Robust Design of Differential Operational Amplifiers by Constrained Optimization

Tapan P Bagchi[a] and Nasiba Mandal[b]
[a]KIIT University, Bhubaneswar, India
[b]NMIMS University, Shirpur, India

Abstract - Two separate solutions to a challenging robust design problem—that of a differential operational amplifier used in telephone equipment—were developed by Phadke (1986, 1989) based on Taguchi's Orthogonal Inner Array ×Outer Array experimental approach and S/N ratios. This present study re-visits that problem in view of several new techniques that have evolved since then. This study finds these newer methods to be remarkably more effective. It is also shown that for such robust design problems marshalling these newer methods is no more complex, and hence could be valuably adapted to similar product or process design challenges.

Keywords - Robust Design, Electronic Devices, Operational Amplifiers, Orthogonal Arrays, Multiple Regression, Constrained Optimization

1. Introduction

In electronics, operational amplifiers (Op Amps) have proven to be especially useful in that by the proper selection of some external components these may be configured for amplification, addition, subtraction, differentiation or integration. Op Amps, originally made of vacuum tubes, subsequently were transistorized, and later formed in CMOS as integrated circuits (ICs). In telecom engineering Op Amps find many uses when power, for instance, must be supplied to devices to near exact specification. The present study involved re-designing an Op Amp designed originally by AT&T engineers (Phadke 1986). Their challenge was twofold—(1) the product must satisfy all design requirements, and (2) it must retain its exacting performance over a *wide range* of operating conditions.

This last characteristic is "robustness" (Taguchi 1086; Phadke 1989). Traditional engineering design typically delivers variation-sensitive performance. To a degree this issue is addressed by designers by a rigorous analysis of component sensitivity on design performance (Wilson and Hannaford, 2001). The requirement of robustness was imposed by Taguchi, who highlighted its value of in the presence of uncontrollable factors in the fabrication process, and also in the environment of operation. To seek robustness *ahead* of going to production, Taguchi's Inner Array × Outer Array (IA_OA) experimental scheme that identifies wrong design factor settings that impair robustness, and constrained optimization (Burmen et al 2002; Fey 2011), are most common tools.

Manufacturing process variations can also cause the device's final performance to deviate. To this end, strategies adopted are parametric yield estimation, parametric yield maximization, worst case analysis, and variability reduction. Taguchi (1978) showed that in many situations prudent manipulation of design factors at the devise design stage can minimize interactions with noise and thus produce a *robust* design. Two near-optimal solutions to a challenging robust design problem—that of a differential operational amplifier used in telephone equipment (Figure 1)—were presented by Phadke (1986, 1989) based on Taguchi's Orthogonal approach. This present study re-visits that problem in view of several new techniques that have evolved since then. But these methods deliver the same goal—on target performance with minimum sensitivity of system performance to noise. The results presented here are superior in quality to the original OA-based solutions. This paper also shows that, the application of these other methods is no more complex than conventional robust design.

Figure 1 shows the circuit mockup for this device that parallels the Op Amp targeted for robust design by Phadke (1986, 1989). The details may be found in the sources referred to. Components are retained at values close to those of Phadke, except that the transistors are 2N3702 and 2N2222 A—shown here only for illustration.

As specified in Section 8.1, Phadke (1989), this device was to be used as a preamplifier with the target value of its offset voltage (output VO) being zero volts. This voltage should be made robust in that VO measured by XMM1 (Figure 1) should remain close to zero under all noise conditions arising out of varying manufacturing and environmental conditions. The control or design factors were five in number—RFM (R5), RPEM (R2), RNEM (R3), and current sources I1 (OCS) and I2 (CPCS)—utilized to achieve two design goals, (a) average VO = 0.0 volts, and also (b) exhibit minimum variance when under the influence of noise.

Phadke simulated the influence of noise on VO by using an L_{36} orthogonal array (Tables 8.1 and 8.2, Phadke 1989). The results of these experiments conducted using a computer simulation model for circuit analysis, the kernel of Phadke's work, are shown in Table 8.6 of Phadke (1989). The present study that is based on constrained optimization to seek robustness exclusively uses this data, utilizing the Mean Offset Voltage numbers from Column 2 and Variance of Offset Voltage from Column 3 of Phadke's Table 8.6.

