Abstract # 011-0788

Strategic Inventories in a two-period Cournot Duopoly

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POMS 20th Annual Conference
Orlando, Florida U.S.A.
May 1 to May 4, 2009
1. Introduction and Literature Review

*Strategic Inventories* are inventories carried by supply chain entities for purely *strategic reasons*, even in the absence of the “traditional” reasons to hold inventory. Traditional reasons for holding inventory at a supply chain entity (e.g., manufacturer, retailer, and distributor) have been economies of scale in production, resulting in *cycle inventories*; to hedge against production or distribution delays and ensure timely availability of goods, resulting in *pipeline inventories*; inventories held as *safety stock*, to hedge against demand and supply uncertainty; inventories held to hedge against price fluctuations, termed *speculative inventory* (Anand et al. 2008); and also inventories held to smooth production and thus lower production costs as in (Holt et al. 1960)

Most of the discussions on inventory, in literature on decentralized supply chain co-ordination, have focused on the development of *coordinating contracts*, which develop optimal inventory strategies in each or a combination of the above situations. Recent literature reviews in the area include Tsay et al. (1998), who review supply chain contracting literature focusing on stochastic and deterministic demand supply chain models. Cachon (2003) presents a review on just uncertain demand models. He reviews five different types of contracts – buyback, quantity flexibility, sales rebate, wholesale price and revenue sharing, elucidating the advantages and drawbacks of each kind of contracting mechanism, using a general newsvendor model example, as well as surveying the extant literature on each contract type and also general trends in contracting literature. He also addresses the important classes of supply chain co-ordination problems such as
coordination with horizontal competition, multi-period ordering, asymmetric information, information sharing, two-location base-stock models with forecast updating and such.

Anand et al. (2008) make an interesting conjecture on the booming literature in the area of supply chain coordination that it largely ignores the effect of inventories and concentrates on motivating the right action under asymmetric information or moral hazard. This paper makes an attempt to bridge this gap, by bringing this literature stream closer to another parallel stream of literature in economics that studies vertical control, horizontal competition and such issues, for example, Tirole (1990) and Deneckere (1996), who both study vertical control issues in a non cooperative setting, as well as papers like Saloner (1996), Rotemberg and Saloner (1999) and Mollgaard et al. (2000), who all study inventories in horizontal competition settings, but decoupled from vertical control.

Anand et al (2008) as well as its predecessor working paper Anand et al. (2002) is one of the first papers in recent times that studies strategic inventories and vertical control together, in a multi-period ordering environment. Keskinocak et al. (2008), extend the basic one-manufacturer, two-retailer, two-period model from Anand et al. (2002) to study the interplay of vertical control and strategic inventories in a situation where the manufacturer’s first period capacity is limited.
We attempt here to contribute to expanding this emerging literature stream studying strategic inventories and vertical control together with downstream competition, using a one-manufacturer, two-retailer model, where the two retailers are a Cournot Duopoly.

A Cournot Duopoly is one of the simplest models of economic competition, owing its existence to its discovery by Antoine Augustin Cournot in the 1800s. It consists of two firms that compete on the quantity of product each is selling, which both decide simultaneously. The most important features of a Cournot Duopoly are that both firms produce a homogeneous product, the number of firms is fixed through the selling seasons, and the firms have market power, i.e., each firm’s quantity procurement decision affects the good’s market price. Also, there is no collusion allowed between firms.

Cournot Duopolies have been studied extensively in Economics literature. Some recent works in the area that are related to our work are the following: Saloner (1987) studies a Cournot Duopoly with two ordering periods. Pal (1991) studies a two-period Cournot Duopoly with two production periods and cost differentials. This model is closest to ours, the significant difference being that the cost differential is endogenous to the competing firms, in Pal’s model compared with our model, where the procurement cost for a retailer will depend on the wholesale price set by the manufacturer for that particular period (vertical control).

