MODELS FOR RETAIL PRICING AND CUSTOMER RETURN INCENTIVE FOR REMANUFACTURING A PRODUCT

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POMS 20th Annual Conference
Orlando, Florida U.S.A.
May 1 to May 4, 2009
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Abstract

Used products can often be collected via customer returns by retailers in supply chains and remanufactured by producers to bring them back into “as-new” condition for resale. In this paper, mathematical models are developed for determining optimal decisions involving order quantity, retail pricing and reimbursement to customers for returns. These decisions are made in an integrated manner for a single manufacturer and a single retailer dealing with a single recoverable item. Numerical example is shown. The study focuses on the sensitivity analysis to explore the relationship between the parameters and the decision variables.

Keywords: supply chain, reverse logistics, sensitivity analysis

1. Introduction

Literature on remanufacturing inventory modeling is very rich. Fleischmann et al. (1997) and Guide et al. (2000) provide thorough surveys of existing research involving remanufacturing. A significant portion of the work on product recovery addresses inventory control and related matters (Schrady 1967, Mabini et al.1992, Richter1996a, 1996b, Koh et al. 2002, Minner et al., 2001, 2004). Most recently, Tang and Teuter(2006) have embellished the economic lot scheduling problem through the incorporation of returns. However, relatively rare efforts has been put towards integrating the decisions of inventory replenishment, product pricing and customer incentive for returning used items (in the form of a cash refund or a discount coupon).
in a remanufacturing environment. As a notable exception, in a recent study Savaskan et al. (2004) have addressed the questions of pricing and return incentives from a game theoretic perspective, in examining alternative reverse logistics structures for the collection of recoverable products. Bhattacharya et al. (2006) conduct the integration of optimal order quantities in different channels, reflecting the various relationships among retailer, manufacturer and remanufacturer. Also, Vorasayan and Ryan (2006) outline procedures for deriving the pricing and quantity decisions for refurbished products.

Inventory control decisions, which are intertwined with such questions, however, have been only superficially treated in the pricing related research. In our previous study (Liu2009), we address this deficiency in the current body of work involving remanufacturing and focus the major issues concerning inventories, pricing, used product collection, materials procurement, product delivery and planning for manufacturing and remanufacturing in an integrated manner. Specifically, we develop an integrated policy to achieve a well-coordinated supply chain via incorporating a lean production process. In this paper, we study the relationship between the parameters involved and the final decisions.

2. Notation and Assumptions

2.1 Notation

We use the following notational scheme throughout the paper:

For the retailer:

\( d \) = demand rate of the product in units/time unit;

\( S_r \) = fixed ordering cost ($/order) for retail stock replenishment;

\( h_r \) = inventory holding cost of the product ($/unit/time unit);

\( h_{r_t} \) = inventory holding cost of the used (returned) product ($/unit/ time unit);
rc = unit reimbursement to customers for returns in $/unit;

ps = unit selling price of the product (new or remanufactured) in $/unit;

x = rate at which customers return the used item to the retailer (units/time unit);

X = total quantity of returns in a replenishment cycle (units).

For the manufacturer:

m = manufacturing or remanufacturing rate of the product (unit/time unit);

Sm = fixed manufacturing/remanufacturing setup cost per replenishment lot ($/setup);

Sr_m = total fixed cost of shipping a replenishment lot of new products to the retailer and
transporting the returned items collected back to the manufacturing facility ($/cycle);

S_i = fixed ordering cost of input materials ($/lot) for manufacturing and/or remanufacturing;

h_m = inventory holding cost of finished product (new or remanufactured) in $/unit/time unit;

h_i = inventory holding cost of input materials necessary for the production of a unit of the
new product in $/unit/time unit;

h_ir = inventory holding cost of input materials necessary for remanufacturing a unit of the used
product ($/unit/time unit);

r_m = transfer price paid to retailer by manufacturer for collecting used products ($/unit);

p_w = wholesale price charged to retailer for the new product ($/unit);

c_s = variable transportation cost of shipping new product to the retailer ($/unit);

c_r = variable cost of transporting, cleaning, preparation, etc for returned items ($/unit);

c_m = variable cost of manufacturing new product ($/unit);

c_r_m = variable cost of remanufacturing a returned used product into a new one ($/unit).

Common to both

T = inventory replenishment cycle time (time units), common to retailer and manufacturer;
$Q =$ total replenishment quantity (units) consisting of new and/or remanufactured items.