Phadke subsequently used S/N ratio data (Column 4 of Table 8.6), factor effect plots generated from this data (Figure 8.4, Phadke 1989) and ANOVA, of experimentally recorded Mean Offset Voltage (VO) and S/N ratio to quantitatively reach the following two "optimal" robust designs for the differential Op Amp device:

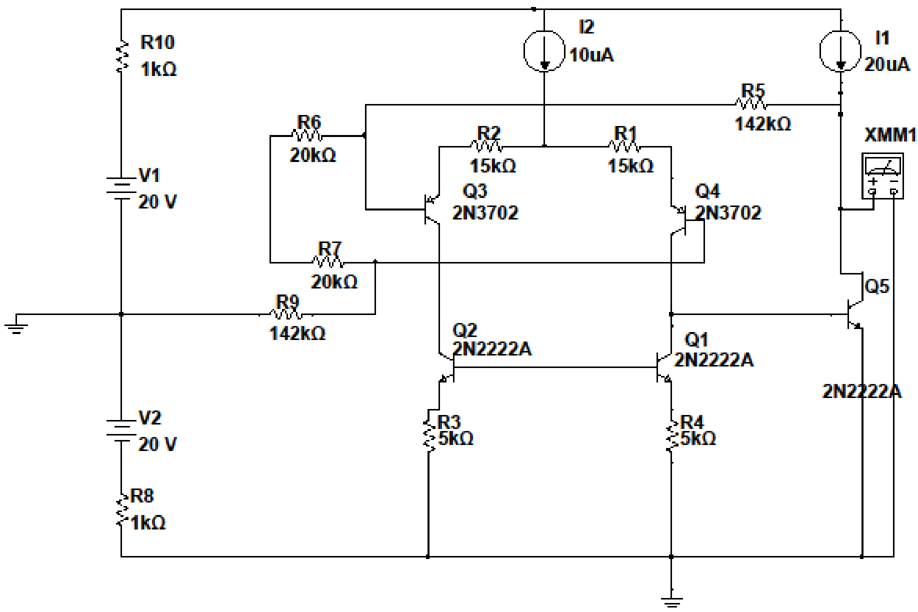

Figure 1 The Differential Operational Amplifier Circuit Mockup paralleling Figure 8.1, Phadke (1989)

Optimum 1: Only change RPEM from 15 kΩ to 7.5 kΩ. By the procedure in "Evaluation of Mean Squared Offset Voltage," the value of η for this design was found to be 33.70 dB compared to 29.39 dB for the starting design. In terms of the rms offset voltage (robustness) this represents an improvement from 33.9 mV to 20.7 mV.

Optimum 2: Change RPEM to 7.5 kΩ. Also change both CPCS and OCS to 10 μA . The η for this design was computed to be 35.82 dB, and rms offset voltage was seen to be 16.2 mV.

To this end Phadke additionally examined the plots (Figure 8.4 Phadke 1989) derived from the analysis of the mean and variance of the data (Table 8.6 Phadke 1989) resulting from the above IA_OA robust design approach. With some mild compromise he successfully reached the two "optimum" designs above. Before Phadke's pioneering work, trial and error alone was the way to produce acceptable designs with little assurance of robustness of performance. Phadke's key results were the identification of control (design) factors D (CPCS) and E (OCS) as good choices for adjusting mean VO without affecting robustness appreciably, and B (RPEM) to maximize η (robustness).

1.1. Intrigues

Phadke's sticking to Taguchi's prescribed method to yield a robust design here was admirable, for it illustrated the high utility of this straightforward method, even if it forces one to ignore design or control factor effect interactions and the possibly existing nonlinearities in the performance domain. The data in Table 8.6 was produced efficiently and it is quite rich in information as we shall shortly discover. Admittedly the final results were accepted at some modest compromise. The same reason is given for the multiple quality characteristics case (Section 6, Phadke 1989). An intrigue that naturally develops is how the solutions reached would change if more exact analytical methods were marshaled to manipulate the *same* data? How far are we from the possibly near-exact robust design? Probing this intrigue at minimum would have some pedagogical value. Furthermore, it might expose some worthy nuggets. To study this we posed the two following hypotheses:

Hypothesis 1: A superior robust design can be reached if constrained optimization method is harnessed.

