Availability Payment Design in Public Private Partnership

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Abstract

This paper presents a bi-level stochastic model to optimize availability payment design. We propose a discrete time-varying linear dynamical system with an affine controller to make the NP-hard program solvable. We use an infrastructure public private partnership project to validate the effectiveness and efficiency of the proposed model.

Keywords: Public-Private-Partnership, Availability Payment, Stochastic Programming

Introduction

Availability Payment-based Public Private Partnerships (PPPs) are long-term contracts where the private sector is allocated the responsibilities of designing, building, financing, operating and maintaining the facilities on a public project. In return for their services the private sector is reimbursed through a predetermined performance-based payment plan. As per this plan, the private sector is entitled to receive predetermined payments called Maximum Availability Payments (MAPs) for each time period, with corresponding adjustment payments. The amount of the adjustment payments, usually realized as a deduction, is based on the performance of the project. The relationship of the deduction amount and the performance are also predetermined in the contract, defined as the deduction terms.

From the public sector’s perspective, once the contract is signed, the behavior of the private sector, as well as the performance of the project is out of their control. And the private sector will make its own decisions through the long-term project operation. Therefore the essential problem in this partnership is the conflict of objectives between the two, and the key purpose in designing the availability payment contract is to unify the benefit through incorporating some incentive and corporative mechanisms.

This paper proposes a bi-level stochastic model to describe the problem. In order to make the NP-hard program solvable, a special discrete-time time-varying linear dynamical system is assumed, and an affine controller and the Genetic Algorithm are used to deal with
the two levels respectively. The solution of the problem provides guidance for better design and negotiation about the PPP contract, and is expected to help both parties understand the essential purpose of the partnership, and seek mutual interest more efficiently.

Public Private Partnerships (PPPs) and Performance-based Availability Payment

Public Private Partnerships (PPPs or P3s) is a long-term contract between public and private sectors for mutual benefits. In this partnership, the public seeks flexibility and efficiency of building, financing and managing infrastructure assets by outsourcing some of the responsibilities to the private sector (Abdel Aziz 2007, Kwak, Chih et al. 2009). PPPs have earned a reputation of a promising project delivery method around the world and have been proved to be successful in various kinds of civil infrastructure projects, such as highways, airports, water treatment plants, etc (FHWA 2007, Kwak, Chih et al. 2009, Parker 2011). The public agencies of United States started using PPPs in the early 90s to deliver public infrastructures. Since then, this type of partnership grew fast and the investment in PPPs increased by five times between 1998-2007 and 2008-2010 (Engel, Fischer et al. 2011).

There are large amounts of literatures elaborating the advantages of using PPP such as maintaining and developing public infrastructure with funding scarcity, and transferring the risks to a private partner (Mallett 2008, PEW 2009, Sharma, Cui et al. 2010) However, literature review also indicates that since the long-term PPPs are relatively new to the United States, the public sector seems to have inadequate qualification for evaluating and designing efficient contracts (Buxbaum and Ortiz 2007, Garvin 2009). Due to poor contract designs, there are some failure cases (Examples: the Chicago Skyway and the Indiana Toll Road) leading to strong criticism proving that public sectors are using PPPs as a revenue generating mechanism rather than achieving operational efficiencies (Bel and Foote 2009, Czerwinski and Geddes 2010). Adverse consequences also contribute towards public burden, such as cases where private sectors have filed bankruptcy (example: SR-125); private sectors have earned super profits (example: Chicago Skyway Project); or the private sector has gained excessive profits from a non-compete clause (example: SR-91) (Sharma, Cui et al. 2010).

Among all the PPP contracts, the non-toll mechanisms, or typically called the performance-based availability payment is the newest type used in the United States. In the availability payment contract, the private sector takes the responsibility of all project phases, which usually is known as Design – Build – Finance – Operate – Maintain (DBFOM) form (FHWA 2007b, Mallett 2008). When the project finishes construction and available for use, the private sector will get started to be reimbursed during the operation and maintenance (O&M) phase, based on the performance, or more specifically, the availability of the project.

