THE RATIONALITY OF STOCHASTIC BREAK-EVEN ANALYSIS

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ABSTRACT:
Break-even analysis is often viewed as simply a useful heuristic for making pricing and capacity decisions. In this paper, we show that stochastic breakeven analysis, with one slight modification, is equivalent to utility-maximization. Hence stochastic breakeven analysis is a fully rigorous approach to making decisions which includes profit-maximization as a special case.

INTRODUCTION

Economic theory and accounting theory seem to approach the theory of the firm in somewhat different ways. Economic guidelines generally flow from assuming that the firm maximizes profit or shareholder wealth; accounting guidelines, for example break-even analysis, generally reflect more target-based principles.   In this paper, we will show that these formulations are compatible, once one recognizes that firms are concerned for risk.

Uncertainty may arise because the prices which firms received for their products are uncertain or because firms are uncertain about the costs of producing various levels of product.   The simplest way to incorporate this uncertainty is to assume that the firm maximizes expected profit(Tisdell,1969).   But this fails to recognize a firm's aversion to risk.   To incorporate risk, Sandmo(19) define a utility function over various levels of profit and assumed that the firm maximized the expected value of this utility function.

In the next section, we will show that the problem of maximizing the expected value of a utility function can always be recast as the problem of maximizing the probability of profit exceeding some random threshold. We can decompose this random threshold into a profit target,t, and a residual error factor. If we interpret this random threshold as reflecting additional uncertainty about the firm's fixed costs, then maximizing expected utility is equivalent to maximizing the probability of exceeding one's profit target, given uncertainty about fixed costs. This establishes an equivalence between maximizing expected utility and stochastic breakeven analysis. This establishes the general compatibility between the economic theory of the firm and the accounting theory of the firm.

Our third section reinterprets some of Sandmo's economic results in terms of stochastic breakeven analysis. Sandmo's result presumed a concave utility function. However breakeven analysis lends itself to more general formulations. We go on to We then go on to extend these results which Sandmo

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The standard economic approach to recognizing uncertainty is to assume that a firm, instead of maximizing profit, maximizes the expected value of profit.

To generalize standard economic formulations to allow for risk, Sandmo defined a firm's utility function over profit and assumed that a firm maximized the expected value of this utility function.

introduces

generally theory generally focuses on various target-based measures like, for example, break-even analysis.

Economic theory generally mandates evaluating various capacity or pricing decisions on the basis of how they impact profit. This approach requires detailed estimates of how product demand depends on price.

Hence in practice, an alternate approach called breakeven or target-profit analysis is often used. Breakeven analysis(Rautenstrauch & Villers,1949;Reinhardt,1973; Gibson,1972; Larimore,1974) focuses on making pricing and capacity decisions so as to either break-even or achieve some profit target. Stochastic breakeven analysis extends this approach to the case of uncertainty. In the stochastic case, one attempts to make pricing and capacity decisions so as to maximize the probability of achieving some profit target.

As Starr(1996) writes:

> The breakeven model may be the most significant basis for making intelligent capacity decisions…Breakeven analysis (BEA) has become one of the most fundamental models of production and operations management, providing important information for capacity decisions.

But despite its widespread use(Schweitzer, Trossman and Lawson,1992), it's generally viewed as a heuristic which is not as technically defensible as profit-maximization. Thus in applying breakeven analysis to pricing, Jagpal(1999) writes:

> Break-even pricing is a heuristic for new product pricing under uncertainty. Although the firm does not require explicit demand estimates for different pricing plans, the firm must nonetheless estimate the subjective probability of bankruptcy for each pricing plan.…In general, however, the firm that uses break-even pricing is likely to make excessively conservative decisions.

This conventional wisdom treats breakeven analysis as a heuristic justified mainly by its ease of use.