2.2 Assumptions

1. The supply chain under study consists of a single retailer and a single manufacturer involved in the production and sale of a single recoverable product. Customers are refunded a part of the purchase price by the retailer as an incentive to return used products, which can be restored to “as new” condition for resale through a remanufacturing process deployed by the manufacturer. The manufacturing/remanufacturing environment is a batch production system where each batch of the product may consist of a mix of remanufactured and new manufactured items within a single setup. The used items, after cleaning, restoration, etc. are completely reincorporated in the existing production process, so that remanufacturing and new product manufacturing rates are the same, although their variable costs may differ.

2. For coordination purposes, the lot-for-lot policy is in effect for input materials ordering, manufacturing and remanufacturing, product delivery and retail inventory replenishment, with a common cycle time of $T$. This lot-for-lot feature is commonly found in JIT based lean manufacturing systems, where minimal levels of material and product inventories are desired.

3. All input materials for manufacturing or remanufacturing are treated as a composite bundle. All of the input materials (for manufacturing and remanufacturing) are ordered on a lot-for-lot basis with a single procurement order prior to the setup of a batch. In each case, the total bundle of inputs necessary for producing (or remanufacturing) a unit of the end product is defined as a “unit”.

4. The retailer is responsible for collecting returned items and holding them in inventory until
picked up by the producer. In our decentralized models, the manufacturer pays the retailer a unit transfer price for the returned items, in order to induce the latter to engage in the collection activity. Without loss of generality, it is assumed that the retailer’s cost of this collection effort is negligibly small, although the cost of holding the returned products in inventory at the retail level is taken into account. Under the centralized scenario, the used product transfer price and the producer’s wholesale price become irrelevant for avoiding double marginalization. In the decentralized models, the retailer sets the item’s selling price and the unit reimbursement to customers for returns. The wholesale price, where applicable, is the same for new or remanufactured items.

5. We assume that the market demand, the customer return rate and all lead times are deterministic. Thus, a production batch of $Q$ units consists of $Q-X$ new items and $X$ units of remanufactured product as shown in Figure 1, which shows the process flow schema of the supply chain under consideration. Figure 2 depicts the various inventory-time relationship involving in the retail and manufacturing facilities. Without loss of generality, these plots are constructed with the assumption that the setup and transit times, as well as the cleaning and refurbishment times for the recovered items are zero. Before setting up a production batch, the $X$ units of returned items collected during the cycle are transported back to the plant for remanufacturing. Therefore, the value of the quantity $Q-X$ is purchased from the supplier prior to each setup. After completion of the manufacturing and remanufacturing process, the replenishment lot of $Q$ is delivered to the retailer for sale. All transportation costs are paid by the producer.
6. We adopt linear structures for both $d$ and $x$ for simplicity of analysis and implementation and assume the retail demand rate, $d$, as a decreasing function of its selling price, $p_s$, i.e. $d = A - Bp_s$. Furthermore, the product’s return rate, $x$, and the total units returned, $X$, during a cycle are expressed, respectively, as $x = ar_c - bp_s$ and $X = Tx = Qx/d$. The parameters $A$, $B$, $a$ and $b$ are given. It is reasonable to assume that the average rate of used product returns is likely to increase as the return incentive, $r_c$, as well as the overall demand level, $d$, increase (or, alternately, as the retail price decreases).

3. Profit Analysis

3.1 The retailer’s profit

The retailer has two sources of revenue, captured by the first two terms in the profit function(1). The first of these represents the revenue from the sales of new products and the second term expresses the net revenue, through reimbursements from the manufacturer for collecting the used items. The next term represents the average ordering cost and the remaining two terms show, respectively, the costs of holding new product and returned item inventories per time unit at the retailer’s end (see Figures 2(a) and (b)). Its profit per time unit can be expressed as

$$\Pi_r = (p_s - p_w)d + (r_m - r_c)x - S_r \frac{d}{Q} - h_r \frac{Q}{2} - h_{rr} \frac{Qx}{2d}. \quad (1)$$