Hypothesis 2: A second order response model of sufficient “explanability” for both offset voltage (VO), and one for its variance (in the particular situation when the design space is constrained to guarantee VO = 0 volt) can be built to serve as the arena where the worthy solutions would be present.

This study exclusively utilized Phadke’s mean VO and variance of VO data to facilitate a comparison.

2. The Present Approach to Robust Design

The response or performance of electronic devices is typically a nonlinear function of the characteristics of the individual components—resistors, capacitors, inductors, Op Amps, digital devices and numerous other parts—that often also have effects that interact with each other. In other words the different effects of these parts on the response are *not* additive (Sedra and Smith 2004). The same holds for the product’s robustness—the device’s sensitivity to variations in environmental and production conditions. Hence, finding the best levels of each design factor is not trivial (Phadke 1989). Mathematical manipulations have been suggested to approach “additivity”, so that the task of driving average performance to the ideal or target value, and also to assure maximum robustness, may be kept simple. Taguchi’s original proposal had included signal-to-noise (S/N) ratios. However, the very OA-based approach is a compromise—it often forces us to not fully exploit nonlinearities and some interactions. This is inescapable, particularly in many electronic device design situations, where the response is a complex function of the values of the resistors, capacitors, inductors, Op Amps, and ICs etc. (Sedra and Smith 2004). The differential Op Amp design problem is no exception: its response or its robustness is not an additive function of factor effects.

The true functional relationship among input or design factors and the responses is often unknown, for the device often uses subassemblies. Again, the differential Op Amp constitutes one such example. However, powerful *empirical* statistical modeling methods have been available for over sixty years to help one develop such relationships empirically. Indeed linear and nonlinear multiple regression now amply facilitates this process (Montgomery 2013), developed using both unplanned or well-planned, collection of data. The details may be found in Box and Draper (1987) or in Montgomery (2013). Once built, such empirical models may serve in the domain of interest as response surfaces, enabling one to optimize the desired characteristics of the response. However, these methods have received less than their deserved share of attention in robust design (Khatree 1996). The present paper evaluates the utility of adopting this stance in the context of the robust design of the differential operational amplifier. As demonstrated in this study, the results obtained are considerably superior to the OA-based method.

2.1. Methodology Adopted

The first step in this stance, applicable when a prototype or a mathematical or simulation model engaging the various design and noise parameters can be set up, is to build an *empirical* (input, response) model; analytically developing such relationship directly may be difficult or impossible. The domain of electronics finds numerous such examples. Even here, empirical models often would deliver the desired system responses as explicit functions of the input (design and noise) factors. Simulation has often been used since the ‘50s to exercise the behavior of systems ever since computing technologies empowered the designer. Such action can yield the required sets of {input, response} data.

If a physical or mathematical prototype can be set up, that also can produce this data set. Empirically establishing the {input} →{response} relationship, a methodology much aided by the development of statistical model building methods, including multiple regression (Box and Draper 1984, Montgomery 2013). Such models are often linear in parameters and possibly non-linear in input variables and their functions. This practice is now a respectable approach to develop a representation of the process or system and often used in Econometrics and physical and social sciences where the required data is obtained by observation or via designed experiments. (When such a model is built, there is no direct implication of “causality”—*regression*, it is well recognized, attempts only to *quantify* “association” among the input and response factors. Causality if implied must be traceable to physical or otherwise established laws underlying such a relationship—independently or experimentally confirmed (Bernard 1982, Holland 1986).)

In designing the differential Op Amp, since a closed form *design factors*→ *response* model is unavailable, we empirically developed a model for offset voltage VO, and another model for the noise-induced variance of VO. The subsequent step, adopting the constrained optimization stance, would be to constrain the VO model's response *at* the target value (here mean VO = 0.0 volt). This identified the set of feasible solutions or designs that met the "on target performance"—Objective #1 of robust design. Next one would search this constrained design space using the variance model of VO to maximize robustness. The goal would be to search for the feasible solution (that already meets the VO = 0.0 objective) that minimizes the variance of VO (Objective #2 of robust design). Note that this approach changes Taguchi's originally recommended strategy for seeking robustness—it does not force one to empirically identify the "adjustment factor(s)" and the robustness-seeking factors, and then to manipulate them (Phadke 1989). Note also that we did not maximize Taguchi's recommended S/N ratio to seek robustness.