Matsumura (2002) studies inventories as a strategic weapon in a two-period Cournot Duopoly, but here, too, as in Pal (1991), vertical control is not considered.
2. Model Description

We formulate a dynamic two-period model of Cournot Duopoly with one upstream manufacturer. Demand is price-dependent, linear and the model is full-information. All information required for each Supply Chain entity to make decisions is known to them. The retailers are allowed to carry inventory from period 1 to period 2 at a finite, but small, holding cost.

Figure 1: Schematic of the two-period ordering Cournot Duopoly with one upstream manufacturer

**Period 1:**

- Manufacturer
- Retailer 1
- Retailer 2
- \( q_{11} \)
- \( q_{12} \)
- \( w_1 \)
- \( I_{11}, I_{12} \)

**Period 2:**

- Manufacturer
- Retailer 1
- Retailer 2
- \( q_{21} \)
- \( q_{22} \)
- \( w_2 \)

The interactions between the retailers and manufacturer in each period are modeled as a three-stage game:
1. Manufacturer announces the wholesale price for that period
2. Retailers quote their respective order quantities
3. Manufacturer delivers the ordered quantity, before the start of the selling season

In the first period, from the received quantity, retailers sell the quantity they have earmarked for sale in the first period and carry the rest forward to the second period.

In the second period, the total selling quantity of each retailer equals the quantity ordered in the first period, plus whatever is carried forward from the first period.

Both retailers sell the same product and it is assumed that each retailer can actually sell whatever quantity he wants to, in both periods.

The price realized per unit of product sold, by either retailer in either period, is given by

\[ p = a - bQ \]

where \( a \) and \( b \) are constant parameters and \( Q \) is the total quantity being sold in the market, by both retailers combined, in that period.

### 3. Analysis

**Nomenclature:**

- \( q_{ij} \): order quantity of retailer \( i \) in period \( j \) \((i = 1,2; j = 1,2)\)
- \( I_{ij} \): inventory carried by retailer \( i \) from period \( j \) to period \( j+1 \)
- \( w_j \): wholesale price set by the manufacturer in period \( j \)
- \( h_i \): holding cost of retailer \( i \), to carry one unit of inventory from period 1 to period 2

We solve for a sub-game perfect Nash equilibrium for this two-period game, with three stages in each period, to get closed form expressions for the order quantities of the
retailer in both periods, the wholesale price set by the manufacturer in each period, as well as the inventory carried by the retailers from period 1 to period 2, in equilibrium. We first start with the second period decisions of the retailers and the manufacturer and use those equilibrium quantities, while solving out the first period game.

**Second period decisions of retailer 1:**

The second period profit function of retailer 1 is given by,

\[ \Pi_{21} = a - b(q_{21} + I_{11} + q_{22} + I_{12}) - w_2 q_{21} \]

For maximizing \( \Pi_{21} \), we have, first order condition, the optimal \( q_{21} \) as

\[ q_{21} = \frac{a - w_2}{2b} - I_{11} - \frac{q_{22} + I_{12}}{2} \]

This is the best-response \( q_{21} \) for any \( q_{22} \) set by retailer 2, and is a function of the second period wholesale-price and the inventories carried by both retailers from period 1.

By Symmetry, we have,

\[ q_{22} = \frac{a - w_2}{2b} - I_{12} - \frac{q_{21} + I_{11}}{2} \]

From equation (1) and equation (2), we obtain

\[ q_{21} = \frac{a - w_2}{3b} - I_{11} \]

and

\[ q_{22} = \frac{a - w_2}{3b} - I_{12} \]

**Manufacturer’s second period wholesale price decision.**

The manufacturer’s second period profit function can be written as:
\[ \Pi_m = w_2 (q_{21} + q_{22}) \] .......................... (5)

and is set as solution to \( \max_{w_2} \Pi_m \).