The availability incentive is designed with regard to the operation and maintenance efforts, and traffic recovery from emergencies. There are two key parameters to be decided during the contract signing and negotiation phase: the Maximum Availability Payments (MAPs) and deduction adjustments for each time period.

The Maximum Availability Payments (MAP) are the upper limits of the payment for each time period (depending on different projects, the time period could be annually, quarterly, or monthly). The MAPs are usually proposed by the public sector in the RFP
document, based on their long-term budget expectation, while are open for negotiation together with some other terms.

The deductions are defined as adjustments to the MAPs. Hence the amount of the deductions can take any value ranging from zero to full MAP (Saage and Ajise 2011). The higher the availability, the smaller the deduction amount will be incurred, and vice versa. The design of deduction terms should be made simultaneously with the MAPs, and fully taking into consideration the uncertainties of private sector’s performance, so that the contract would protect public interests and would provide public sector with a reliable estimate of its financing obligations (Sharma, Cui et al. 2010). Together, the MAPS and the deductions can be regarded as the performance incentives.

Since the availability payment PPP is a relative new concept, there still lack of proper methodologies to systematically analyze the strategies of the two parties, and quantitatively determine the contract parameters. The traditional cost-benefit and Value for Money (VfM) analysis for other PPP type, such as toll-based PPP, does not apply well. Overall, the design for availability payment PPP is complicated and should take the stochastic environment and dynamic decision making process into consideration, while is subject to various constraints from both the public and private parties.

Recognizing the above challenges, we put forward a stochastic two level optimization model with dynamic control and feedback process to depict the behavior of the two parties in the PPP relationship.

**Bi-level Multi-objective Problem for Availability PPP Design**

In the process of contract negotiation and contract execution, the objectives of the parties in the partnership, the public sector and the private sector, are different. The private sector has full authority to decide how to obtain a maximized long-term profit, specifically, how to control the cost of maintenance while reach good performance level so as to get better payment. Hence, the profit not only relies on the detailed contract terms, but also depends on the private sector’s Operation and Maintenance (O&M) strategy during the contracted life time of the project. The public sector, on the other hand, is trying to incentivize the private party to sustain a good performance level of the project through the contract, given the long-term budget constraint.

The following *Figure 1* illustrates the relationships of the two parties as one system. Accordingly, the private sector will operate the project based on the highway condition and the contract terms, in order to maximize its overall profit. The public sector, given the full information of the private sector’s reaction, will be able to design the detailed contract terms so as to obtain an optimized performance level for the sake of social welfare, as well as minimizing the expense. Ideally, the solution of the problem will efficiently guide the private sector to a strategy that performs the best, gains the most, and is within the limited long term budget of the public sector.
The overall problem can be generally expressed as:

\[
\max \sum_{t=1}^{T} y_t^* - \mu \cdot \sum_{t=1}^{T} [MAP_t - Deduction(y_t^*)] \\
\text{subject to} \\
MAP_t - Deduction(y_t^*) \leq Budget(t), t = 1, \cdots, T
\]

\( y_t^* \) solves problems \((t = 1, \cdots, T)\)

\[
\begin{align*}
\max E \left[ \sum_{t=1}^{T-1} \frac{MAP_t - Ded(y_t) - Cost(u(t))}{(1 + i)^{t-1}} + \frac{MAP_T - Ded(y_T)}{(1 + i)^{T-1}} \right] \\
\text{subject to} \\
Ded(y_t) + Cost(u(t)) \leq \eta MAP_t, t = 1, \cdots, T - 1 \\
Ded(y_T) \leq \eta MAP_T \\
x(t + 1) = A(t)x(t) + B(t)u(t) + w(t), t = 0, \cdots, T - 1 \\
y(t) = C(t)x(t) + v(t), t = 0, \cdots, T - 1
\end{align*}
\]

