Seat allocation and pricing in a duopoly - a theoretical perspective

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Abstract

In this paper, we study the simultaneous problem of seat inventory allocation and pricing in a duopoly in the airline industry. Our problem deals with the setting of booking limits and prices in a two fare-class scenario when passenger overflow from one airline to the other is allowed in the same fare-class in response to price differences. Our game allows the competing airlines to follow two pricing strategies. At each strategy, the seat allocated and the revenue earned is calculated. We then develop the conditions under which one or both these strategies achieve Nash equilibrium.

Keywords: Revenue management, passenger overflow, Nash equilibrium

Introduction and related literature

The practice of managing perishable assets by controlling their availability and/or prices with an objective to maximize the total revenue is known as Revenue Management (RM) or Yield Management (YM). According to Talluri and van Ryzin (2005), the RM related decisions can be categorized into three basic classes: (i) Structural decisions, (ii) Price decisions, and (iii) Quantity decisions. Seat inventory allocation falls in the purview of quantity-based RM. The practice of limiting the number of seats made available to different fare-classes that share a common cabin in an aircraft is known as airline seat inventory control. Its objective is to balance the number of passengers in each fare-class in order to maximize total revenues.

The most important early works are those by Littlewood (1972), Belobaba (1978) and Belobaba and Weatherford (1996) that center around the development of the expected marginal revenue analysis techniques. These methods are utilized in many heuristic approaches to the RM problem and include many highly restrictive assumptions (Tan, 1992). The issue of finding optimal nested booking limits under the existing forecasting archetype of estimating the total demand to come was resolved in Brumelle and McGill (1993), Curry (1990), Robinson (1995), and Wollmer (1992). Brumelle and Walczak (2003) studied the dynamic nature of the RM problem with multiple
demands. The seat allocation problem in a flight network was studied by Bertsimas and Popescu (2003). They also proposed approximate dynamic programming approach for network RM.

However, the early works, cited above, make the assumption of a monopolistic market environment. This assumption was later relaxed by Netessine and Shumsky (2005) to consider competition. This stream of work was then expanded by others including Li et al. (2008), Raza and Akgunduz (2008), and Gao et al. (2010) to consider customer overflow between competing airlines. However these works do not consider price differentiation between the competing airlines and the effect of price on the customer overflow.

Game theory provides general mathematical techniques for analyzing situations in which two or more individuals (called players or agents) make decisions that influence one another’s welfare. According to Mascollel et al. (2005), a game can be described by four elements, viz. (i) Players or agents which are individuals or a group of individuals that make a decision, (ii) Rules that specify who moves when, what they do, what they know when they make their move and so on, (iii) Outcomes that show what happens when each player plays in a particular way and (iv) Payoffs i.e. the players preferences over the possible outcomes. Game theory can be classified as cooperative or non-cooperative. In a cooperative game, the players operate under an axiom. As per the Websters Dictionary axiom is a proposition that is not susceptible of proof or disproof; its truth is assumed to be self-evident. Thus, in cooperative game theory there are no set external rules. The players follow a natural order that is accepted as a rule. In a non-cooperative game, however, all choices are decided by the players based on their own self-interest, presumably without sharing knowledge. Here, the players do not have the option of planning, as a group, prior to taking the action. In the strategic form the payoff for a given player depends on the strategy of that player and all other participating players. The rules and all available strategies are assumed to be common knowledge. There are no unfair advantages or insider knowledge.

Nash equilibrium is a solution concept of a non-cooperative game involving two or more players, in which each player is assumed to know the equilibrium strategies of the other players. In game theory, the best response is the strategy (or strategies) which produces the most favorable outcome for a player, taking other players’ strategies as given.

In this paper we attempt to tackle simultaneously the problems of seat inventory allocation and pricing in a duopoly where each of the competing airlines operate two fare-classes. We consider the possibility that the similar fare-class may be priced differently, allow for overflow of passengers in the similar fare-class and define a non-cooperative game that the two players i.e. the two competing airlines, play by. For a two player, two strategy game, we prove that pure strategy Nash equilibrium exists when both players follow the equal pricing strategy.

The problem

In this paper we deal with seat allocation and pricing in a duopoly for competitors who offer direct flights between the same origin and destination. We evaluate different pricing strategies that maybe adopted by the competitors and locate the price points and corresponding seat allocation at which the revenues earned by the competitors achieve Nash equilibrium.
The main features of our problem are: (i) we consider a duopoly where each airline operates two fare-classes only; (ii) we model a static realization of the simultaneous seat allocation and pricing problem (iii) price differentiation in the same fare-class between the competing airlines is considered, i.e. $p^i_n \neq p^j_n$, where $n = 1, 2$ represent the high-fare and low-fare classes respectively of a particular airline and $i, j$ represent the competing airlines; (iv) overflow of passengers, in the same fare-class, is considered when the price of tickets in their first choice airline is higher than a pre-defined threshold price; (v) buy-up, buy-down and cancellations are not considered.

