Price Game Analysis of Leader-Follower Service Providers with Service Delivery Time Guarantees

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Abstract: Under the situation that customers have the same time sensitivity and obey the uniform distribution on a linear city with length one, we study the price game of leader-follower service providers denoted as sp1 and sp2) with two types of service delivery time guarantee. The study shows that if the number of potential customers is fixed, when the customer reservation payment (CRP) is smaller, the two service providers are both local monopoly. With the increase of the CRP, if sp2 choose the longer service delivery time guarantee, the two service providers are still local monopoly and if sp2 choose the shorter service delivery time guarantee, they would compete for some customers. When the CRP is larger, no matter which types of service delivery time guarantee they choose, they will compete for some customers. When the two service providers compete with each other, the follower has the second-move advantage. If the per unit service cost and the service capacity cost are the same, the per unit time profit of sp2 is higher than the per unit time profit of sp1.

Keywords: service delivery time guarantee; service price; service capacity; linear city

1 Introduction

In traditional industries, the price and yield are often the focus of competition between enterprises. Researchers always assume that each production of the products is identical. However, besides price and yield, there are some other differences. In 1929, Hotelling established a price competition model of the two companies with spatial different products\(^1\). In the model, he assumes that the two companies have different locations in the linear city and in order to get the product, consumers should not only pay the price but also spend some transport expenditure. In the modern service industry, consumers always pay their attention to price and the waiting time spending on the service. In fact, geographical differences of service providers also
have some influence on the customer choices. In this paper, based on the Hotelling model, we study the price game of leader-follower service providers with two types of service delivery time guarantee. Our main aim is to answer the question: “how do the two service providers choose their service time guarantee and price the service in order to achieve profits maximization?”

The Existing research results related to this paper mainly have two aspect: one is the price, service capacity and service delivery time guarantee competition of service providers; the other is to study the service price and capacity decision with consideration the locations of service providers. On the first aspect, Chen and Wan (2003) studied the price game of two make-to-order firms with different service capacity, which indicates that the firm with higher service capacity or lower per unit operation cost could take a relatively larger market share with premium price [2]. Based on this, Chen and Wan (2005) studied the service price and service capacity competition of two make-to-order firms with fixed market capacity. The study proved the existence of Nash equilibrium and show that in the equilibrium, the number of firm operating in a market depends on the capacity of the market and the duopoly Nash equilibrium is socially optimal if and only if there is one firm operating in the equilibrium[3]. So.KC (2000) studied the service price and service delivery time guarantee competition of firms with demands sensitive to both price and delivery time guarantees. The research indicated that firms will exploit their distinctive firm characteristics to differentiate their services. Assuming all other factors being equal, the high capacity firms provide better time guarantees, while firms with lower
operating costs offer lower prices, and the differentiation becomes more acute as demands become more time-sensitive[4]. Zhang and Tan (2009) studied the service price competition between two web service providers offering functionally the same web services with service level guarantees. The study shows that in the long term, the two providers intend to choose different service levels and service prices. However, in the shorter term, they may incline to the similar service levels and service prices[5]. Fan and Kumar (2009) studied the service price and software quality short-term and long-term competition between two software service providers (SasS: software as a service, SWS: shrink-wrap software) with the service time guarantee[6]. Pekgun and Griffin[7] analyzed the service price and lead-time competition of two firms under centralized decision making and decentralized decision making[7].

On the second aspect, Dobson and Stavrulaki (2006) developed a model of simultaneous price, location and capacity decisions of a service provider with time-sensitive customers uniformly distributing on a line city. They worked out the optimal solution with the consideration of the shipping delay[8]. On this basis, Kwasnica and Stavrulaki (2008) studied the service capacity and location two-stage competition between the two service providers who can choose their location on a linear city. The study indicates that unless the cost of service capacity obtained is particularly low, two service providers will choose a limited service capacity and focus on the location competition. When the market capacity is large, the two service providers will not compete and service the customers closer to their own facilities while the market demand is small, two service providers will compete for some
customers \cite{9}. Under the assumption that when customers face with a congested facility, they may be balking, reneging and veering, Mohammad and Ebrahim (2010) studied the influences of the customer behaviors on the market shares of multiple service providers. The study shows that the provider with higher service capacity can occupy a higher market share. If the balking ratio is high, the market share of all the providers will be reduced. Price volatility will not have great impact on the market share of each service provider \cite{10}.