where \( y_t \) is the availability of the project at each time period \( t \); \( MAP_t \) and deduction matrix \( Ded(\cdot) \) are decision variables of contract design for the public sector (level one) problem (1). Given the detailed contract, the private sector (level two) are assumed to decide on the best O&M strategy \( u(t), t = 0, \cdots, T - 1 \) for each time \( t \) to optimize its total discounted profit and output the corresponding project availability \( y_t^* \), according to problem (2). Meanwhile, the contractor should prevent itself from bankrupting with an annual check. The solution of the general problem is expected to provide the contract terms and annual project performance.

In order to achieve the optimized contract terms, the public sector need to iteratively search the different combinations of the \( MAP_t \) and \( Ded(\cdot) \), and check the optimality based
on the second level reaction. Solving the problem requires a generalized heuristic multi-level programming since this is an NP-hard problem. We are going to use the Genetic Algorithm (GA) to deal with the stochastic search. The GA has been applied in many optimal control problems, and is able to obtain the global optimal solution fairly especially when the optimization problem has multimodal objective functions or irregular search spaces (Holland 1975, Glodberg 1989, Liu 2009).

As for the dynamic decision making of the private sector through the term of the contract, we consider some typical systems and assume them to be dynamic discrete-time time-varying linear systems, where $x(t)$ is the system state, $y(t)$ is the system output, and $w(t)$ and $v(t)$ are respectively the stochastic variables for the system operation and output measurement error. More specifically, the system feedback controller $u(t)$, defined as the O&M strategy of the private sector, is assumed to be affine to the system output $y(t)$. Using this so-called affined controller, the second-level problem can be transformed to an equivalent linear optimization problem, thus can be efficiently solved (exactly or approximately) with analytical solutions (Skaf and Boyd 2010).

**Numerical Analysis for Highway Projects**

In order to conduct a more reasonable analysis, we refer to a real project undergoing development with an availability PPP contract, the Presidio Parkway project in California, USA. Some data are directly referred from the project, while some of the information are assumed from a broader perspective. Meanwhile, some assumptions and simplifications are made as follows:

1. The availability of the project $y(t)$ is measured by a metric called Pavement Condition Index (PCI), which ranges from 0 to 100, with 100 representing the best possible condition and 0 representing the worst possible condition (ASTM 2009). For safety concern, the PCI of highway and arterial systems is usually required to be larger than 50.

2. Pavement Condition Index (PCI), literally defined as the highway condition, is also assumed to be the state variable $x(t)$, hence simplify the system model with $y(t) = x(t) = PCI_t$.

3. The deduction function is in the form of percentage of MAPs, and assumed to be a decreasing linear function of the PCI. Meanwhile, we assume that when the $PCI=100$, the deduction is 0. We also assume that the most strict deduction is 100% at PCI=50 (safety threshold), which makes the slope $m \geq -2$. Therefore, the deduction function can be expressed as:

$$Ded(PCI)_t = (m \cdot (x(t) - 100)/100) \cdot MAP_t, -2 \leq m \leq 0$$

(3)

4. The O&M strategy is decided dynamically by the contractor by the end of each year, and is assumed to be operated with no lead time. O&M strategy has different types, and are usually classified according to the condition of the road. As to a large highway system, the annual O&M strategy could be the combination of any of the classes, hence we assume that the controller $u(t)$ equals the $PCI$ jump with the average O&M operation taken for the year $t$. The higher the $u(t)$, the more O&M actions are taken for that year.
(5) We approximate the O&M cost as linear to the controller $u(t)$, which makes

$$\text{cost}(u(t)) = c \cdot u(t)$$  \hspace{1cm} (4)$$

(6) As to the system transition procedure, the PCI annual deterioration and PCI jump due to the O&M operation are both assumed to be linear functions, which makes parameters $A(t)$ and $B(t)$ scalars $a$ and $b$.