In this paper, we challenge the conventional wisdom by showing that stochastic breakeven analysis is actually more general and more technically defensible than profit-maximization. To do so, we first follow Sandmo(1971) in assuming the firm wishes to act rationally and therefore acts as if it had a utility function. We then adapt arguments from Borch, Castagnoli, LiCalzi and Bordley showing that a firm interested in maximizing a utility function will act as if it were maximizing the chances of outperforming a stochastic benchmark. This establishes the equivalence between utility-maximization and stochastic break-even analysis.

It's well-known, of course that breakeven analysis and profit maximization can be viewed as special cases of utility-maximization. This paper's contribution is to show that for **any** utility function, there is an equivalent form of stochastic break-even analysis. Hence both profit-maximization and standard break-even analysis can be viewed as special cases of stochastic break-even analysis. We then illustrate this equivalence using a linear profit model.

2. BORCH'S MODEL OF THE FIRM

Borch(1968) considered an insurance firm with some fixed wealth S at the start of some time period. Suppose the firm knows that a certain amount of insurance claims, y, will need to be paid off in the course of the year. The firm is then offered an alternative which will give it a payoff of v. What is the probability of the firm being solvent if it accepts this alternative? This probability is the probability that v exceeds y-S.

In reality, of course, the firm does not know the amount of the insurance claims,Y, which it might receive over the course of the year. Hence instead of thinking of the claims, y, as a known amount, we represent them as a random variable Y where Y can assume a range of possible values with various probabilities. Since we no longer know what value Y will actually assume, we no longer know whether the firm will achieve its objective. Instead we assess the probability that v exceeds Y-S.

In addition, the firm may not know about the payoffs it will attain for accepting the alternative. Hence we need to replace the fixed quantity, v, with a random variable V that likewise can assume a range of possible values with various probabilities. Hence the probability of being solvent if we make this decision is the probability that V exceeds Y-S.

Now Borch(1968) additionally showed that one could define the firm's utility for a consequence v as the probability that v exceeds Y-S. He likewise showed that one could define the firm's utility for uncertain consequences v as the probability that v exceeds Y-S.

He further showed that this did define a utility function by showing that the utility of the uncertain consequence X was the expected value of the utility of the possible consequences x.

(2.2) Extending the Borch Result

The Borch insurance problem can easily be generalized. Suppose the firm has a goal of achieving a target profit of $G=-S$. Let x denote actions which the firm might take. These actions lead to a net profit of $V(x)$. However there are other uncertainties unrelated to x (e.g., lawsuits, accidents, strikes,etc.) which can affect that net profit. We denote the impact of these uncertain events on net income by $Z=-Y$. Then the probability of the firm reaching its target profit of G is the probability that $V(x)>G-Z$.

Following Borch, we define the utility of earning v from actions, x, by the probability that v exceeds $G-Z$. Hence the probability that the firm achieves its target profits given an action x is

$$Pr(V(x)>G-Z) = \sum_v Pr(v>G-Z)Pr(V(x)=v) = \sum_v u(v)Pr(V(x)=v)$$

(2.3) Castagnoli and LiCalzi's Breakthrough

Note that we can define the random variable $T=G-Z$ as the uncertain amount which must be achieved through pricing if the firm is to achieve its goal, G. Hence the firm is trying to maximize the probability that $V(x)$ exceeds T.

Now Castagnoli and LiCalzi observed that for any utility function, u, there exists a random variable T such that $u(v)=Pr(v>T)$. Furthermore if $V(x)$ is a random variable and is independent of T, we have

$$\sum_v u(v)\, Pr(V(x)=v) = \sum_v Pr(v>T)\, Pr(V(x)=v) = Pr(V(x)>T)$$

Thus acting so as to maximize expected utility is the same as acting so as to maximize the chance of meeting our profit target, T.

3. BREAK-EVEN ANALYSIS

(3.1) Linear Breakeven Analysis for Capacity Decisions

This section illustrates these results in the context of a classical breakeven analysis problem with a linear profit function. Specifically suppose that the profit associated with an action x is dx-K where d represents the contribution margin(or marginal profits) and K represents the fixed costs. Then setting $x=K/d$ will allow the firm to break even. If there is a target profit-threshold of t, then setting $x=[K+t]/d$ will allow the firm to achieve this threshold. It's common to refer to d as the contribution margin and $K+t$ as the contribution block.