Substituting $x = ar_c - bp_s$ and $d = A - Bp_s$ into (1), the retailer’s average profit per time unit can be rewritten as

$$\Pi_r = (p_s - p_w)(A - Bp_s) + (r_m - r_c)(ar_c - bp_s) - S_r \left[ \frac{A - Bp_s}{Q} \right] - \frac{Q}{2} \left[ h_r + h_{rr} \left( \frac{ar_c - bp_s}{A - Bp_s} \right) \right]. \quad (2)$$
3.2 The manufacturer’s profit Decentralized Models with exogenous wholesale price

In order to develop the manufacturer’s profit function, we need to determine the average inventories at the manufacturing facility. From Figure 1(c), the average inventory of the finished product at the manufacturer’s end

\[
= \left( \frac{Q}{2} \right) \left( \frac{Q}{m} \right) / \left( \frac{Q}{d} \right) = \frac{Qd}{2m}.
\]  

(3)

Also, from Figure 1(d) it can be shown that the average inventories of the input materials necessary for remanufacturing and manufacturing purposes, respectively

\[
= \frac{x^2 Q}{2md} + \frac{Q}{2m} \left[ d - 2x + x^2 / d \right].
\]  

(4)

Incorporating these results, the profit per time unit for the manufacturer can be expressed as

\[
\Pi_m = (p_w - c_x) d - \frac{d}{Q} (S_m + S_{rm} + S_i) - (r_m + c_r)x - \frac{h_m Qd}{2m} - \frac{h_{ir} Qx^2}{2md}

- \frac{h_i Q}{2m} (d - 2x + x^2 / d) - c_m (d - x) - c_{rm}x.
\]  

(5)

The first term in (5) shows the manufacturer’s revenue based on the wholesale price, less the variable shipping cost to the retailer. The second term includes the fixed costs involving production set up, transportation of new products to and used items from the retailer and ordering of input raw materials. The third term expresses the reimbursement cost to retailer, as well as the variable transportation, cleaning and preparation costs for the returned items. The next three terms represent the holding costs, respectively, for the finished product and input materials inventories necessary for remanufacturing and manufacturing. The final two terms in (5) are the variable costs per time unit for manufacturing and remanufacturing, respectively.

Substituting for \(d\) and \(x\) into (5), and collecting terms, the manufacturer’s profit per time unit is rewritten as follows:
\[ \Pi_m = (p_w - c_m - c_s)(A - Bp_s) - \frac{A - Bp_s}{Q} (S_m + S_{rm} + S_i) \]

\[-(r_m + c_r + c_{rm} - c_m)(ar_c - bp_s) - \frac{Q}{2m} \{h_m(A - Bp_s) + \frac{h_{ir}(ar_c - bp_s)^2}{(A - Bp_s)} \} \]

\[+ h_i[A - Bp_s - 2(ar_c - bp_s) + \frac{(ar_c - bp_s)^2}{A - Bp_s}] \]  \hspace{1cm} (6)

3.3 Supply Chain Profit

Suppose that the retailer and the manufacturer agree to cooperate towards formulating a jointly optimal integrated policy, involving inventory replenishment, retail pricing and customer return reimbursement decisions, for the supply chain as a whole. The focus of such a centralized policy, where both parties are willing to freely share their cost and other relevant information, is to maximize the profitability of the entire system, rather than that of either party. We illustrate in the next section that this centralized joint optimization approach can be economically attractive from the standpoint of both the parties through an equitable profit sharing methodology. In this centralized approach, we propose that in order to avoid double marginalization, the parameters wholesale price \( p_w \) and manufacturer’s rebate for returned items \( r_m \) need not be considered and are omitted. Thus, combining (1) and (5), without an explicit wholesale price and a direct manufacturer’s reimbursement to the retailer for product returns, the total supply chain profit is

\[ \Pi_s = (p_s - c_m - c_s)(A - Bp_s) - (r_c + c_r + c_{rm} - c_m)(ar_c - bp_s) - \frac{(A - Bp_s)}{Q} (S_r + S_m + S_{rm} + S_i) \]

\[- \frac{Q}{2m} \left\{ m \left( h_r + h_{rr} \frac{(ar_c - bp_s)}{(A - Bp_s)} \right) + (h_{ir} + h_i) \left\{ \frac{(ar_c - bp_s)^2}{(A - Bp_s)} \right\} \right\} \]

\[+ (h_m + h_i)(A - Bp_s) - 2h_i(ar_c - bp_s) \]  \hspace{1cm} (7)

4. Development of Hierarchical Decision Making Models and Analysis
4.1 Decentralized Models with exogenous wholesale price