Substituting equilibrium \( q_{21} \) and \( q_{22} \) from (3) and (4) into (5) and taking first order condition, with respect to \( w_2 \), we have the optimal \( w_2 \) for the manufacturer in period 2 as

\[ w_2 = \frac{a}{2} - \frac{3b}{4} (I_{11} + I_{12}) \] .......................... (6)

Substituting (6) into (3) and (4), we get the equilibrium profit function of retailer 1 in period 2 as

\[ \Pi_{r1} = \frac{a^2}{36b} + \frac{a}{12} (I_{11} + I_{12}) - \frac{3b}{16} (I_{11} + I_{12})^2 \] ........... (7)

Similarly from symmetry of retailers,

\[ \Pi_{r22} = \frac{a^2}{36b} + \frac{a}{12} (I_{11} + I_{12}) - \frac{3b}{16} (I_{11} + I_{12})^2 \]

Retailers’ first period decisions:

Retailer 1’s 1st period profit function is

\[ \Pi r_{11} = (a - b(q_{11} + q_{12}))(q_{11} + q_{12}) - w_1(q_{11} + I_{11}) - h_1I_{11} \]

Where \( q_{11} \) and \( I_{11} \) are set by retailer 1, in period 1, to maximize \( \Pi r_{11} + \text{second period equilibrium profit} \).

Writing this out, we have the first period problem as

\[ \max_{q_{11} > 0, q_{12} > 0} [a - b(q_{11} + q_{12})(q_{11} + q_{12}) - w_1(q_{11} + I_{11}) - h_1I_{11} + \frac{a^2}{36b} + \frac{a}{12} (I_{11} + I_{12}) - \frac{3b}{16} (I_{11} + I_{12})^2 \] \ldots (8)

Grouping terms, we have
\[ \max_{q_{11}, 0, I_{11}, 0} \]

\[ -bq_{11}^2 - \frac{3b}{16} I_{11}^2 + (a - bq_{12} - w_1)q_{11} + \left( \frac{a}{12} - w_1 - h_1 - \frac{3b}{8} I_{12} \right) I_{11} + (aq_{12} - bq_{12}^2 + \frac{a^2}{36b} - \frac{3b}{16} I_{12}^2 + \frac{c}{6} \right) I_{12} \]

Now, setting the partial derivative of the above function with respect to \( q_{11} \) to zero and solving for \( q_{11} \), we have:

\[ q_{11} = \frac{(a - w_1)}{2b} - \frac{q_{12}}{2} \text{………………………….. (9)} \]

Similarly, taking partial derivative of the above function with respect to \( I_{11} \) and solving for \( I_{11} \)

\[ I_{11} = \frac{2a}{9b} - \frac{8}{3b} (w_1 + h_1) - I_{12} \text{…………… (10)} \]

Similarly, from symmetry of the retailers

\[ q_{12} = \frac{(a - w_1)}{2b} - \frac{q_{11}}{2} \text{………………………….. (11)} \]

\[ I_{12} = \frac{2a}{9b} - \frac{8}{3b} (w_1 + h_2) - I_{11} \text{…………… (12)} \]

Also, since the retailers are complementary, \( I_{11} = I_{12} \) in equilibrium.

Using the above in (10) and (12), and substituting (9) into (11), we get

\[ q_{11} = q_{12} = \frac{a - w_1}{3b} \]

\[ I_{11} = \frac{a}{9b} - \frac{4}{3b} (w_1 + h_1) \]

\[ I_{12} = \frac{a}{9b} - \frac{4}{3b} (w_1 + h_2) \]

Because, \( I_{11} = I_{12} \), for this to hold, we should have \( h_1 = h_2 \)

Then we can write
The first period wholesale price is decided by the manufacturer as the solution to the problem

\[ \max_{w_1} (q_{11} + q_{12} + I_{11} + I_{12})w_1 \]

Taking first order condition with respect to \( w_1 \) and solving, we get the optimal \( w_1 \) as

\[ w_1 = \frac{2}{5} \left( \frac{a}{3} - h \right) \]

