Comparison of periodic-review inventory control policies
in a serial supply chain

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POMS 23rd Annual Conference
Chicago, Illinois, U.S.A.
April 20 to April 23, 2012

ABSTRACT
This study aims to compare (R, S) and (R, S, Qmin) inventory control policies in a serial supply chain. We develop a simulation based genetic algorithm (GA) in order to find the optimal numerical "S" value that minimizes the total supply chain cost (TSCC) and compare our results between two methods.

Keywords: inventory management, serial supply chain, simulation-based genetic algorithm
1. Introduction

In today's global economy, firms need to manage their supply chains effectively, in order to survive in the markets and gain competitive advantage in the growing markets where customer expectations have been rising. Supply chain management aids companies reduce their costs, represent the products in right times, right amounts and right places by performing in better and faster conditions, thus getting the advantage against their competitors.

Supply chain management is very different than the management of one site. The inventory stockpiles at the multiple sites, including both incoming materials and finished products, have complex interrelationships. Effective and efficient management of inventory in the supply chain process has a significant impact on improving the ultimate customer service provided to the customer. (Lee, Billington, 1992: 65)

In order to satisfy customer demand timely, firms need to hold the right amount of inventory. While inventory can protect firms against unpredictable market conditions, can be very costly in a supply chain. Given the primary goal of reducing system-wide cost in a typical supply chain; it is important to take a close look at interaction between different facilities and the impact it has on the inventory policy that should be employed by each facility. (Simchi-Levi, Kaminsky, Simchi-Levi, 2000: 61)

The main contribution of this paper is two-fold; first, to develop the simulation part of the solution methodology using the Microsoft Excel spreadsheet for the sake of implementation simplicity and second, to implement (R, S, Q_{min}) inventory policy developed by Keismüller et.al. (2011) in a serial supply chain and compare it with the classic (R, S) policy. We use simulation based GA to determine “S” numerical value which will minimize the TSCC. In this model, the TSCC consists of two cost components which are holding and shortage costs.
The remainder of the paper is organized as follows. Section 2 considers inventory control in multi-echelon systems and describes two corresponding inventory control policies which are used in this study. In section 3, the solution methodologies are defined. In section 4, the numerical example is presented to test the performance of those policies. Lastly, conclusions are summarized in section 5.

2. Inventory Control in Multi-echelon systems

Inventory has a significant role in a supply chain’s ability to support a firm’s competitive strategy. If a very high level of responsiveness is required by the firm’s competitive strategy, this can be achieved by locating large amounts of inventory close to the customer. On the other hand, a company can use inventory to become more efficient by reducing inventory through centralized stocking. The responsiveness that results from more inventory and the efficiency that results from less inventory is the main trade-off implicit in the inventory driver. (Chopra, Meindl, 2010: 26)

Finding the best balance between such goals is often trivial, and that is why we need inventory models. In most situations some stock is required. The two main factors are economies of scale and uncertainties. “Economies of scale” means we need to order in batches. Uncertainties in supply and demand together with lead-times in production and transportation inevitably create a need for safety stocks. Organizations can reduce their inventories without increasing other costs by using more efficient inventory control tools. (Axsater, 2006: 2)

Multi-echelon inventory models are central to supply chain management. The multi-echelon inventory theory began when Clark and Scarf (1960) published their seminal paper. (Chen, 1999: 73) Clark and Scarf (1960) consider multi-echelon inventory systems for the first time in their study and they also use simulation to evaluate corresponding
dynamic inventory model. Their study is a starting point for an enormous amount of publications on multi-echelon systems.

There are two different decision systems used in multi echelon inventory systems and those are centralized decision system (echelon stock) and decentralized decision system (installation stock). Decentralized decision systems only require local inventory information, while centralized systems require centralized demand information. (Chen, 1999: 75) The centralized decision system, in which an optimal decision to send a batch from one site to another, may depend on the inventory status at all sites and has several disadvantages. In order to use that kind of a decision system, the firm needs to spend an additional cost for data movement despite the advanced information technology. In addition to this, it is difficult to derive complete general centralized policies. As a result of this, it is more suitable to limit the degree of centralization. (Axsater, 2006: 195) On the other hand, relatively independent organizations often control their inventory systems and make their own replenishment decisions, since different facilities are normally situated at locations far from each other in a supply chain. (Petrovic, Roy, Petrovic, 1998: 302-303) (Axsater, 2006: 195) Decentralized decision system does not require any information about the inventory situation at other sites and it is not necessary to explicitly keep track of the stocks at the downstream installations. These are the obvious advantages of this type of decision systems. (Axsater, Rosling, 1993: 1274)

It is natural to think of the physical stock on hand when talking about the stock situation. However, stock on hand cannot be the sole determinant in ordering decision. The outstanding orders that have not yet been delivered should also be included in the equation. Therefore, the stock situation is characterized by the inventory position in inventory control.
Inventory position = stock on hand + outstanding orders – backorders.