To solve the second level affine controller problem, we use the classic $Q$-design procedure discussed in (Skaf and Boyd 2010) to transform the complicated non-convex problem to a solvable convex problem. Meanwhile, due to the stochastic property of the parameter $w(t)$, a Monte Carlo simulation process will be utilized as an approximation for the objectives and the constraints whichever are uncertain in nature, for both the two level problems. Therefore, the ultimate bi-level stochastic model can be expressed as problem (5):

$$\begin{align*}
\max \frac{1}{\bar{\Omega}} \sum_{k=1}^{\bar{\Omega}} \sum_{t=1}^{T} & \left[ x^{(k)}(t) - \mu \cdot MAP_t + \mu \cdot \left( \frac{m \cdot (x^{(k)}(t) - 100)}{100} \right) \cdot MAP_t \right] \\
\text{subject to} \quad & MAP_t - \left( \frac{m \cdot (x^{(k)}(t) - 100)}{100} \right) \cdot MAP_t \leq Budget(t), \quad t = 1, \cdots, T \\
& -2 \leq m \leq 0 \\
\end{align*}$$

\begin{align*}
x(t) &= (I + HQ)GW + (I + HQ^*)x_0 + Hr^* \\
Q^*, r^* &\text{ solve the second level problem}
\end{align*}$$

$$\begin{align*}
\min \frac{1}{M} \sum_{j=1}^{M} & \left[ \sum_{t=1}^{T-1} \left( \frac{1}{(1 + i)^{t-1}} \cdot \left( \frac{m \cdot MAP_t}{100} \right) \cdot x^{(j)}(t) + c \cdot u^{(j)}(t) \right) \\
& + \frac{1}{(1 + i)^{T-1}} \cdot \left( \frac{m \cdot MAP_T}{100} \right) \cdot x^{(j)}(T) \right] \\
\text{subject to} \quad & (\frac{m \cdot MAP_t}{100}) \cdot x^{(j)}(t) + c \cdot u^{(j)}(t) \leq (\eta + m) \cdot MAP_t, \quad t = 1, \cdots, T - 1, \quad j = 1, \cdots, M \\
& (\frac{m \cdot MAP_T}{100}) \cdot x^{(j)}(T) \leq (\eta + m) \cdot MAP_T, \quad j = 1, \cdots, M \\
& 50 \leq x^{(j)}(t) \leq 100, \quad t = 1, \cdots, T, \quad j = 1, \cdots, M \\
& u^{(j)}(t) \geq 0, \quad t = 1, \cdots, T - 1, \quad j = 1, \cdots, M \\
& x^{(j)}(t) = (I + HQ)GW^{(j)} + (I + HQ)x_0 + Hr \\
& u^{(j)}(t) = Q(GW^{(j)} + x_0) + r
\end{align*}$$

Specifically, all the parameters are summarized in the following Table 1 to initialize the model.
<table>
<thead>
<tr>
<th>Parameter for Model Initialization</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contract concession period</td>
<td>$T=10$</td>
</tr>
<tr>
<td>Highway deterioration</td>
<td>$x(t+1)=0.9259 \cdot x(t) + u(t)$</td>
</tr>
<tr>
<td>PCI jump due to O&amp;M</td>
<td>$\text{Cost}(u(t)) = 3280.6 \cdot u(t)/\text{mile} \cdot \text{lane}$</td>
</tr>
<tr>
<td>Budget for Public</td>
<td>$$100M$</td>
</tr>
<tr>
<td>Stochastic Condition Impact $w(t)$</td>
<td>Normal distribution with $\mu = -5, \sigma^2 = 1, w(t) \leq 0$.</td>
</tr>
<tr>
<td>Bankrupt threshold</td>
<td>$\eta = 0.8$</td>
</tr>
<tr>
<td>Risk Free Rate</td>
<td>$i = 3%$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters for Genetic Algorithm Setup</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Variable</td>
<td>$N=T+1$</td>
</tr>
<tr>
<td>Size of Population</td>
<td>$K=10^N$</td>
</tr>
<tr>
<td>Number of Generation</td>
<td>$G=K$</td>
</tr>
<tr>
<td>Number of Sampling</td>
<td>$\Omega = 100$</td>
</tr>
<tr>
<td>Probability of Crossover</td>
<td>$P_c = 0.7$</td>
</tr>
<tr>
<td>Probability of Mutation</td>
<td>$P_m = 0.1$</td>
</tr>
</tbody>
</table>