In our research, considering the situation that two service providers (sp1, sp2) are located at both ends of a straight line with length 1 and the customers are time-sensitive and uniformly distribute on the linear city, we study the price game of leader-follower service providers with two types of service delivery time guarantee. The distinguishing feature of our work is to consider the spatial difference of the two service providers and two types of service delivery time guarantees constraint. This has significant differences with the research of Zhang (2009). Their research is based on fixed service capacity and does not considering the spatial differences of service providers.

2 Notations and Assumptions

Notations:
\[ \lambda_i \] — customer arrive rate of service provider \( i \), \( i = 1, 2 \)
\[ \mu_i \] — service rate of service provider \( i \), \( i = 1, 2 \)
\[ p_i \] — service price of service provider \( i \), \( i = 1, 2 \)
\[ \gamma_i \] — the unit operation cost of service provider \( i \), \( i = 1, 2 \)
$s_i$—service delivery time guarantee ($i = H, L$ and $s_H > s_L$)

$g(x)$—the function of traveling cost, $x$ is the distance

c — the unit service capacity cost

$\pi_i$—the per unit time profit of service provider $i$ $i = 1, 2$

$v$—customer reservation payment

$c_0$—per unit time waiting cost of customer

**Assumptions:**

(1) For each service provider, the service system is M/M/1 type. Each service provider aims to maximize profit and each customer is target to maximize utility.

(2) The two service providers are located at the two ends of a straight line. Customers uniformly distribute on the linear city with a density $l$. Customer reservation payment and the per unit time waiting cost of customer are the same for each customer.

(3) The two types of service delivery time guarantee are $s_H, s_L$ ($s_H > s_L, s_H$ or $s_L$, they do not include the traveling delay). The two service providers must ensure that the probability of meeting the time guarantee for each service provider must be at least $\alpha$ ($\alpha$ can be 0.95 or 0.98). Each service provider can not occupy the entire market.

**3 Model Formulation**

In this paper, we write the customer utility function as:

$$U = v - c_0 s_j - g(x) - p_j \ (j = H, L, i = 1, 2).$$

Only when it satisfies $U \geq 0$, the customer could choose the service. Otherwise, they will not choose it. We denote the function of transportation cost as: $g(x) = ax$. $x$ is the distance that a customer should travel in order to get the service and $a$ is the per unit
travelling cost of each customer.

As for a M/M/1 queuing system, the requirement that the probability of meeting the time guarantee for each service provider must be at least $\alpha$ can be written as follows (So 2000):

$$1 - e^{-(\mu - \lambda)s_j} \geq \alpha \quad j = H, L$$ (1)

Sp1 is the leader and sp2 is the follower. Based on optimal price and service time guarantee of sp1, sp2 draw its own optimal solutions. If each service provider can not occupy the entire market, then in equilibrium there must exist a point (denote as A, the distance from A to sp1 is denoted as $x$ and then the distance from A to sp2 is $1-x$). Then we can draw an equation as follows:

$$v - c_0s_j - ax - p_1 = v - c_0s_j - a(1-x) - p_2 \quad j = H, L$$ (2)

Sp1 as the leader, he can choose $s_L$ or $s_H$, and formulate the optimal service price. Sp2 as the follower, his choice have a significant impact on the equilibrium solution and the profit of sp1. So sp1 need to analyze the response of sp2 when he choose the each type of service delivery time guarantee and compare the per unit profit. Therefore, we discuss in two cases.