Various criteria could guide the data collection step—to assure that the empirically developed regression model (Montgomery 2013) would be of sound character, especially in respect to the parameters empirically estimated. A common approach is to conduct specially designed statistical experiments that aim at statistical optimality. Phadke (1989) chose an L_{36} design to determine only the main effects of the five design factors RFM, RPEM, RNEM, CPCS and OCS to move toward a robust design. For the present we incorporated second order terms and factor interactions into a model that we developed using the data given in Phadke's Table 8.6, albeit settling for a practical compromise between using Phadke's Table 8.5 and running fresh experiments for instance, with center points (Montgomery 2013).

3. Approaching Robustness of the OpAmp

In Taguchi's design methodology two distinct objectives are sought—(a) on-target performance, and (b) robustness of such performance. Both are quantifiable using Taguchi's "quality loss functions." Various methods have been devised to accomplish (a) and (b), including as stated earlier the use of signal to noise (S/N) ratios. In contrast, the procedure presently adopted was induced by response surface method (Montgomery 2013), as follows.

Step 1. Generate a response surface for mean offset voltage (VO) using the starting level data for each factor from Table 8.5 and response data from Table 8.6 of Phadke (1989). Find a subset of solutions (designs) that satisfy the first criterion (a) above. This is the "feasible" solution space in which every design would deliver mean VO = 0.0.

Step 2. Apply constrained optimization. To do this use the expression of the offset voltage response surface from Step 1 to constrain the solutions at VO = 0.0. Next, pick a reasonable trial value(s) for one or more design factor(s) leaving the others to assume values so as to keep the solution exactly on the VO = 0.0 surface. This is often possible for a single performance objective when there are two or more factors in the response surface equation.

Step 3. On the VO = 0.0 surface conduct a search using the response surface expression for Var(VO) constructed in Step 1, aimed at minimizing Var(VO). The solution with the smallest Var(VO) will be the final design—assuring VO = 0.0 and also maximum robustness.

Step 4. Compare the solution found in Step 3 with the "Optimum 1" and "Optimum 2" solutions shown in the introductory section above. This comparison would help test the acceptability of Hypotheses 1 and 2.

4. Development of the Response Surface for offset voltage VO

To accomplish this we used a recursive procedure aimed at developing a multiple regression model for VO in terms of the design ("control" in Phadke's work) factors A (RFM), B (RPEM), C (RNEM), D (CPCS) and E (OCS) shown in Table 8.4, Phadke (1989). Generally speaking, however, expressions relating design factors and system performance are nonlinear, quite complex involving interaction (Gaekwad 1993, Franco 2002), and often unavailable. In general, such relationships are often not linear or additive. For OpAmps such expressions do not yet exist. Still, two features—simplicity and parsimony—would be desirable for such models. For this first a simple model (1) for the OpAmp—an additive linear regression model without involving interaction effects—was fitted to the 36 mean VO data shown in Phadke's Column 2 of Table 8.6.

$$VO = \alpha_0 + \alpha_A A + \alpha_B B + \alpha_C C + \alpha_D D + \alpha_E E \qquad (1)$$

Regression results show that the simple regression model is significant ($F < F_{crit}$) at $\alpha = 0.05$, hence a quantitative association between A, B, C, etc. and VO does exist that may be utilized to predict VO given values of A to E. The adjusted R^2 is 0.79—respectable. However, only factors D and E are significant (P-value is > P_{crit} at $\alpha = 0.05$ while the other factors are not. The value of adjusted R^2 also indicates that over 20% of the variation in response in the mean VO data is unexplained—possibly due to higher order terms that the linear model has not included. For this reason the next higher order model shown in model (2) was tested. The second order model comprised single factors, their squares and two factor product terms to capture interactions. The hypothesized model parameters (coefficients$\alpha_{i/j}$) are shown in (2).

$$VO = \alpha_0 + \alpha_A A + \alpha_B B + \alpha_C C + \alpha_D D + \alpha_E E + \alpha_{A2} A^2 + \alpha_{AB} AB + \alpha_{AC} AC + \alpha_{AD} AD + \alpha_{AE} AE + \alpha_{B2} B^2 + \alpha_{BC} BC + \alpha_{BD} BD + \alpha_{BE} BE + \alpha_{C2} C^2 + \alpha_{CD} CD + \alpha_{CE} CE + \alpha_{D2} D^2 + \alpha_{DE} DE + \alpha_{E2} E^2 \quad (2)$$