Summarizing the results we have obtained so far, we have

**First Period Decisions:**

\[ w_1 = \frac{2}{5} \left( \frac{a}{3} - h \right) \]

\[ q_{11} = q_{12} = \frac{a - w_1}{3b} = \frac{1}{15b} \left( \frac{13a}{3} + 2h \right) \]

\[ I_{11} = I_{12} = \frac{1}{15b} (8h - a) \]; is positive for only \( a \leq 8h \)

**Second Period Decisions:**

\[ w_2 = \frac{3a - 4h}{5} \]

\[ q_{21} = q_{22} = \frac{a - w_2}{3b} - I_{11} = \frac{1}{15b} (3a - 4h) = q_{22}; \text{ is positive only when } a \geq \frac{4h}{3} \]

**4. Discussion:**

We observe two insights from the analysis section. One is that the strategic inventory quantity is positive only if the reservation price “a” of the product is less than or equal to 8\( h \). This implies that strategic inventory is recommended in a two-period ordering.
Cournot Duopoly, only when the reservation price \((a)\) of the product being sold is reasonably small. Another observation is that the second period ordering quantity of the retailers is positive only for \(a \geq 4h/3\). This also is significant in the sense that, when \(a < 4h/3\), no second period ordering is needed and all second period demand is met from inventory carried from the first period. So, when \(a < 4h/3\), the inventory carried from the first period is “enough” to fully meet the second period demand.

Combining both the results, we can say that for \(4h/3 \leq a \leq 8h\), it is optimal for the retailers in a Cournot Duopoly to carry strategic inventory to the second period, and this inventory is actually fully sufficient to meet second period demand. This also intuitively seems like a good strategy in that price range, since we observe that the second period equilibrium wholesale price set by the manufacturer is much greater than the first (comparing eqn. (6) to eqn. (14)), but this information is not known to the retailer in period 1. However this is only a narrow band of reservation prices, and generally a low range, since \(h\) is a small positive quantity.

As such we can say that strategic inventory is optimal in a two-period ordering Cournot Duopoly, only for low-priced products and too only in a narrow range.

Anand et al. (2008) show that the strategic inventory quantity is positive, when \(a > 4h\), in a one-retailer, one-manufacturer, two-period ordering problem, without any competition. So, we can conclude that, when we expand the said model to consider a two-retailer duopoly, most of this incentive to hold strategic inventory disappears. Also, we observe
that the equilibrium wholesale prices set by the manufacturer in the two periods is higher in our case (Cournot Competition), in both periods, than the corresponding prices considered by Anand et al. (2008), which has no competition, (all other conditions similar), implying that retailer competition forces a regime of higher wholesale prices.

We also observe an interesting trend with the equilibrium wholesale prices under Cournot Competition. If we compare the prices we have obtained to those obtained by Anand et. al. (2008), under a two-period ordering dynamic contract, but without competition, (Compare Equations (14) and (15) to Table 1 of Anand et. al. (2008)) we see that the prices we obtain are lower in both periods, possibly implying that retailer competition forces the manufacturer to lower his wholesale price. The second period equilibrium wholesale price is higher than the first, in Cournot Competition, which is similar to the result under a dynamic contract without competition obtained in Anand et. al. (2008)

Competition also seems to have interesting effects on the total system output. We define the total system output as the total quantity of product sold in the two periods, which is the sum of the quantities ordered in both periods (because, we are assuming that the retailers can sell everything they want to sell in each period, it is only the price-per-unit that varies, depending on the total quantity in the market). Comparing the total system output across the two periods in competition (\(q_{11} + q_{12} + q_{21} + q_{22}\) from above, to \(q_1 + q_2\) from Table 1 of Anand et al (2008), we observe that the system output is higher in a Cournot Duopoly, than for the case with a single monopolistic retailer, implying that
competition promotes each retailer to order more and sell more, thereby increasing the system output.

5. References:


