Strategic and Tactical Planning of a Closed-Loop Supply Chain Network: A Linear Physical Programming Approach

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ABSTRACT
In this paper, a single phase Linear Physical Programming (LPP) model is formulated that explores the strategic and tactical planning stages of a Closed-Loop Supply Chain Network (CLSC). The model when solved identifies simultaneously the most economical used-product to re-process in the closed-loop supply chain, the efficient production facilities and the right mix and quantity of goods to be transported across the supply chain. A numerical example is considered to illustrate the methodology.

Keywords: Linear Physical Programming, Closed-loop Supply Chain, Strategic Planning, Tactical Planning, Remanufacturing.

INTRODUCTION

Rapid developments in the technological world are rendering the life span of electronic equipment short, well below their functional limits. It is estimated that 14 to 20 million PCs are retired annually in the US, of which, 20 to 30% may be resold, while the rest are discarded [1], [2]. The current growth in consumption results in increased use of raw material and energy which results in the depletion of world’s finite natural resources and increasing the amount of waste generated. This environmental degradation is considerable and not sustainable by the earth’s eco-system [3]. These environmental issues, in addition to government regulations and economic opportunities are driving many original equipment manufacturers (OEM) to engage in additional series of activities stemming from the reverse supply chain.

A reverse supply chain consists a series of activities that involves retrieving used products from consumers and remanufacture (closed-loop) or recycle (open-loop) them to recover their left-over market value. The combination of forward (traditional) and reverse supply chains is called a closed-loop supply chain (CLSC). While this process is common in majority of the European companies, it is still in its infancy in American companies.

A Supply Chain involves three stages of planning: Strategic, Tactical and Operational planning. Strategic planning primarily deals with the design (what products should be processed/produced in what facilities etc) of the supply chain that is typically a long-range planning performed every few years when a supply chain needs to expand its capabilities. Tactical planning involves the optimization of flow of goods and services across the supply chain and is typically a medium-range planning performed on a monthly basis. Finally, Operational planning is a short-range planning that deals with the day-to-day production planning and inventory issues on the factory floor.

Much work is done in the area of designing forward and reverse supply chains (for example see [2], [3], [4]). However, not many models deal with both the forward and reverse supply chains together. While the forward supply chain models don’t address the issue of EC, the models dealing with reverse supply chain assume that each incoming used product is economical to re-process and each available production facility is efficient enough to re-process the incoming used products. As a result, there is a risk of re-processing uneconomical used products in inefficient facilities. Pochampally and Gupta [5] address these drawbacks in a reverse supply chain and propose a three phase mathematical programming approach for its strategic planning.
This paper extends Pochampally and Gupta’s [5] work and concentrate on the strategic and tactical planning of a CLSC that ideally should involve the following phases:

1. Selection of the most economical product to reprocess in the CLSC
2. Identification of potential production facilities operating in the region
3. Transportation of the right mix and quantities of goods across the CLSC

In this paper, we propose a single phase mathematical model that addresses the three major issues, mentioned above, in the strategic and tactical planning of a CLSC.

**METHODOLOGY**

We formulate a single phase mathematical model for the strategic and tactical planning of a CLSC and employ linear physical programming technique to solve it. The model when solved identifies simultaneously the most economical used-product to re-process in the closed-loop supply chain, the efficient production facilities that remanufacture used-products and/or produce new products and the right mix and quantity of goods to be transported across the supply chain. A numerical example is considered to illustrate the methodology.

We consider the following scenario in our model. Suppose that the manufacturer has incorporated a remanufacturing process for used products into her original production system, so that products can be manufactured directly from raw materials, or remanufactured from used-products. The final demand for the product is met either with new or remanufactured products.