Inventory control models that are subject to uncertainty have two types: periodic review and continuous review. In periodic review, the inventory level is known only at discrete points in time, whereas, in continuous review, the inventory level is known at all times. (Nahmias, 2009: 252) In this study, periodic review inventory control policies are considered.

2.1. Periodic review, (R,S) Inventory control policy

This system is commonly seen when companies order from the same supplier, or have shared resources. The control procedure is that every “R” units of time (that is, at each review instant), enough inventory is ordered to raise the inventory position to the level “S”, which is a desirable property when the demand pattern is changing with time. The disadvantage of (R, S) system is that it has higher carrying costs than continuous review systems. A typical behavior of this type of system is shown in Figure 1 (Silver, Pyke, Peterson, 1998: 240)

Figure 1: The (R,S) system (Silver, Pyke, Peterson, 1998: 240)
Most of the time, “R” and “S”, two decision variables, are not independent, that is, the best value of “R” depends on the “S” value, and vice versa. Assuming that “R” has been predetermined without knowledge of the “S” value is still quite reasonable for practical purposes when dealing with B items. (Silver, Pyke, Peterson, 1998: 278) In this study, we assume that the value of “R” is predetermined.

2.2. (R, S, Qmin) Inventory control policy

This simple periodic review policy, called (R, S, Qmin) is proposed by Keismüller et.al. (2011). In this policy, the inventory position is monitored every “R” units of time and if the inventory position is above the level “S”, then no order is placed. In case the inventory position is below the level “S”, an amount of order is placed which equals or exceeds Qmin (minimum order size). An amount larger than Qmin is ordered if the minimal order size Qmin is not sufficient to raise the inventory position up to level S. This policy is a special case of (R, s, t, Qmin) policy which is developed by Zhou et.al. (2007) where s=S−Qmin and t=S−1. (Keismüller, Kok, Dabia, 2011: 281)
Figure 2: The \((R, S, Q_{\text{min}})\) policy (lead time equal to zero) (Keismüller, Kok, Dabia, 2011: 281)

If the demand is always larger than the minimum order quantity, which may happen in case of small values of \(Q_{\text{min}}\), then the order constraint is not restrictive anymore and in that case, the \((R, S, Q_{\text{min}})\) policy is similar to a base-stock level policy \((R, S)\) with base-stock level \(S\). For large values of \(Q_{\text{min}}\) the parameter \(S\) functions as a reorder level only, the policy is equal to an \((R, s, Q_{\text{min}})\) policy with a re-order level \(s\).

Kiesmüller et. al. (2011) prove that the proposed policy is simple to compute and it has an efficient cost performance, which is close to the more sophisticated two-parameter policy developed by Zhou et.al. (2007). Although the proposed policy cannot derive better solutions than \((R, s, t, Q_{\text{min}})\) policy in terms of the cost (since it is a special case of that policy), it is still practical with its computational simplicity.

3. Solution Methodology

3.1. Genetic Algorithm

Genetic algorithm (GA) is a mathematical search technique based on the principles of natural selection and genetic recombination which is firstly proposed by Holland (1975). (Chambers, 1995: 1) The original motivation for the GA approach was a biological
analogy. In the selective breeding of plants or animals, for example, offspring that have certain desirable characteristics are sought — characteristics that are determined at the genetic level by the way the parents’ chromosomes are combined. In the case of GAs, a population of strings is used, and these strings are often referred to in the GA literature as *chromosomes*, while the decision variables within a solution (chromosome) are *genes*. The recombination of strings is carried out using simple analogies of genetic crossover and mutation, and the search is guided by the results of evaluating the objective function *f* for each string in the population. Based on this evaluation, strings that have higher fitness (i.e., represent better solutions) can be identified, and these are given more opportunity to breed. (Glover, Kochenberger, 2003: 58)

The GA search starts with the creation of a random initial population of *N* individuals that might be potential solutions to the problem. Then, these individuals are evaluated for their so-called fitnesses, i.e. of their corresponding objective function values. A mating pool of size *N* is created by selecting individuals with higher fitness scores. This created population is allowed to evolve in successive generations through the following steps: (Marseguerra, Zio, Podofillini, 2002: 158)