The second order model was also significant as indicated by ANOVA and it had an adjusted $R^2 = 0.83$. Factors significant at $\alpha = 0.05$ were A (RFM), E (OCS), D^2, E^2 and BD indicating the presence of a B_D interaction. This model—without the intercept term—had an adjusted $R^2 = 0.90$—quite satisfactory, and contained A, B, C, E, A^2, AE, BC and BD as significant regression terms or explanatory variables. Presence of BD in (2) would require us to specify a value for B to use the mean VO model as a predictor, even if factor B would not appear by itself in the variance model as indicted by its *P*-value. This indicates that there is indeed a strong contribution in mean VO of some kind of interaction between B and D. Figure 2 qualitatively shows the fit of the second order model built without the intercept term to the observed data. No further improvement of fitting was attempted.

To implement Step 3 a second order response model (3) for variance of VO (Var_VO) was similarly built. Adjusted R^2 for this model was 0.73. The unexplained part of Var_VO would be traceable to higher order or omitted terms, and, in Taguchi's assertion, to the interaction between control factor settings and noise. It would be this last part of Var_VO that a robust design procedure would attempt to minimize.

$$Var\_VO = \beta_0 + \beta_A A + \beta_B B + \beta_C C + \beta_D D + \beta_E E + \beta_{A2} A^2 + \beta_{AB} AB + \beta_{AC} AC + \beta_{AD} AD + \beta_{AE} AE + \beta_{B2} B^2 + \beta_{BC} BC + \beta_{BD} BD + \beta_{BE} BE + \beta_{C2} C^2 + \beta_{CD} CD + \beta_{CE} CE + \beta_{D2} D^2 + \beta_{DE} DE + \beta_{E2} E^2 \quad (3)$$

Note most significantly in that in the Var_VO model the only model coefficient that is significant is β_{BD}, which corresponds to the factor interaction effect BD controlled by the values of factors B and D. Thus BD may now be used as the *only* quantity to manipulate *inside* the feasible design space where VO=0 to maximize the robustness of the Op Amp device. This operationalizes "constrained" optimization to seek robustness. Figure 3 displays the BD two-factor interaction (cf. Montgomery 2013, Figure 9.4).

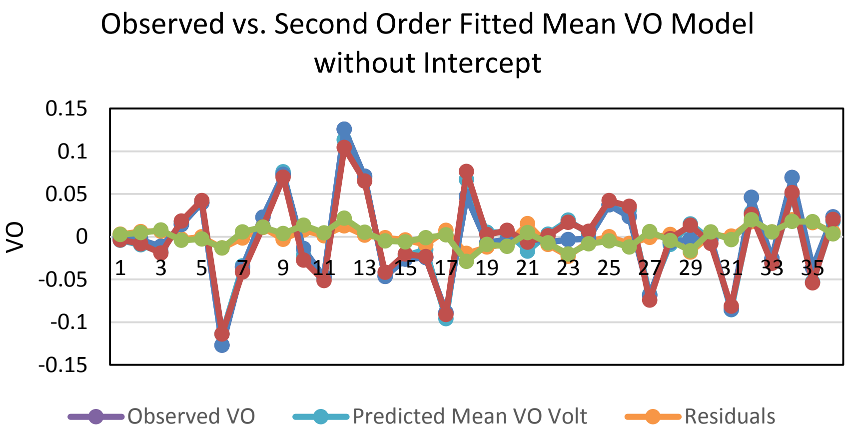

Figure 2. The Second Order VO Model without Intercept vs. Observed VO Data

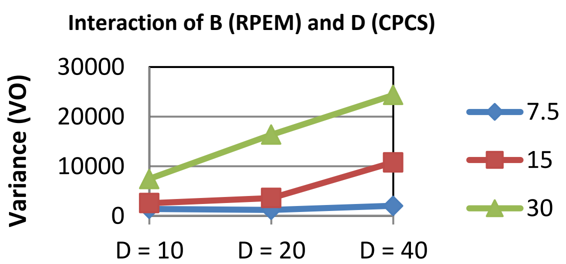

Figure 3 Graphic visualization of BD interaction

5. Delivering the Robust Design—The Final Step

The steps to reach the robust design for the differential Op Amp are now traced.