**Nomenclature used in the methodology**

- $A_{iuv}$ = decision variable representing number of used-products of type $i$ transported from collection center $u$ to remanufacturing facility $v$
- $B_{ivw}$ = decision variable representing number of used-products of type $i$ transported from production facility $v$ to demand center $w$
- $b_i$ = probability of breakage of product $i$
- $TA_{uv}$ = cost to transport 1 unit from collection center $u$ to remanufacturing facility $v$
- $TB_{vw}$ = cost to transport 1 unit from remanufacturing facility $v$ to demand center $w$
- $CC_u$ = cost per product retrieved at collection center $u$
- $CNP_v$ = cost to produce 1 unit of new product at production facility $v$
- $CR_v$ = cost to remanufacture at production facility $v$
- $C_{di}$ = disposal cost of product $i$
- $DI_i$ = disposal cost index of component $y$ in product $x$ (0 = lowest, 10 = highest)
- $DT_i$ = disassembly time for product $i$
- $DC$ = disassembly cost/unit time
- $i$ = product type
- $MINTPS$ = minimum through-put per supply
- $N_{ivw}$ = decision variable representing number of new products type $i$ transported from production facility $v$ to demand center $w$
- $Nd_{iw}$ = net demand for product type $i$ (remanufactured or new) at demand center $w$
- $PRC_i$ = % of recyclable contents by weight in product $i$
- $RCYR_i$ = total recycling revenue of product $i$
\[
R_{SRi} = \text{total resale revenue of product } i
\]
\[
RCRI_y^x = \text{recycling revenue index of component } y \text{ in product } x
\]
\[
S_{1v} = \text{storage capacity of remanufacturing facility } v \text{ for used products}
\]
\[
S_{2v} = \text{storage capacity of remanufacturing facility } v \text{ for remanufactured and new products}
\]
\[
S_u = \text{storage capacity of collection center } u
\]
\[
SP_i = \text{selling price of a unit of new product of type } i
\]
\[
SU_{iu} = \text{supply of used product } i \text{ at collection center } U
\]
\[
SF_v = \text{supply of used products at production facility } v, \text{ different from } SU_i, \text{ these are products that are fit for remanufacturing, after accounting for recycled and disposed products + new products}
\]
\[
TP_v = \text{through-put (considering only remanufactured products) of production facility } v
\]
\[
U = \text{collection center}
\]
\[
V = \text{remanufacturing facility}
\]
\[
W = \text{demand center}
\]
\[
W_i = \text{weight of product } i
\]
\[
x_1 = \text{space occupied by 1 unit of used product (square units per product)}
\]
\[
x_2 = \text{space occupied by 1 unit of remanufactured or new product (square units per product)}
\]
\[
Y_v = \text{decision variable signifying selection of production facility } V (1 \text{ if selected, 0 if not)}
\]
\[
Z_{iu} = \text{decision variable representing number of units of product type } i \text{ picked for remanufacturing at collection center } u \text{ (}SU_{iu} - Z_{iu} = \text{recycled or disposed)}
\]
\[
\delta_v = \text{factor that accounts for un-assignable causes of variations at production facility } v
\]

**LINEAR PHYSICAL PROGRAMMING**

Majority of the real world decision making problems are multi-objective in nature and many mathematical programming techniques, such as linear programming (LP), require an aggregate objective function that is subjected to constraints. However, the decision-makers would benefit immensely from a general framework that possesses two key features. First, the framework must allow the problem to retain its multiobjective nature, i.e., the decision-maker should not be forced to form a weighted sum of several criteria for the sole purpose of confirming to the requirements of the technique used. Second, the problem should allow for the possibility of deliberate imprecision in the statement of preferences [8]. Linear physical programming (LPP) possesses both these features making it a versatile tool for multi-criteria optimization problems.

In LPP method [8], four distinct classes, each class comprising of two cases, hard and soft, referring to the sharpness of the preference, are used to allow the decision maker to express his/her preferences for values of criteria not in the traditional form of weights, but in the form of ranges of different degrees of desirability. The classes are defined as follows:

**Soft:**

Class-1S: Smaller-is-better, i.e. minimization
Class-2S: Larger-is-better, i.e. maximization
Class-3S: Value-is-better
Class-4S: Range-is-better

**Hard:**

Class-1H: Must be smaller
Class-2H: Must be larger
Class-3H: Must be equal
Class-4H: Must be in a range

The value of the $p$-th criterion, $g_p$, is categorized according to the preference ranges. For example, preference ranges for Class-1S are:

- **Ideal range**: $g_p \leq t^+_{p1}$
- **Desirable range**: $t^+_{p1} \leq g_p \leq t^+_{p2}$
- **Tolerable range**: $t^+_{p2} \leq g_p \leq t^+_{p3}$
- **Undesirable range**: $t^+_{p3} \leq g_p \leq t^+_{p4}$
- **Highly Undesirable range**: $t^+_{p4} \leq g_p \leq t^+_{p5}$
- **Unacceptable range**: $g_p \geq t^+_{p5}$

The LPP application procedure consists of the following four steps:

**Step 1**: Identify the criteria for evaluation
**Step 2**: For each criterion, specify the preferences based on one of the four classes and determine which of the two cases, *hard* or *soft* applies to it.
**Step 3**: Calculate the incremental weights using the LPP weight algorithm [14], these weights are used in the next step.
**Step 4**: Minimize the function $J$ constructed as a weighted sum of deviations over all ranges and criteria, subjecting each criterion to the constraint that falls into either one of the four soft classes or four hard classes.

$$J = \sum_{p=1}^{P} \sum_{s=2}^{5} \left[ \Delta w_{ps}^+ d^-_{ps} + \Delta w_{ps}^- d^+_{ps} \right]$$

where $P$ represents the total number of criteria, $\Delta w_{ps}^+$, $\Delta w_{ps}^-$ are the incremental weights for the $p$-th criterion, $d^+_{ps}$ and $d^-_{ps}$ represent the deviations of $p$-th criterion from its corresponding target value.

**PROBLEM FORMULATION**

We consider the following cost and revenue criteria in our methodology.

**Costs: Class 1S (Smaller the Better)**

1. **Collection/Retrieval Cost**
   $$\sum_u \sum_i CC_u SU_{iu}$$

2. **Processing Cost = Disassembly cost of used-products + Remanufacturing cost of used products + New Products production cost in the forward supply chain**
   $$\left( DC \sum_i \sum_u \sum_v DT_i A_{inv} \right) + \sum_i \sum_u \sum_v CR_v B_{ivw} + \sum_i \sum_u \sum_v CNP_v N_{ivw}$$

3. **Transportation Costs = Cost of transporting used-products from collection centers to production facility + remanufactured and new products from production facilities to demand centers.**
   $$TA_{inv} \sum_i \sum_u \sum_v A_{inv} + TB_{vw} \sum_i \sum_v \sum_w (B_{ivw} + N_{ivw})$$
4. Disposal Cost: Products that can’t be remanufactured (broken products) or recycled are disposed.
\[
\sum_i \sum_u \{(SU_{iu} - Z_{iu}) DI_i W_i (1 - PRC_i)\}
\]

**Revenues: Class 2S (Larger the Better)**

1. Reuse Revenue
\[
\sum_i \sum_u \{Z_{iu} RSR_i\}
\]

2. Recycle Revenue
\[
\sum_i \sum_u \{(SU_{iu} - Z_{iu}) RCYI_i W_i PRC_i\}
\]

3. New Product Sale Revenue
\[
SP_i * N_{ivw}
\]

**System Constraints**

1. The number of used-products sent to all production facilities from a collection center \( u \) must be equal to the number of used-products picked for remanufacturing at that collection center.
\[
\sum_v A_{inv} = Z_{iu}
\]

2. Demand at each center \( w \) must be met with either by new or remanufactured goods.
\[
\sum_v (B_{iwv} + N_{ivw}) = Nd_{iw} \forall w
\]

3. Number of remanufactured products transported from a production facility \( v \) to a demand center \( w \) = (Number of used products fit for remanufacturing, transported from collection center \( u \) to that production facility)*\( \delta_v \) \( v \) i.e., no loss of products in the supply chain due to reasons other than common cause variations, over which there’s no control. \( \delta_v \) is a factor that accounts for the un-assignable causes of variation at the production facility \( v \).
\[
\sum_w B_{iwv} = \sum_u A_{inv} * \delta_v \forall v
\]