1. Selection of a pair of individuals as parents;

2. Crossover of the parents, with generation of two children;

3. Replacement in the population, so as to maintain the population number *N* constant;


The genetic operators of crossover and mutation are applied at this stage in a probabilistic manner which results in some individuals from the mating pool to reproduce. (Chambers, 1995: 1) In general, the parent selection is fitness proportional and the survivor selection
is a generational replacement. The crossover operator is based on exchange of sub trees and the mutation is based on random change in the tree. (Talbi, 2009: 203) Setting values for various parameters, such as crossover rate, population size, and mutation rate is a critical process in implementing a GA. (Mitchell, 1998: 175)

GAs are stochastic search methods that could in principle run for ever, unlike simple neighborhood search methods that terminate when a local optimum is reached. In practice, a termination criterion is needed; common methods are to set a limit on the number of fitness evaluations or the computer clock time, or to track the population’s diversity and stop when this falls below a certain threshold. (Glover, Kochenberger, 2003: 64)

3.2. Simulation Based Genetic Algorithm

Simulation is proved to be one of the best means to analyze and deal with stochastic facets existing in supply chain. Its capability of capturing uncertainty, complex system dynamics and large-scale systems makes it very well suited for supply chain studies. It can help the optimization process by evaluating the impact of alternative policies. (Ding, Benyoucef, Xie, 2005: 610)

![Diagram](image)

**Figure 3: The simulation based optimization framework** (Ding, Benyoucef, Xie, 2005: 613)

Simulation is preferred to compute numbers for real world situations. Simulation successfully handles certain flexibility that decision makers would prefer. A validated
simulation has a better chance of being accepted by end users compared to complicated models. (Kapuscinski, Tayur, 1999: 11)

The analytical objective function and constraints are replaced by one or more discrete event simulation models in simulation optimization. Decision variables are the conditions the simulation is run under, and the performance measure becomes one (or a function of several) of the responses derived by a simulation model. Simulation optimization techniques have generally been applied to systems where the decision variables are quantitative and associated with the amount of some resources available in the model. (Azadivar, Tompkins, 1999: 169-170)

A general simulation-based optimization method includes two essential components: an optimization module that guides the search direction and a simulation module that is used to evaluate performances of candidate solutions (network configuration + operational rules and parameters). In comparison with MP techniques, simulation-based optimization methods employ one or more simulation models as a replacement to the analytical objective function and constraints. The decision variables are the conditions under which the simulation is run. Iterative output of the simulation is used by the optimization module to provide feedback on progress of the search for the optimal solution. (Ding, Benyoucef, Xie, 2005 : 612)

In industrial applications, several search algorithms such as, pattern search, simplex, simulated annealing and GA, have been linked with simulation. These search algorithms successfully bring simulation model to near-optimal solutions. Developed algorithms in the literature showed that GA has the capability to robustly solve large problems and problems with nonnumeric variables. It performed well over the others in solving a wide
variety of simulation problems. (Ding, Benyoucef, Xie, 2005: 612) Thus, in this study we will consider these systems as a combination of GA and simulation.

4. Numerical Example

4.1. The Model

We consider a four stage serial supply chain in which random customer demand occurs at stage 1, retailer; stage 1 orders from stage 2, distributor; stage 2 orders from stage 3, manufacturer; stage 3 orders from stage 4, supplier; and stage 4 orders from an outside raw material supplier that has unlimited supply.

![Serial Supply Chain Model](image)

Figure 4: Serial Supply Chain Model

We develop a simulation based genetic algorithm (GA) in order to find the optimal numerical "S" value that minimizes the total supply chain cost (TSCC), comprising holding and shortage costs, and compare our results between two inventory control methods. Simulation is used to evaluate “S” numerical values generated by the GA.

The objective function of the problem can be formulated as below.

\[
\text{Min (Total Supply Chain Cost)} = \sum_{i=1}^{T} \sum_{j=1}^{N} (h_i I_{i,j} + b_i B_{i,j})
\]

\[h_i = \text{unit inventory holding cost at member } i \quad (i=1 \text{ to } N)\]
\( I_{it} \) = the quantity of on hand inventory at member \( i \) \hspace{1cm} (i=1 \text{ to } N) \\
\( b_i \) = unit shortage cost at member \( i \) \hspace{1cm} (i=1 \text{ to } N) \\
\( B_{it} \) = the quantity of backordered inventory at member \( i \) \hspace{1cm} (i=1 \text{ to } N) \\
\( L_{ti} \) = replenishment lead time with respect to member \( i \) \hspace{1cm} (i=1 \text{ to } N) \\
\( D_{it} \) = demand per unit time at member \( i \) \hspace{1cm} (i=1 \text{ to } N)

In this study, we use a four-stage serial supply chain model which is developed by Daniel and Rajendran (2005). The assumptions of the model are given below.