**Results and Discussion**

The bi-level stochastic model is run with the parameters according to the assumptions. With the solutions evolving, the model obtains 79 different feasible solutions. It is worth noticing that the heuristic algorithm such as GA is not guaranteed to obtain an optimized analytical solution, instead, it is to search the feasible region, and meant to evolve towards a better solution through each iteration. Therefore, it is more important to analyze the evolving trend of the solutions. The results are summarized in Figure 2 through Figure 5, with observations and analysis discussed thereafter.

**Observation 1:** With contract design improves, the performance of the highway will be increased, and the public expense is reduced (Figure 2).

![Figure 2 Solutions of the bi-level stochastic contract design model](image)

**Observation 2:** A better contract doesn’t simply mean higher MAPs or stricter deduction terms. Instead, it obtains a combination and trade-off between the two.
As shown in the Figure 3, along with the optimizing of the problem, the better solutions converge with the mean MAPs ranging from $40M to $48M, and deduction slope ranging from -1.5 to -1.3 (the smaller, the stricter the deduction function). Meanwhile, an above-average MAP usually comes together with a stricter deduction term, and vice versa.

**Observation 3**: Better contract tend to incentivize contractors to conduct proactive O&M strategy, which will bring about a constant high performance of the highway condition. On the contrary, worse contract design only force contractors to conduct reactive O&M strategy, resulting a decreasing performance along with the contract period.
**Observation 4:** Improved contract not only increase the highway performance, but also enhance the benefit of both parties.

As shown in Table 2, better contract design tends to increase the highway performance, hence reduce the deduction amount. Meanwhile, both parties benefit from the well-designed combination of MAPs and deductions, in terms of their annual cash flows.

| Table 2 Comparison between 10 Better Contracts and 10 Worse Contracts—Cash Flow |
|---------------------------------|----------------|----------------|
|                                 | 10 Better Contracts | 10 Worse Contracts |
| Public Annual CF ($M)           | -42.985          | -47.405          |
| Private Annual CF ($M)          | 42.659           | 35.296           |
| Annual Deduction ($M)           | 1.490            | 2.666            |

**Observation 5:** Summarized from the 10 best contract of the solutions, the MAP for each year should not be a constant number, or with an escalation. Instead, in order to maximize the highway performance, the MAPs should follow the pattern as in the Figure 5.

In fact, if the MAPs remains constant and is valued as the average of the 10 best contracts, $44.48M, the average PCI will be 95.12, which is less than the average performance of the 10 better contract, which is 96.17.

![Figure 5 MAP Design of the 10 Better Contracts](image)

**Conclusions**

In this paper, we consider a complicated bi-level procedure of the availability payment PPP design. The contractor’s O&M strategy is regarded as a dynamic decision making process, which is solved by a time-varying affine controller program. And contract design from the public side is then iteratively solved through heuristic Genetic Algorithm. Applying the methodology on the numerical case study of the Presidio Parkway project in California, the contract design is dramatically improved through 100 runs of the GA program. The evolving trend shows that the contractor’s behavior is improved from ‘reactive’ O&M towards ‘proactive’ O&M strategy, and the performance of the highway project is increased and is remained at a high standard. Both parties benefit from better design of the contract in terms of their annual cash flows. And the solved pattern of annual MAP is recommended to replace the common practice of constant MAPs or annual escalation.
References