Now stochastic break-even analysis commonly treats d and K as uncertain and attempts to specify x so as to maximize the probability that dx-K exceeds the corporate profit-threshold,t. Stochastic break-even analysis devotes special attention to E[K/d], i.e., the expected break-even point.

(3.2) Linear Breakeven Analysis with Arbitrary Utility Functions

Now as the previous section suggested, a firm which maximizes its utility for profit is equivalent to a firm which maximizes the probability that profit exceeds some uncertain profit threshold, T. Such a firm will maximize the probability that dx-K exceeds T. Since $T=t+e$, this corresponds to standard stochastic break-even analysis where we add the noise term, e, to the contribution block $K^*=K+t$.

The utility of v, $u(v)$, is the probability that v exceeds T. By choosing different specifications of T, we get different utility functions:

(1) Conventional breakeven analysis corresponds to the case in which T is a known quantity, t

(2) Conventional profit maximization sets $u(v)$ proportional to v which corresponds to assuming that T is uniformly distributed.

(3) Sometimes analysts assume that $u(v)$ is an exponential function of v. This corresponds to assuming that T is exponentially distributed.

Hence generalizing stochastic breakeven analysis to be consistent with arbitrary utility functions simply involves adding a noise term, $e=t-T$, to the contribution block. Since stochastic breakeven analysis already treats K^* as a random variable, adding this noise term does not require any modifications in conventional breakeven analysis.

To ascertain how it affects the expected breakeven point, $E[K^*/d]$, note that the covariance between (e/d) and d is clearly negative. Since

$$\mathit{Covariance}(\{e/d],d\} = E[(e/d)d] - E[e/d]E[d] = -E[e/d]E[d].$$

we conclude that $E[e/d]>0$ if $E[d]>0$. Hence

$$E[(K+t+e)/d] = E[(K+t)/d]+E[e/d]>E[(K+t)/d]$$

Thus the expected breakeven point increases as the uncertainty in e increases.

In other words, the conventional firm interested in breaking even(with no uncertainty about e) will have a lower expected breakeven point than a profit-maximizing firm(which has considerable uncertainty about e.)

(3.3) Modeling Different Attitudes toward Risk

We now present some empirical evidence suggesting that *e* can be treated as normally distributed with the variance of *e* being proportional to the individual's risk-neutrality.

Friedman and Savage(1948) observed that individuals simultaneously buying insurance and lottery tickets seem to be risk-averse for payoffs exceeding some threshold, *t*, and were risk-prone for payoffs less than *t*. Hence they postulated an S-shaped utility distribution. Kahneman and Tversky's celebrated `reference point' work likewise reaffirms the reasonableness of this S-shaped utility model. This corresponds to assuming a unimodal probability distribution over *T* where the reference point is the mode of *T*.

One natural choice of unimodal distribution over *T* is the normal distribution. In this case, the mean of *T* reflects the individual's reference point, the point at which the utility function switches from being convex to concave. Now let u_o denote the utility function of a hypothetical individual whose *T* has a mean of *0* and a standard deviation of one. Let u denote the utility function of our decisionmaker whose *T* has a mean of *T* and a standard deviation of *s*. Then

$$u(x) = Pr(x>T) = Pr((x-ET)/s>(T-ET)/s) = u_0((x-ET)/s)$$

Hence the utility of x to our real individual is just the utility of (x-ET)/s to our hypothetical individual. If we differentiate this by x, we get

$$u'(x) = (1/s)u'_0((x-ET)/s)$$

Differentiating again gives $u''(x) = (1/s^2)\, u''_0((x-ET)/s)$. Hence the index of absolute risk-aversion for our individual (*or* $r(x)=-u''(x)/u'(x)$) is defined by

$$R(x) = (1/s)\, R_0(x)$$

Thus $s = R_0(x)/R(x)$. In other words, the standard deviation of the target is inversely proportional to the individual's risk-aversion.

(3.4) Break-even Analysis:The Normal Case:

In conventional stochastic breakeven analysis, K is the sum of the fixed costs plus t while d is the uncertain contribution margin. Hence the firm breaks even if $dx+K+e>0$ where the mean of e is zero. We assume that d,K and e are normally distributed with means of Ed, EK and 0 and variances of $v(d)$, $v(K)$ and $v(e)$. For simplicity, we will assume that d and K are uncorrelated. By construction, e must be uncorrelated with d and K.

Following Ekern(1979), we define a \`slightly generalized' Geary-Hinckley transformation by

$$z = (x-E(d)-E(K))/[v(K)+x^2v(d)+v(e)\}^{1/2}$$

which will be normally distributed with mean zero and variance one. We also define the Sharpe ratio by

$$S=(xEd-EK]/[v(K)+x^2 v(d) + v(e)]^{1/2}$$

Then the probability of breaking even is the probability that S exceeds Z.

There will be a 50% chance of breaking even when $[xEd-EK]=0$. Hence we can define $x^*=E[K]/E[d]$ as the median breakeven point. This will generally be less than $E[(K+e)/d]$ which is the expected breakeven point. Since the probability that $1>Z$ is 84%, there will be an 84% chance of breaking even when

$$(xEd-EK) = [x^2 v(d) + Var(K)+Var(e)]^{1/2}$$

Defining $v'(d)=v(d)/E^2(d)$, $v'(K)=Var(K)/E^2(d)$ and $v'(e)=Var(e)/E^2(d)$ implies

$$x - x^* = [x^2\, v'(d) + v'(K) + v'(e)]^{1/2}$$

Solving the quadratic equation gives

$$x = [x^* +/- \{[v'(e)+v'(K)][1-2v'(d)] + 2(x^*)^2 v'(d)\}^{1/2}\,]/[1-2v'(d)]$$

This formulation shows that increasing *v'(e)*, which increases the uncertainty in the contribution block, will increase the `high' estimate of the breakeven point.

4. IMPLICATIONS

In this paper, we presented an argument for why breakeven analysis may be technically more defensible than profit-maximization. We can paraphrase the argument as follows:

(1) A firm wishes to act rationally and therefore should act as if it were maximizing the expected value of some utility function. When the utility function is linear, this leads to profit-maximization. When the utility function is the probability of solvency, this leads to breakeven analysis, i.e., to maximizing the probability of achieving some profit target.

(2) Since the firm cannot know about all the possible factors that impact long-run profit, the firm cannot know what short-term profit target must be achieved in order to meet a long-term profit target. Hence breakeven analysis should be modified to allow for an uncertain profit target.

(3) For any utility function, there always exists some random threshold, T, such that the utility function is the probability of exceeding *T*. If we take *T* to be the uncertain profit target, then maximizing that utility function is equivalent to breakeven analysis.

Hence our firm has a choice between utility-maximization or our more general approach toward breakeven analysis. There are several reasons why one might prefer breakeven analysis:

(1) Ease of Use: The standard form of breakeven analysis is generally considered easier to use than the profit maximization criterion because it doesn't require explicit estimation of how sales depend upon price. For similar reasons, breakeven analysis with an uncertain net income target will be easier than utility-maximization---which is even more complicated than the profit-maximization criterion.

(2) Ability to Motivate: A critical problem with many methods for making decisions is that individuals often aren’t motivated to implement those decisions. But the psychological literature establishes that individuals are more motivated to achieve goals.

(3) Familiarity to Individuals: Simon’s work on bounded rationality attests to the effectivenessness and widespread use of goal-setting in organizations. Since individuals are accustomed to thinking in terms of goal-setting, the bankruptcy formulation might be easier to get people to use.

Hence stochastic breakeven analysis, properly used, is not a heuristic and potentially offers a viable, and theoretically rigorous, alternative to profit-maximization.

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