4. Total number of used products of type \( i \) picked for remanufacturing @ \( u \) must be at most equal to the total # of used products fit for remanufacturing.
\[
Z_{iu} \leq SU_{iu} (1 - b_i)
\]

5. Total number of used products of all types collected @ all collection centers must be at least equal to the net demand.
\[
\sum_i \sum_u SU_{iu} \geq \sum_i \sum_w Nd_{iw}
\]

6. Number of remanufactured products must be at most equal to the net demand; this is to avoid excess remanufacturing.
\[
\sum_i \sum_u Z_{iu} \leq \sum_i \sum_w Nd_{iw}
\]

7. Space constraints for used products at production facility \( v \),
\[
x_1 \sum_i \sum_u A_{inv} \leq S_{iv} Y_v
\]

8. Space constraint for new and remanufactured products at production facility \( v \), assuming new and remanufactured products occupy the same space.
\[ \sum_{i} \sum_{w} x_i (B_{iw} + N_{iw}) \leq S_{2v} \cdot Y_v \]

9. Space constraint for used products at collection center

\[ x_i \sum_{v} a_{iw} \leq S_{w} \]

Production Facility’s Potentiality Constraints, valid only for remanufactured products:

10. \((TP_v / SF_v) Y_v \geq \text{MINTPS}\) (MINTPS = minimum throughput per supply)

**Non-Negativity Constraints:**

\[ A_{iwv}, B_{iwv}, N_{iwv}, Z_{iu} \geq 0, \quad \forall \ u, v, w \]

\[ Y_v \in [0, 1] \quad \forall \ v, \ 0 \text{ if facility } v \text{ not selected, } 1 \text{ if selected} \]

**NUMERICAL EXAMPLE**

We consider a Closed-Loop Supply Chain with three collection centers, two production facilities to choose from, two demand centers to be served and three brands of similar products. We consider the collection cost, transportation cost, disposal cost, and recycling revenue, re-use revenue and the revenue from the sale of new product criteria in our numerical example.

The example data we take to implement the LPP model are:

\[
\begin{align*}
CCu &= 0.01; \ SU_{11}=20; \ SU_{12}=25; \ SU_{13}=15; \ SU_{21}=25; \ SU_{22}=18; \ SU_{23}=15; \ SU_{31}=17; \ SU_{32}=9; \\
SU_{33}=15; \ TA_{11}=0.001; \ TA_{12}=0.009; \ TA_{21}=0.01; \ TA_{22}=0.002; \ TA_{31}=0.004; \ TA_{32}=0.003; \\
TB_{11}=0.004; \ TB_{12}=0.003; \ TB_{21}=0.009; \ TB_{22}=0.005; \ DL_{1}=4; \ DL_{2}=6; \ DL_{3}=5; \ W_{1}=0.8; \ W_{2}=1.0; \\
W_{3}=0.9; \ PRC_{1}=0.65; \ PRC_{2}=0.6; \ PRC_{3}=0.75; \ \text{Cd}_{1}=0.02; \ \text{Cd}_{2}=0.05; \ \text{Cd}_{3}=0.03; \ \text{RSR}_{1}=80; \\
\text{RSR}_{2}=80; \ \text{RSR}_{3}=65; \ \text{RCYR}_{1}=5; \ \text{RCYR}_{2}=7; \ \text{RCYR}_{3}=10; \ \text{RCRI}_{1}=7; \ \text{RCRI}_{2}=4; \ \text{RCRI}_{3}=6; \\
\text{SP}_{1}=100; \ \text{SP}_{2}=110; \ \text{SP}_{3}=95; \ \text{Nd}_{11}=20; \ \text{Nd}_{12}=15; \ \text{Nd}_{21}=16; \ \text{Nd}_{22}=22; \ \text{Nd}_{31}=25; \ \text{Nd}_{32}=20; \\
\delta_{1}=0.85; \ \delta_{2}=0.75; \ b_{1}=0.2; \ b_{2}=0.4; \ b_{3}=0.3; \ X_{1}=0.7; \ S_{11}=400; \ S_{12}=400; \ S_{1}=150; \ S_{2}=150; \ S_{3}=150; \\
X_{2}=0.7; \ S_{21}=500; \ S_{22}=500; \ \text{MINTPS}=0.25.
\end{align*}
\]

The target values for the criteria are given in table 1 (target values are scaled by a factor of 10) and table 2 shows the incremental weights obtained using the LPP weight algorithm [8].