In some industries, due to intense competition, the wholesale price for the manufacturer is determined the existing market conditions and is, consequently, treated as a constant parameter. The exposition in this subsection pertains to such cases.

\[ \max_{Q, r_c, p_s} \Pi_r \]

The first order optimality solution can be obtained by setting

\[ \frac{\partial \Pi_r}{\partial Q} = 0 \]  
(8)

\[ \frac{\partial \Pi_r}{\partial r_c} = 0 \]  
(9)

\[ \frac{\partial \Pi_r}{\partial p_s} = 0 \]  
(10)

\[ 0 \leq ar_c - bp_s \leq A - Bp_s \]  
(11)

Due to the calculation, we need to make sure that \( d \geq x \) because the return rate cannot exceed the demand rate of the item. Meanwhile, all the negative solutions are disregarded in this and subsequent models for computational purposes.

4.2 Retailer controlled situation with exogenous wholesale price

If the manufacturer, instead of the retailer, is in a position of dictating supply policy, it would prefer to implement a production and delivery policy (assuming the lot-for-lot operating framework) that is optimal from its own perspective. In this case, the supplier’s wholesale price is treated as a given parameter. The retailer, nevertheless, is likely to be free to set its own selling price and the level of incentive to induce customers to return the used products, given the manufacturer’s preferred replenishment lot size. Thus, if the manufacturer, instead of the retailer,
has control of the order quantity, the model above may be written as a bilevel problem, as shown below:

\[
\begin{align*}
\max_{Q} & \quad \Pi_m \\
\text{s.t.} & \\
\frac{\partial \Pi_r}{\partial r_c} &= 0 \quad (9) \\
\frac{\partial \Pi_r}{\partial p_s} &= 0 \quad (10) \\
0 &\leq ar_c - bp_s \leq A - Bp_s \quad (11)
\end{align*}
\]

This constrained nonlinear problem may be solved by one of several widely available optimization software packages, such as MATLAB.

4.3 Manufacturer controlled model with wholesale price endogenous

Under monopolistic market conditions, manufacturers may lower the wholesale price in order to encourage retailers to increase their order quantities. As discussed before, under a decentralized policy, the retailer determines its selling price and the customer return incentive. It will make these decisions after the observation of a wholesale price set by the manufacturer. Initially, the manufacturer would anticipate the optimal response from the retailer when it decides on the wholesale price, resulting in the following model:

\[
\begin{align*}
\max_{p_w, Q} & \quad \Pi_m \\
\text{s.t.} & \\
\frac{\partial \Pi_r}{\partial r_c} &= 0 \quad (9) \\
\frac{\partial \Pi_r}{\partial p_s} &= 0 \quad (10) \\
0 &\leq ar_c - bp_s \leq A - Bp_s \quad (11)
\end{align*}
\]
4.4 Centralized Model for Supply Chain Optimality

The first order optimality conditions yield the optimal values of the replenishment lot size, $Q$, unit customer reimbursement for returns, $r_c$ and the unit selling price, $p_s$, which maximize the total supply chain profit under the proposed centralized policy, as shown below:

$$\max_{p_s, Q_c, r_c} \Pi_s$$

$$0 \leq ar_c - bp_s \leq A - Bp_s$$  \hfill (11)

Once again, first order conditions can be solved via any appropriate equation solving software, such as MATLAB, for determining the centrally controlled inventory replenishment, retail pricing and return reimbursement decisions.

4. A Numerical Illustration and Discussions

To illustrate our models outlined above, a numerical example is provided below. The following information pertaining to the two parties in the supply chain are available.

Retailer:

$S_r = \$50/\text{order}$, $h_r = \$0.015/\text{unit/day}$, $h_{rr} = \$0.002/\text{unit/day}$, $A = 120$ $B = 3.0$ $a = 15$ $b = 0.1$

That is, the daily demand rate is $d = 120 - 3p_s$ and the daily return rate is $x = 15r_c - 0.1p_s$.

Manufacturer:

$m = 100 \text{ units/day}$, $S_m = \$300/\text{batch}$, $S_{rm} = \$200/\text{batch}$, $S_i = \$30/\text{batch}$

$h_m = \$0.01/\text{unit/day}$, $h_i = \$0.009/\text{unit/day}$, $h_{ir} = \$0.007/\text{unit/day}$, $p_w = \$20/\text{unit}$

$r_m = \$2.80/\text{unit}$, $c_s = \$2/\text{unit}$, $c_m = \$8/\text{unit}$, $c_{rm} = \$2/\text{unit}$, $c_r = \$1.20/\text{unit}$.