We assume that there are no other flights operating within this time-frame for the same route and both flights have the same capacity; when the price of tickets in a passenger’s first choice airline is higher than a pre-defined threshold price, the passenger overflows to the second-choice airline in the same fare-class i.e. the customers are driven away by increasing prices; if all passengers who overflow cannot be accommodated by the competitor, then the passengers do not return to the higher priced airline but are lost; a ticket purchased at either fare gives access to the same product viz. a coach-class seat on one flight leg; the service restrictions placed on the low-fare tickets is the same for both the airlines; the two flights offer the same fare structure and appeal to the same market.

The non-cooperative game is defined as follows: there are only two players, airlines $i$ and $j$, who operate between two points in a specific time interval; the airlines know the times of their own flights and that of their competitor; the capacity of their planes and that of their competitor is known; the no. of fare classes in their planes and that of the competitor at the time of booking is known; the exact fare of each of those fare-classes for their own as well as that of the competitor at the time of booking is known; the market share enjoyed by themselves and their competitor is known; the average demand for high and low fare-class tickets for themselves and for their competitor is known; the airlines know the no. of seats that they have reserved for the lower fare-class in their planes; the no. of seats that the competitor has reserved for the lower fare-class is not known.

The two pricing strategies adopted by the competing airlines are:

S1: Both classes are priced at the base price
S2: Class 1 is priced at a predetermined percentage higher than the base price, Class 2 is priced at the base price

**The method**

We determine the revenue earned by the two airlines when they follow the two different pricing strategies as explained below. Table 1 may be referred to for a clearer understanding.

- When the airlines follow strategy 1
  - Class 1, the higher fare-class, is priced at $p$ or $\alpha p$, where $\alpha > 1$ and,
  - Class 2, the lower fare-class, is priced at $\beta p$, where $0 < \beta < 1$. 

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When the airlines follow strategy 2

- Class 1 for airline $i$ is priced at $p$ and for airline $j$ is priced at $\alpha p$ or vice versa and,
- Class 2 for both airlines is priced at $\beta p$.

Table 1: Strategies employed & revenues earned

<table>
<thead>
<tr>
<th>Airline</th>
<th>$p, \beta p$</th>
<th>$\alpha p, \beta p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p, \beta p$</td>
<td>Case a: $R_i^a, R_j^a$</td>
<td>Case d: $R_i^d, R_j^d$</td>
</tr>
<tr>
<td>$\alpha p, \beta p$</td>
<td>Case b: $R_i^b, R_j^b$</td>
<td>Case c: $R_i^c, R_j^c$</td>
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The Expected Marginal Seat Revenue (EMSR) method is used to calculate the seat protection level of Class 1 customers of an airline assuming that the demand, $D_n^k \sim U(a, b)$, where $n = 1, 2$, $k = i, j$, and $a$ and $b$ are the highest & lowest demands respectively for a particular fare-class for an airline. The highest demand for a particular fare-class of an airline is considered to be the base demand plus the overflow of passengers from the higher-priced airline, when the price is higher than a pre-determined threshold price. The overflow equation is adapted from that proposed by Feichtinger and Dockner (1985) where the overflow is a function of market share of the competitors and the behavior of customers when there is a difference in price between competitors for similar products. According to the authors, the overflow of customers from firm $j(i)$ to firm $i(j)$ increases more than proportionally with increasing difference between its price and the threshold price, $(\alpha - \alpha_t)p$, and linearly with the current market share of firm $j(i)$. Here $\alpha_t p$ is the predetermined threshold price above which the overflow occurs. The revenue for each class is determined as the product of the class fare and the number of seats booked at that fare, including any overflow, if applicable. The total revenue earned is calculated as the sum of the revenues earned for individual fare-classes.

Once the total revenue functions for each competitor are computed, the Nash equilibrium points are determined using the best response method. It is proved that Nash equilibrium exists when Strategy 1 is followed i.e. when $R_i^a \geq R_i^d \Rightarrow R_j^a \geq R_j^d$ and when $R_i^c \geq R_i^b \Rightarrow R_j^c \geq R_j^b$. We also show that Nash equilibrium does not exist when Strategy 2 is followed. Applying the Fourier-Motzkin elimination method we show the non-feasibility of the associated equations and prove that when $R_i^b \geq R_i^c$, $R_j^b \not\geq R_j^c$ and when $R_i^d \geq R_i^c$, $R_j^d \not\geq R_j^c$, or vice versa.

Conclusion

In this paper, we compute the seat allocation and revenue earned by the competitors in a duopoly when they adopt different pricing strategies. Our problem considers the overflow of passengers in the same fare-class in response to a price differential between the competitors. For a
2-strategy case, we prove the existence of Nash equilibrium when both fare-classes of the competing airlines are equally priced. We also prove that Nash equilibrium does not exist when one of the fare-classes is differentially priced. We can conclude from this that differential pricing does not hold any significance for the competing airlines from an operational perspective, when the market shares are equal.

**Bibliography**