The first step was to locate the feasible solution space and to identify solutions (designs) that were inside it, i.e., that met the "on target" criterion of robust design. For each of these designs, mean VO would be 0.0, and this delineated the constrained design space within which the optimal design with maximum robustness would be sought. To do this model (2) was used retaining in it only its statistically significant coefficients. The VO = 0.0 constraint was imposed as follows:

$$\alpha_A A + \alpha_B B + \alpha_C C + \alpha_E E + \alpha_{A2} A^2 + \alpha_{AE} AE + \alpha_{BC} BC + \alpha_{BD} BD = VO = 0.0 \quad (4)$$

Next, a second order model (hypothesized by model (3)) was developed for Var_VO using the Variance of data given in Column 3 of Table 8.6, Phadke (1989). Adjusted R^2 for this regression step is 0.73, indicating that almost a quarter of this variance (this now includes the variance induced by the Inner × Outer Array operation) is due to factor and other effects not included in model (3). It was found that the only statistically significant contributor to Var_VO was the BD term in (3), its coefficient α_{BD} being 0.004823253. This was not totally unexpected, keeping in perspective the plot of control factor effects on η, the S/N ratio in Figure 8.4, Phadke (1989). This incidentally simplified searching the multidimensional design space to seek maximum robustness. In fact, from (4) one finds

$$BD = -(\alpha_A A + \alpha_B B + \alpha_C C + \alpha_E E + \alpha_{A2} A^2 + \alpha_{AE} AE + \alpha_{BC} BC)/ \alpha_{BD} \quad (5)$$

The quantity BD (the product of the values of factor settings for B and D) involves two decision variables—B and D, the other design factors A, C and E being pre-set reasonably—only to enable the satisfaction of "on-target" constraint (4). If B is now independently set at some value, the factor that may be manipulated so as to maximize Var_VO is D. (This choice of B vs. D could be reversed without affecting the procedure.) Tables 1 and 2 display part of the calculations involved. The final robust design reached was as follows:

A RFM = 71000 Ω

B RPEM = 1760 Ω

C RNEM = 2500 Ω

E OCS = 0.00002 Amp
D CPCS = 1.5444E-08 Amp

The resulting predicted mean offset voltage VO for this design was 0.0 Volt and the resulting predicted variance of offset voltage would be nearly 0 Volt^2, *very robust* at least theoretically for the conditions specified. A close inspection of the above solution, which is noticeably different from "*Optimum 1*" and "*Optimum 2*" given by Phadke suggests that the absolute value of the offset voltage VO and its variance *rise together and fall together*.

Indeed this is what is visible in a scatter plot of the data given in Column 2 and Column 3 of Table 8.6, Phadke (1989). Figure 4 displays this. An early discovery of this relationship would have reduced the circuitous route taken to reach the robust design as done above. However, this would not be the situation always. Nonetheless, the results above demonstrate the high value of using RSM combined with constrained optimization to seek robustness for similar devices. As for the data collection step (this study used data from Phadke's Table 8.6 which he had obtained by his Inner × Outer Array experimental runs). In order to build superior response surface models, better performing (optimal) experimental designs rather than OAs should be used in place of the Inner Array (Table 8.5, Phadke 1989). One candidate is the D-Optimal experimental design (see Montgomery (2013), page 468). D-optimal designs minimize the volume of the joint confidence region on the vector of regression coefficients.

The solution shown in Table 1 is a very robust design while it meets the target requirement, offset voltage VO = 0.0 Volt. But VO's variance 1178E-6 V^2 can be *further* reduced by adjusting design factor B. This may be done here by a simple search (What If analysis) as shown in Table 2. The result is a differential Op Amp that has A = 71,000 Ω, B = 1760 Ω, C = 2500 Ω, D = 1.54E-08 Amp, E = 0.00002 Amp, VO = 0.0 and variance of VO ~ 0.

Thus this section has effectively demonstrated the utility of adopting the constrained optimization approach to develop robust designs for electronic devices such as the differential operational amplifier. With various computing, simulation and optimization tools and aids now being available, it is certainly worth one's expending some effort by engaging such gear to deliver higher quality robust design, as shown in this paper.