All the results obtained from the various perspectives are summarized in Table 1. It can be easily verified that the chosen parameters satisfy the joint concavity conditions (Liu et al. 2009).
From this table it is clear that if the retailer has sufficient policy implementation power in the supply chain, it attempts to keep the replenishment lot size comparatively small (i.e. 428.359 units), in view of its relatively low fixed ordering cost. Furthermore, through its retail pricing ($p_s = $30.032/unit), in conjunction with a customer return reimbursement price of $1.493/unit, it prefers to achieve daily market demand and customer return rates of 29.904 and 19.391 units, respectively, that attempt to balance the gains from sales and returns against the ordering and inventory carrying (for both new and used items) costs. The maximum attainable daily profit for the retailer is, thus, $318.361, resulting in a profit of $299.918/day for the manufacturer. Note that as every unit of the returned product represents a net gain of $1.307 (i.e. the difference between the amount, $r_m$, compensated by the manufacturer and the customer reimbursement, $r_c$) for the retailer, it attempts to achieve a relatively high used item return rate about 64.845%.

If, on the other hand, the manufacturer is in a position to exert a greater level of negotiating power in the supply chain, its individual optimal policy would dictate a significantly larger replenishment batch of 2683.3 units, due to the relatively high fixed setup and transportation costs. In spite of a more than six-fold increase in the lot size, however, the selling price and return reimbursement, set by the retailer in response, are both only slightly lower than their values under its own optimal policy, i.e. $29.961 and $1.455 per unit, respectively. It is interesting to note that, consequently, the retail demand rate increases slightly to 30.118 units/day and the average product returns decline slightly to 18.834 units/day. The returns, however, now decline slightly to 62.525% of sales. Not unexpectedly, implementing the manufacturer’s optimal replenishment policy reduces the retailer’s profit to $302.957/day, whereas the manufacturer’s profit increases to $327.245/day. Nevertheless, in terms of total supply chain profitability, the difference between adopting any one party’s optimal policy over the other’s amount to only
about 1.928%.

Table 1 shows that if the retailer and the manufacturer decide to cooperate through the sharing of necessary information and adopt a jointly optimal policy that maximizes the total supply chain profit, instead of optimizing either party’s position, both parties stand to gain considerably from such an approach. As mentioned earlier, the centralized model attempts to avoid double marginalization, i.e. the manufacturer does not explicitly charge the retailer a wholesale price, nor does it explicitly offer the latter a reimbursement for collecting the returns (implying that $p_w = r_m = 0$). Without these cost factors, the centrally controlled approach results in a maximum supply chain profit of $720.876/day, representing a more than 25.728% improvement in total system profitability, compared to the retailer’s optimal policy, or over 16.594% improvement vis-à-vis the manufacturer’s optimal policy. As expected, the jointly optimal replenishment quantity now is 1496.510 units, which is less than the manufacturer’s optimal batch size, but larger than the retailer’s optimal order quantity. More interestingly, the retail price is reduced to $25.179/unit and the return reimbursement is decreased to $2.443/unit, respectively, resulting in a considerably larger demand rate of 44.464 units/day, as well as a smaller average product return rate of 34.121 units/day (i.e. about 76.739% of items sold are returned by customers). The implication of our centralized model is that under a jointly optimal policy, relatively fewer products sold are remanufactured items. Under the given set of problem parameters, it appears desirable to increase the overall market demand through a lower retail price. Also, there is a lesser emphasis on collecting customer returns for remanufacturing. The centralized model reduces the incentive for customer returns, which maximizes the total supply chain profitability.