3.1 sp1 choose $s_L$

Based on the analysis above-mentioned, the optimization problem of sp2 can be written as:

$$\max_{\tilde{p}_j, \mu_L, j} \pi_2 = (p_2 - \gamma_2)\tilde{\lambda}_2(p_2, s_j) - c\mu_2$$ (3)

$$\begin{align*}
1 - e^{-(\mu - \lambda)s_j} & \geq \alpha \\
\text{st} \\
v - c_0s_L - ax - p_1 & = v - c_0s_j - a(1-x) - p_2
\end{align*}$$ (4)
\[ \nu - c_o s_j - a(1-x) - p_j \geq 0 \quad (6) \]

Simplifying (4), we obtain:

\[ \mu_j \geq \frac{k}{s_j} + \lambda_j \ln k = -\ln(1-\alpha) \quad j = H, L \quad (7) \]

We do not consider inequality constraint (6) and when \( \pi_j(p_j, \mu_j, j) \) is at optimality, constraint (7) must be binding. So we have:

\[ \mu_j = \frac{k}{s_j} + \lambda_j \ln k = -\ln(1-\alpha) \quad j = H, L \quad (8) \]

Proof: If there is an optimal solution \((p_j^*, \mu_j^*, j^*)\) which make constraint (7) to be strict inequality, then reducing \( \mu_j^* \) can increase the \( \pi_j^* \). So \((p_j^*, \mu_j^*, j^*)\) is not the optimal solution. So when \( \pi_j(p_j, \mu_j) \) is at optimality, constraint (7) must be binding.

As \( s_j \) is either \( s_L \) or \( s_H \), we can regard \( s_j \) as a constant. When we get the solution, we substitute \( s_L \) and \( s_H \) into \( \pi_j(s_j) \) and have a comparison. So we can get the optimal service delivery time guarantee.

Substituting (5) and (8) into (3), we obtain:

\[ \pi(p_j) = (p_j - \gamma_j - c) \frac{1}{2} \left( \frac{p_j - p_1 + c_o(s_j - s_L)}{2a} \right) l - c \frac{k}{s_j} \quad (9) \]

It is not difficult to get the solution of \( \pi(p_j) \):

\[ p_j^* = \frac{a + p_1 + \gamma_j + c - c_o(s_j - s_L)}{2} \quad \mu_j^* = \frac{k}{s_j} \frac{\ln s_j}{2} \quad j = H, L \]

As for \( s_{11} \), its optimization problem can be written as:

\[ \max_{p_j, \mu_j} \pi_1 = (p_j - \gamma_1) \lambda_1(p_j) - c \mu_j \quad (10) \]

\[ 1 - e^{-(\mu_j - \lambda_j)s_L} \geq \alpha \quad (11) \]
\[ s.t. \quad v - c_0 s_L - ax - p_1 = v - c_0 s_L - a(1-x) - p_2 \]  
\[ v - c_0 s_L - ax - p_1 \geq 0 \]  
(13)

For the same method above-mentioned, we can obtain:

\[ \pi(p_1) = (p_1 - \gamma_1 - c)(\frac{p_2 - p_1 + c_0(s_j - s_L)}{2a})l - c \frac{k}{s_L} \]  
(14)

Substituting \( p_2^* \) into (14), we obtain

\[ p_1^* = \frac{3a + \gamma_1 + \gamma_2 + 2c + c_0(s_j - s_L)}{2} \quad j = H, L \]  
(15)

\[ x = \frac{\gamma_2 - \gamma_1 + 3a + c_0(s_j - s_L)}{8a} \quad \mu_i^* = \frac{k}{s_L} + \lambda_i(p_i) \bigg|_{p_i = p_i^*} \]  
(16)

Substituting (15) into \( p_2^* \), we obtain

\[ p_2^* = \frac{5a + \gamma_1 + 3\gamma_2 + 4c - c_0(s_j - s_L)}{4} \quad j = H, L \]

Substituting (15) and (16) into (13) which is equal to (6), we can obtain:

\[ v - c_0 s_L - ax - p_1 = v - c_0 s_L - \frac{5\gamma_2 + 3\gamma_1 + 15a + 8c + 5c_0(s_j - s_L)}{8} \]  
(17)

Denoting \( v_1 = \frac{5\gamma_2 + 3\gamma_1 + 15a + 8c}{8} + c_0 s_L \), \( v_2 = \frac{5\gamma_2 + 3\gamma_1 + 15a + 8c + 5c_0(s_j - s_L)}{8} + c_0 s_L \),
then we have:

(1) If \( v < v_1 \) the utility of customers on the boundary point is negative which means (17) is negative. At this time, the two service providers are local monopoly. They have no need to compete with each other. The optimization problem of sp1 can be written as:

\[ \max_{p_1, \mu_i} \pi_1 = (p_1 - \gamma_1) \lambda_i (p_1) - c \mu_i \]
\[
\begin{aligned}
\text{s.t.} & \quad 1 - e^{-(\mu - \lambda)v} \geq \alpha \\
& \quad v - c_0 s_L - ax - p_1 = 0
\end{aligned}
\]

It is not difficult to get the optimal solution:

\[
\hat{p}_1 = \frac{v - c_0 s_L + \gamma_1 + c}{2} \quad \hat{\mu}_1 = \frac{k}{s} + \lambda_1(p_1) \bigg|_{n=\hat{p}_1}
\]

In the same method, we can get \( \hat{p}_2 \), \( \hat{\mu}_2 \). So we can obtain the optimal per unit time profit of sp1 and sp2:

\[
\pi_1 = \frac{(v - c_0 s_L - \gamma_1 - c)^2 l}{4a} - \frac{ck}{s_L} \quad \pi_2 = \max\left\{\frac{(v - c_0 s_j - \gamma_2 - c)^2 l}{4a} - \frac{ck}{s_j}, \quad j = H, L\right\}
\]

Because of the customer reservation payment is small, the market capacity is large enough for them to realize local monopoly. So the two service providers can choose their optimal solution without considering the choice of the other. The optimal \( \hat{p}_1 \), \( \hat{p}_2 \) is increasing in \( v \). The profits of two providers are decreasing in the unit service capacity cost and the unit operation cost.

(2) If \( v > v_2 \), \( p_1^*, p_2^* \) are the optimal service prices of sp1 and sp2. The per unit time profits are:

\[
\pi_1 = \frac{(3a + \gamma_2 - \gamma_1 + c_0(s_j - s_L))^2 l}{16a} - \frac{ck}{s_L}, \quad \pi_2 = \max\left\{\frac{(5a + \gamma_1 - \gamma_2 - c_0(s_j - s_L))^2 l}{32a} - \frac{ck}{s_j}, \quad j = H, L\right\}
\]

In this case, the two service provider compete with each other for some customers and each customer can get positive utility. From the expression of \( \pi_2 \), we can see that if \( \gamma_1, a, c_0 \) are unchanged, the service delivery time guarantee of sp2 depends on service capacity cost. If the service capacity cost is relatively large, sp2 will choose \( s_H \). Otherwise, he will choose \( s_L \). From the expression of \( \pi_2 \), we can
observe that when sp2 choose $s_H$ the per unit time profit of is relatively larger. This means sp2 choose relatively higher service delivery time guarantee is conducive to sp1. As for sp2, his choice of service delivery time guarantee depends on the specific parameter values. From the expression of $p_1^*, p_2^*$ we can see that $p_1^*$ is increasing in $s_j$ and $p_2^*$ is decreasing in $s_j$. In order to compare the profits of the two service providers, we assume that sp2 also choose $L_s$ and $\gamma = \gamma_2$. We can obtain:

\[
p_1^* = \frac{3a}{2}, \quad p_2^* = \frac{5a}{4}, \quad x = \frac{3}{8}, \quad \pi_1^* - \pi_2^* = -\frac{7a}{32}
\]

Obviously, sp2 occupies a relatively large market share with a lower service price and get higher profit. From this, we can see that when the conditions are the same, the follower sp2 can obtain a second-move advantage and its per unit time profit is on less than the profit when it choose the same service delivery time guarantee with sp1.

