.503		183015.82
20	18877698	22585179.85	18304660.5			3707481.384		573037.97
21	18789560	23317348.84	18606301.88			4527789.211		183257.755
22	18794540	26339895.78	20708699.75			7545356.209		1914160.18
23	21149140	22421482.95	17380100.38			1272343.146		3769039.425
24	20763433	26728701.23	20440332			5965267.918		323101.31
					MAD=	**3149967.079**		**1063622.78**