Table 1 A Typical Robust Op Amp design with Target Constrained at VO = 0.0 Volt

Variable D selected as dependent variable whose value is constrained by VO = 0 Volt;
All other variables A, B, C and E are independently fixed in this search for robustness.

	Coefficients	Factor Value
Intercept	0	0
A RFM	-8.34838E-07	71000
B RPEM	1.92636E-06	15000
C RNEM	-5.79532E-06	2500.0
E QCS	1897.742888	0.00002
A^2	3.65258E-12	5041000000
AE	0.008871285	1.42
BC	3.20613E-10	37500000
BD	-0.147958254	0.244132104
D	0	**1.62755E-05**
Resulting VO from Model (2)		0.000000
Resulting Variance of VO from Model (3)		1178 10^{-6} V^2 = 0.001177511 $Volt^2$

Table 2 Constrained *What If* analysis that seeks maximum robustness of VO by varying factor B

	B Ω -->				
	15000	**10000**	**5000**	**2000**	**1760**
Target VO Volt =	0	0	0	0	0
D Amps	1.62755E-05	1.51947E-05	1.19526E-05	2.22602E-06	1.5444E-08
Model's VO Volt	0	0	0	0	0
Projected Var VO 10^{-3} V^2	1178	733	288	21	0

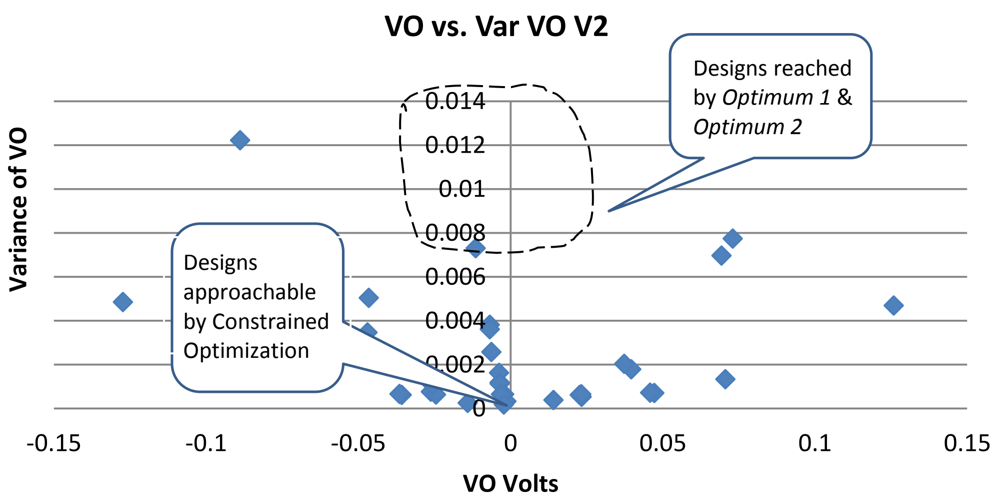

Figure 4 Scatter Plot of mean VO vs. variance of VO data (data source Table 8.6, Phadke 1989)

6. Discussion and Conclusions

The present study involved an Op Amp designed by AT&T engineers at component level from transistors, resistors, and DC supply and current sources (Phadke 1986). Their challenge was twofold—(1) the product must satisfy all design requirements, and (2) it must sustain its exacting performance over a wide range of *operating conditions* that may occur during their use and process variations that may occur during the *fabrication* of the components and the final device itself. The challenge was amplified by the constraint that no closed or explicit form expression for such devices was available relating performance and the device's design parameters.

To this end design methods such as parametric yield estimation, parametric yield maximization, worst case analysis, and variability reduction etc. have been pursued. However, worst-case type of approach is very conservative as the predicted worst-case performance is overly pessimistic and it also often leads to higher product cost. Taguchi (1978) showed that prudent manipulation of design factors during design can also produce a *robust* design. Phadke (1989) pursued this path. The present study retained Phadke's original experimental data to develop response surface models that were then coupled with constrained optimization. Still, as shown here, this produced substantially improved robustness of the final design.

Work in progress by the authors involves the robust design of CMOS-based devises for which again analytical response models are not easy to devise.

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