- There is no lead time for information or order processing, however, there is a combined lead time consisting of processing and transportation at each stage and it is called replenishment lead time. Every member has its respective replenishment lead time and they are 1, 3, 5, 4 days respectively for retailer, distributor, manufacturer and supplier.
- When there is enough on-hand inventory to meet the order of the downstream member, the demand is fully replenished. Otherwise, the unsatisfied demand is backlogged, in other words, placed in the back-order queue.
- Every member has infinite capacity.
- The most downstream member, retailer, faces random customer demand which is assumed to be constant.
- The source of supply of raw materials to the most upstream member, supplier, has infinite raw material availability.
4.2. Application of the inventory control policies

We aim to observe different impacts of the relative inventory control policies in terms of cost reduction on a specific serial supply chain model.

(R,S) inventory control policy application

Inventory level at every member is periodically monitored and if the relative inventory position falls below the pre-specified “S” level, a replenishment order is placed for a quantity that will bring the inventory position back to the pre-specified “S” level. Base-stock level at every member in the supply chain takes integer values.

(R, S, Qmin) inventory control policy application

In this policy, the inventory position is monitored periodically and if the inventory position is above the level “S”, then no order is placed. In case the inventory position is below the level “S”, an amount of order is placed which equals or exceeds Qmin (minimum order size). An amount larger than Qmin is ordered if the minimal order size Qmin is not sufficient to raise the inventory position up to level S.

Since (R,S,Qmin) policy differs most from the order-up-to policy (R,S) or fixed order size policy (R,s,Q) when the numerical values of Qmin is close to the mean period demand, a non-dimensional parameter, m=Qmin/ E [D], is introduced. In our study, we assume that Qmin value is predetermined and it is 38 for all supply chain members while m=0.95.

4.3. Proposed Solution Methodology

Simulation-based GA is used as an experimental method to evaluate the models performance. The supply chain simulation is run for given customer demands generated from a uniform distribution for a specified run length over which the statistic TSCC is collected. Random customer demand is generated uniformly within the range [20, 60] per
unit time. Simulation experiments are carried out with a run length of 1200 days and TSC is noted.

![Excel Link Diagram]

**Figure 5: Excel Link**

The GAtool in MATLAB 7 is used to run the GA. We generate an Excel Link between MATLAB and Microsoft Excel in order to evaluate the performance of the “S” values which are generated by GA and we make a decision about which members of one generation are forced to leave the population in order to make room for an offspring to compete. Additionally, we derive 100 different uniform random number sets and take the average of the objective function (TSCCk) value which is obtained through simulation in order to avoid computational errors that might arise due to the usage of random numbers. A macro is developed in Excel to calculate the average of the objective function (TSCCk) value. Thus, GA derives the “S” values and sends them to the Excel simulation as an input data and the output data of the simulation which is the fitness value (fk) of the chromosome, is sent to GA as an input data.

*Chromosome representation*

This study uses gene-wise chromosome representation. Each chromosome is coded with a set of “S” values representing every member in the chain. In the numerical example, string
length is taken as four and each gene in a chromosome represents the respective installation’s “S” value as shown below in figure 6.

![Figure 6: Chromosome representation](image)

**Initial Population Generation**

The initial population is created by following procedure.

$$S_i^{UL} = \text{Max } D_{i,t} \times \text{Max } L_i$$

$$S_i^{LL} = \text{Min } D_{i,t} \times \text{Min } L_i$$

A random number between [20, 780] is generated, which is assigned as the “S” value for that member and same procedure is repeated for the remaining members. For the retailer considered in this model, the maximum and minimum customer demands are 60 and 20 per unit time respectively. The minimum replenishment lead time is predetermined as 1 day for the retailer. However, if the distributor doesn’t have enough on hand inventory at the time, to fulfill the order of retailer, the lead time will be longer than 1 day. And, in case all upstream members don’t have enough on hand inventory, the replenishment lead time for retailer will be the maximum replenishment lead time, which is the sum of replenishment lead times of the retailer, the distributor, the manufacturer, and the supplier, i.e. 13 days. (i.e. 1+3+5+4 days) Therefore, the initial “S” value for retailer is generated randomly between [20, 780]. According to that procedure, the lower limit and upper limit vectors \([S_i^{UL}, S_i^{LL}]\) for all supply chain members are determined as \([20 60 100 80]\) and \([780 720 540 240]\), respectively.
Selection

In this study, we use the roulette-wheel selection procedure. In roulette selection process, chromosomes are grouped together based on their fitness function values. First, MATLAB sends each chromosome in the initial population over to the simulation in Microsoft Excel via the M-file and the simulation calculates fitness values of those chromosomes. Then those fitness values are again sent from Excel to GATOOL in MATLAB via the M-file. Fitness values for each chromosome are summed up to reach a cumulative fitness value. The process continues by dividing each chromosome’s fitness value by the cumulative fitness value, thus calculating a percentage value for each chromosome. Then, those percentages are lined up in order around a roulette wheel and the selection process starts; a random uniform number between 0 and 1 is selected and whichever chromosome falls into this number is selected to be passed on to the next generation.