The absence of a wholesale price and an explicit incentive for the retailer to collect returned
items raises some interesting questions concerning a fair and equitable sharing of the total gain resulting from the centralized cooperative policy shown in Table 1. Although this can be achieved in several possible ways, we propose a profit sharing plan under a scenario where the retailer is the more powerful member of the supply chain and can dictate the implementation of its own optimal policy. The task of the manufacturer is then to offer sufficient incentive to the retailer in order for the latter to adopt the results of this procedure. Note that under its own individual optimal policy, the retailer’s share is 51.492% of the total profit for both the parties. Therefore, it would be reasonable if the retailer is allocated the same percentage of the total supply chain profit of $720.876/day yielded by the centralized model. In other words, the retailer’s share of the total profit is $371.190/day and that of the manufacturer is $349.686/day. With this profit sharing arrangement, each party’s daily profit is more than 16% larger than that achieved under the retailer’s optimal policy. Thus, it is economically attractive for both parties to adopt the jointly optimal policy yielded by our centralized model. If the manufacturer is more powerful of the two parties, the terms of a corresponding profit sharing arrangement, can also be derived easily along similar lines.

Finally, Table 1 also shows the results for the manufacturer controlled models where under monopolistic competition, the manufacturer can set its wholesale price, which is now treated as a decision variable. Compared with the results for a given wholesale price, the retailer’s individual optimal policy dictates increasing the selling price from $29.961/unit to $32.565/unit and decreasing the customer return reimbursement from $1.455/unit to $1.444/unit. Consequently, the order quantity is reduced from 2683.300 units to 2894.360 units. These changes indicate that the retailer would expend less effort to increase market demand and would tend to compensate by attempting to increase its revenue from returns. This appears to be a rational response to a
higher wholesale price. Also, as expected, the manufacturer’s share of the total supply chain profit now increases from 51.927% to 69.266%, while, the total profits for the supply chain declines to $531.645/day. These effects, not unexpectedly, tend to be magnified when the supplier is in a position to dictate the adoption of its own optimal policy by the retailer. Now the total supply chain profit shrinks, although the manufacturer’s relative share of this, as well as its own daily profit go up substantially, albeit at the expense of the retailer.

5. Sensitivity Analysis

With the numerical example illustrated above as the base case, we conduct sensitivity analysis to explore the influence of various parameters on the decision variable, the total supply chain profits and the retailer’s share in the whole supply chain under different scenario. In each test, we vary all the parameters from -50% or -40% to 50% and record the percentage changes in the decision variables and the objective values. The results indicate which parameters with significant effect. We define “significant” as the change in these parameters lead to at least 10% increase or decrease on these given variables. The economic underlying insights also are demonstrated.

5.1 Centralized situation

In the centralized scenario, the change in the selling price of the new product ($p_s$) strongly depends on the change in the parameters of linear demand function ($A$ and $B$). The optimal reimbursement price changes significantly with the change in the constant of the demand function ($A$), the coefficient of returning price in linear return function ($a$) and the manufacturing cost of producing new products. Reimbursement price ($r_c$) is an increasing function of manufacturing cost ($c_m$), which means that the supply chain would pay more to collect the
returns back if the unit manufacturing cost is higher. However, the supply chain would not necessarily pay more to collect unit return back if the customer returns are price sensitive because the high volume of the return has a side effect on the whole supply chain profits. The positive relationship between reimbursement price and the constant of the demand function comes from the rationale that the bigger constant creates a larger demand leading to a larger amount of return due to the higher reimbursement price which encourages returns.

The major factors take in effect on order quantity in the centralized situation are manufacturing setup cost ($s_m$), manufacturing rate ($m$), retailer’s return holding cost ($h_{rr}$) and the constant in demand function($A$). The order quantity is an increasing function of manufacturing setup cost, manufacturing rate and retailer’s return holding cost and instead, a decreasing function of $A$. The major factors influencing the supply chain profits are the parameters in the demand function($A$ and $B$). The Figure 6 shows the reverse relationship between the changes in the price coefficient of demand function ($B$) and those in the supply chain profits as well as a positive relationship between the changes in supply chain profits and those in the constant.
5.2 Retailer controlled situation

In the retailer controlled situation, the retailer determines the selling price of the new product \( (p_s) \), the reimbursement price to the customers \( (r_c) \) and the order quantity \( (Q) \).

All the decisions, \( r_c, p_s \) and \( Q \), are determined by the parameters \( (A, B) \) in the demand function and the wholesale price \( (p_w) \). The selling price \( (p_s) \) is a decreasing function of the coefficient \( (B) \) in the new product demand function. If the lower product elasticity is, the higher selling price would be. On the contrary, the selling price has to set lower if the demand is price sensitive. The selling price increases with the increases of the constant \( (A) \) and the wholesale price \( (p_w) \).