(3) If $v_1 \leq v \leq v_2$ then we have: When sp2 choose $s_H$, then the optimal solutions are $(\hat{p}_1, \hat{\mu}_1)$ and $(\hat{p}_2, \hat{\mu}_2)$ and their respective per unit time profits are as follows:

\[
\pi_1'' = \frac{(v - c_0 s_L - \gamma_1 - \gamma_2 - c)^2 L}{4a} - \frac{ck}{s_L}, \quad \pi_2'' = \frac{(v - c_0 s_H - \gamma_1 - \gamma_2 - c)^2 L}{4a} - \frac{ck}{s_H}
\]

When sp2 choose $s_L$, then the optimal solutions are $(\hat{p}_1, \hat{\mu}_1)$ and $(\hat{p}_2, \hat{\mu}_2)$ and their respective per unit time profits are as follows:

\[
\pi_1' = \frac{(3a + \gamma_2 - \gamma_1)^2 L}{16a} - \frac{ck}{s_L}, \quad \pi_2' = \frac{(5a + \gamma_1 - \gamma_2)^2 L}{32a} - \frac{ck}{s_L}
\]

In this case, if sp2 choose $s_L$ then they will be in competition and if sp2 choose $s_H$, then they will be in local monopoly. Sp2 would compare the profit of the two conditions and make his decision. So the per unit time profit of sp2 can be denoted
as $\pi^*_2 = \max(\pi^H_2, \pi^L_2)$ 

Comparing $\pi^H_2$ with $\pi^L_2$ we can obtain:

$$v^* = \sqrt{(5\alpha + \gamma_1' - \gamma_2') / 8 - 4ck(a(s_H - s_L)/s_Hs_L + c_0s_H + \gamma_2' + c)}$$

If $v^* \in (v_1, v_2)$, then we have 

If $v_1 \leq v < v^*$ sp2 will choose $s_L$. 
If $v^* < v \leq v_2$ sp2 will choose $s_H$. 
If $v^* \geq v_2$ sp2 will choose $s_L$.

3.2 sp1 choose $s_H$

The process is similar with the situation when sp1 chooses $s_L$. Denoting

$$v_3 = \frac{5\gamma_2 + 3\gamma_1 + 15a + 8c + 5c_0(s_L - s_H)}{8} + c_0s_H, \quad v_4 = \frac{(5\gamma_2 + 3\gamma_1 + 15a + 8c)}{8} + c_0s_H,$$

we have

(1) If $v < v_3$ the two service providers are local monopoly. The respective per unit time profits of the two service provider are as follows:

$$\pi_1 = (v - c_0s_H - \gamma_1' - c)^2 - \frac{ck}{s_H} \quad \pi_2 = \max(\frac{(v - c_0s_j - \gamma_2' - c)^2}{4a} - \frac{ck}{s_j}, j = H, L)$$

(2) If $v > v_4$ the two service providers are in competition. The respective per unit time profits of the two service provider are as follows:

$$\pi_1 = \frac{(3\alpha + \gamma_2' - \gamma_1' + c_0(s_j - s_H))^2}{16a} - \frac{ck}{s_H} \quad \pi_2 = \max(\frac{(5\alpha + \gamma_1' - \gamma_2' - c_0(s_j - s_H))^2}{32a} - \frac{ck}{s_j}, j = H, L)$$

(3) If $v_3 \leq v \leq v_4$: When sp2 choose $s_H$, the two service providers are local monopoly and their respective per unit time profits are as follows

$$\pi^H_1 = (v - c_0s_H - \gamma_1' - c)^2 - \frac{ck}{s_H} \quad \pi^H_2 = \max(\frac{(v - c_0s_H - \gamma_2' - c)^2}{4a} - \frac{ck}{s_H})$$

When sp2 choose $s_L$ the two service providers are in competition and their
respective per unit time profits are as follows

\[ \pi_i^* = \frac{(3a + \gamma_2 - \gamma_1 + c_0(s_L - s_H))^2}{16a} - \frac{ck}{s_H^i} \]
\[ \pi_2^* = \frac{(5a + \gamma_2 - \gamma_1 + c_0(s_L - s_H))^2}{32a} - \frac{ck}{s_L} \]

The optimal per unit time profit of sp2 is \( \pi_2^* = \max(\pi_2^H, \pi_2^L) \)

4 A Numerical Example

Given the parameters \( \gamma_2 = \gamma_1 = 1 \), \( a = 1 \), \( c = 2 \), \( c_0 = 0.75 \), \( s_H = 4 \), \( s_L = 3.6 \), \( l = 4.5 \), we will compare the optimal service prices and the profits of the two service providers.