**Table 1. Target values of criteria**

<table>
<thead>
<tr>
<th>Criteria</th>
<th>( t_{p1}^+ )</th>
<th>( t_{p2}^+ )</th>
<th>( t_{p3}^+ )</th>
<th>( t_{p4}^+ )</th>
<th>( t_{p5}^+ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g_1 )</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>( g_2 )</td>
<td>5</td>
<td>10</td>
<td>13</td>
<td>17</td>
<td>20</td>
</tr>
<tr>
<td>( g_3 )</td>
<td>2</td>
<td>2.5</td>
<td>5</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>( g_4 )</td>
<td>2</td>
<td>5</td>
<td>7</td>
<td>9</td>
<td>13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Criteria</th>
<th>( t_{p1}^- )</th>
<th>( t_{p2}^- )</th>
<th>( t_{p3}^- )</th>
<th>( t_{p4}^- )</th>
<th>( t_{p5}^- )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g_5 )</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
<td>30</td>
</tr>
<tr>
<td>( g_6 )</td>
<td>10</td>
<td>15</td>
<td>17</td>
<td>19</td>
<td>22</td>
</tr>
<tr>
<td>( g_7 )</td>
<td>15</td>
<td>17</td>
<td>20</td>
<td>25</td>
<td>35</td>
</tr>
</tbody>
</table>
Table 2. Output of PP weight algorithm

<table>
<thead>
<tr>
<th>Criteria</th>
<th>( \Delta w_{p2}^+ )</th>
<th>( \Delta w_{p3}^+ )</th>
<th>( \Delta w_{p4}^+ )</th>
<th>( \Delta w_{p5}^+ )</th>
<th>( \Delta w_{p2}^- )</th>
<th>( \Delta w_{p3}^- )</th>
<th>( \Delta w_{p4}^- )</th>
<th>( \Delta w_{p5}^- )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g_1 )</td>
<td>0.05</td>
<td>0.17</td>
<td>0.748</td>
<td>3.291</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( g_2 )</td>
<td>0.02</td>
<td>0.12667</td>
<td>0.337</td>
<td>2.355467</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( g_3 )</td>
<td>0.2</td>
<td>0.024</td>
<td>1.344</td>
<td>4.285867</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( g_4 )</td>
<td>0.033</td>
<td>0.24667</td>
<td>1.288</td>
<td>2.822</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( g_5 )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.02</td>
<td>0.024</td>
<td>0.052</td>
<td>0.11616</td>
</tr>
<tr>
<td>( g_6 )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.02</td>
<td>0.09</td>
<td>0.132</td>
<td>0.112933</td>
</tr>
<tr>
<td>( g_7 )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.05</td>
<td>0.03</td>
<td>0.035</td>
<td>0.023</td>
</tr>
</tbody>
</table>

Upon solving the LPP model using LINGO, we get the following optimal solution:

\[
A_{211}=10, A_{212}=5, A_{222}=11, A_{231}=9, A_{311}=3, A_{321}=6, B_{211}=16, B_{222}=12, B_{312}=8, N_{111}=20, N_{112}=15, N_{212}=10, N_{321}=25, N_{322}=12, Z_{21}=15, Z_{22}=11, Z_{23}=9, Z_{31}=3, Z_{32}=6, Z_{33}=6, Y_1=1; Y_2=2.
\]

It is obvious from the above solution that both the production facilities were chosen for the network design.

**CONCLUSIONS**

In order to address the critical issues in the strategic and tactical planning stages of a closed-loop supply chain, in this paper, we formulated a unified single-phase mathematical model. We employed linear physical programming technique to solve the model. The model when solved identifies simultaneously the most economical used-products and their quantities to re-process in the supply chain, the efficient production facilities and the right mix and quantities of goods to be transported across the supply chain. A numerical example is considered to illustrate the methodology.

**REFERENCES**