Figure 7: Selection flow chart

In the next step, some random changes are made on chromosomes with the help of the genetic operators, in order to obtain better results. Various trials are conducted when determining which genetic operators to use in order to generate the optimum results.
**Crossover**

The crossover operator, by combining the chromosomes of two parents, helps to obtain one or two offspring which have a better fitness function. A single point crossover operator is used in this study. This type of crossover operator generates a random number between 1 and the length of chromosome (N) and this becomes a cut point. Parts of two parents after the cut point are exchanged to form the two offspring.

![Figure 8: A single point crossover representation](image)

**Mutation**

The mutation operator randomly modifies a parent to generate an offspring who will replace it. Since every gene in a chromosome represents the “S” value of the corresponding member, a gene-wise mutation is used in this study.

**Elitism**

This operator aims to ensure the offspring that have the best fitness scores evolve into successive generations. In this study, the number of the offspring that will be allowed to evolve into successive generations is determined to be 2.
**Termination Criteria**

These criteria are determined in order the complete genetic algorithm calculations based on user preferences. Termination criteria include number of generations, time limit, fitness value limit, maximum number of generations in case genetic algorithm generates an identical value. In this study, there is no time limit set and the number of generations is determined to be 100.

As a result of MATLAB GATOOL calculations, and using aforementioned operators, (R,S) policy “S” numerical values are determined as [52 147 227 185] for the retailer, the distributor, the manufacturer and the supplier, respectively. TSCC generated by GATOOL is calculated as 415.832. On the other hand, (R,S, Qmin) policy “S” numerical values are determined as [47 144 224 191], for the retailer, the distributor, the manufacturer and the supplier, respectively. TSCC generated by GATOOL is calculated as 439.951.

5. **Summary and Conclusion**

Supply chain management provides customers with the right product or service at a reasonable price, in the right place, at the right time, and with the best quality possible, thus increasing customer satisfaction. Supply chain managers strive to deliver products or services at the right price in order for customers to gain competitive advantage over competitors. At this point, reducing inventory cost, which is a major part of total supply chain costs, will help provide products or services at a better price. Since demand is stochastic in real life cases, and there is certain replenishment lead time for every member, supply chain members do not have an option to apply lean production techniques, in which the inventory levels are zero. However, the trade-off between the quality of customer service level and the costs should be taken into account carefully while determining the appropriate level of on-hand inventory. Thus, insufficient inventory level
might lead to inferior customer service level and satisfaction albeit a lower product cost. In this study, we aim to determine the optimal level of on-hand inventory in order to minimize supply chain inventory costs. In the decision process, in order to save on time and costs, supply chain managers should prefer a method such as simulation, which better reflects uncertainties of real life situations. In addition to this, they can use a heuristic optimization method, such as genetic algorithm which derives optimal solutions in a short time period. Using a combination of those two methods, thus placing results generated by genetic algorithm into the simulation, they can observe results in several different realistic circumstances.

In our study, we examine the application and measure the performance of the inventory policy (R, S, Qmin) developed by Keismüller et.al.(2011) on the four stage serial supply chain model. This policy was considered on a single item single echelon system with stochastic demand in a previous study. Our study extends (R, S, Qmin) inventory control policy implementation by applying it in a multi echelon system. We develop a simulation model using Microsoft Excel spreadsheet for the sake of implementation simplicity. This simulation model can be used to evaluate the performance of the (R, S, Qmin) inventory policy on various supply chain scenarios.

Afterwards, we compare the relative inventory policy with the classic (R, S) policy. According to our experimental results, the (R, S, Qmin) policy costs slightly more than the classic (R,S) policy, for the given scenario. However, it leads to a better customer service level by avoiding inventory shortages. Also, (R, S, Qmin) policy is more efficient when economies of scale exist. We use a simulation based GA to determine the “S” numerical value which will minimize the TSCC. In this model, the TSCC consists of two cost components which are holding and shortage costs. The solution methodology used in this study is easy to implement and doesn’t require cumbersome mathematical endeavors,
which makes the process practical for users who don’t have advanced level of analytical skills.
6. References