(1) When the two service provider in local monopoly, their optimal service price is the same due to the same the unit operation cost. When they are in competition, the optimal service price and the equilibrium point (denote as \( x \)) are provided in Table 1.

From Table 1, we can see that sp2 as the follower can occupy a relatively large market share with a lower service price.

\[
\begin{array}{|c|c|c|}
\hline
(p_1, p_2, x) & s_L (sp2) & s_H (sp2) \\
\hline
s_L (sp1) & (4.5, 4.25, 0.375) & (4.65, 4.175, 0.413) \\
\hline
s_H (sp1) & (4.35, 4.325, 0.338) & (4.5, 4.25, 0.375) \\
\hline
\end{array}
\]

(2) The profit comparison

Provided sp1 choose \( S_L \) the respective profit of sp1 and sp2 when sp2 choose \( s_H, s_L \).
From Figure 1, we can see that if $v > v_1$, then sp2 choosing $s_H$ is beneficial to sp1. From Figure 2, we can observe that if $v \leq v_1$ or $v \geq v_2$, sp2 will choose $s_L$ in order to attain profit maximization and if $v_1 < v < v_2$, there is a point $v^*$: if $v_1 < v < v^*$, then sp2 will choose $s_L$; if $v^* < v < v_2$, then sp2 will choose $s_H$.

When sp1 chooses $s_H$, the respective profit of sp1 and sp2 when sp2 chooses $s_H, s_L$.

From Figure 3, we can see that if $v > v_3$ sp2 choosing $s_H$ is beneficial to sp1.

From Figure 4, we can observe that if $v \leq v_3$ or $v \geq v_4$, sp2 will choose $s_L$ in order to attain profit maximization and if $v_3 < v < v_4$, there is a point $v^*$: if $v_3 < v < v^*$, then sp2 will choose $s_L$; if $v^* < v < v_4$, then sp2 will choose $s_H$.

We know that sp2 can draw its optimal service price, service capacity and service delivery time guarantee after he know the decision of sp1. However, as for sp1, he should study the customer reservation payment, market capacity and other parameters and the optimal response of sp2 when he choose different service delivery time guarantee. Only in this way, he can draw his optimal solution. From the four figures, we can see that if $v \geq v_4$, the optimal service delivery time guarantee of sp1 is $s_L$. 
Because if $v \geq v_4$, form Figure 2, sp2 must choose $s_L$; if sp1 choose $s_H$, form Figure 4, sp2 must choose $s_L$. Then we compare the $\pi_{iL}^L$ (the profit of sp1 when sp1 and sp2 choose $s_L$) with $\pi_{iH}^H$ (the profit of sp1 when sp1 choose $s_H$ and sp2 choose $s_L$). From Figure 1 and Figure three, we can see $\pi_{iL}^L > \pi_{iH}^H$. So the optimal service delivery time guarantee of sp1 is $s_L$ when $v \geq v_4$. With the same method, we can find the optimal service delivery time guarantee of sp1 given the specific value of $v$. If we know the service delivery time guarantee, we can draw the optimal service price and service capacity.

5 Conclusions

In this paper, we study the price game of leader-follower service providers with two types of service delivery time guarantee constraints and changeable service capacity. We mainly resolve the problem that how the leader-follower service providers choose the service delivery time guarantee and pricing the service in order to attain maximum profit. Our research shows that if the number of potential customers is fixed, when the customer reservation payment is smaller, the two service providers are both local monopoly. With the increase of the customer reservation payment, if sp2 choose the longer service delivery time guarantee, the two service providers are still local monopoly and if sp2 choose the shorter service delivery time guarantee, they would compete for some customers. When the customer reservation payment is larger, no matter which types of service delivery time guarantee they choose, they will compete for some customers. When the two service providers compete with each other, the follower has the second-move advantage. If the per unit service cost and the service
capacity cost are the same, the per unit time profit of sp2 is higher than the per unit time profit of sp1. Sp2 can draw its optimal solution after he know the decision of sp1. However, as for sp1 he should study the customer reservation payment, market capacity and other parameters and the optimal response of sp2 when he choose different service delivery time guarantee. Then he can make his best decision.

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