The reimbursement price \( (r_c) \) paid by the retailer to the customer is directly affected by the
incentive \((r_m)\) paid by the manufacturer. The increasing of market size indicated by \(A\) would not make much difference in \((r_c)\). However, the decreasing of market size would cause a big change in the setting of reimbursement price from the retailer’s perspective.

If the retailer has the bargain power to decide the order quantity \((Q)\), according to the principle of the EOQ model, the changes in the parameters such as the unit holding cost and the set up cost account for the change in the retailer’s decision on the order quantity.

As results, the total supply chain profit relies greatly on the parameters in the demand function of the new product and the wholesale price specified by the retailer in the contract. However, these influences are relatively significant, compared with the effect on the changes in retailer’s profit share of the supply chain.
5.2.1 Manufacturer controlled situation with $p_w$ given

If manufacturer has the power to set the order quantity, the factors influence the decision of the retailer’s decision in price are still the same factors as we have discussed in the retailer controlled situation. However, other factors, such as unit manufacturing cost($c_m$), remanufacturing cost($r_m$) and production rate ($m$) also have influence on the change in the order quantity. They are all closely related to the production.

The supply chain profit and the share that the retailer can get from the total profits are changed with the paramters ($A$ and $B$) in the demand function and the price parameters ($p_w$ and $r_m$) specified in the contract. The supply chain’s profits is an increasing function with $A$ and $r_m$ but a decreasing function of $B$ and $p_w$. 
5.2.2 Manufacturer controlled situation without pw given

Similar as before, selling price determined by the retailer are mainly controlled by the parameters in the linear demand function while the reimbursement price relies heavily on the incentive the manufacturer gives to the retailer to collect the returns($r_m$), the constant in the demand function($A$) and the coefficient in the linear return function. However, all these influence are not strictly linear relationship. We can observe that the more incentive the manufacturer gives the retailer to collect the returns, the greater proportion of the incentive is likely to be passed to customers by the retailer.

Figure 17 $p_r$ in the Manufacturer Controlled Situation ($p_w$ not given)  
Figure 18 $r_c$ in the Manufacturer Controlled Situation ($p_w$ not given)

Figure 19 showes the effects of multiple variables on the optimal order quantity decided by the manufacturer. It is easy to observe that there is no strong influence within in the certain range.
However, beyond the certain range, for example, below -20% or above 20%, the level of change in the optimal quantity is exaggerate.

Figure 19 Q in the Manufacturer Controlled Situation (p_w not given)

Figure 20 Π_s in the Manufacturer Controlled Situation (p_w not given)  Figure 21 Π_r / Π_s in the Manufacturer Controlled Situation (p_w not given)

Figure 20 and Figure 21 display the parameters influencing the changes in total profit and profit share. A and B in the demand function are still the major factors. In this case, the profit share for the retailer depends on the manufacturing cost c_m, which makes sense in reality. Especially when the retailer has less power in the supply chain franchise, the share it can get from the supply chain is determined by the manufacturing cost.

6. Summary and Conclusions

In this study, we have developed mathematical models under deterministic condition, for simultaneously determining the production/delivery lot size, the retail price and the customer return reimbursement level for a single recoverable product in a two-echelon supply chain.
consisting of a single retailer and a single lean manufacturer. Items returned by customers at the retail level are refurbished and totally reintegrated into the manufacturer’s existing production system for remanufacturing and are sold eventually as new products. As in many lean manufacturing (a JIT) environments, we assume a lot-for-lot operating mode for production, procurement and distribution, as an effective mechanism for supply chain coordination.

Decentralized models are developed and solved for determining profit maximizing optimal policies from the perspectives of both members of the supply chain. A centralized, jointly optimal procedure for maximizing total supply chain profitability is also presented. A numerical example illustrates that the centralized approach is substantively superior to individual optimization, due to the elimination of double marginalization. The example also outlines a fair and equitable proportional profit sharing scheme, which is economically desirable from the standpoint of either member of the supply chain, for the purpose of implementing the proposed centrally controlled model.

Of necessity, the simplifying assumptions made here (e.g. deterministic parameters and the lot-for-lot modality), are the major limitations of this study. Embellishments by future researchers, such as relaxation of the lot-for-lot assumption, incorporation of uncertainty, more realistic and complex demand and product return functions, multiple products, manufacturers, etc. will, undoubtedly, lead to more refined remanufacturing and related models. Furthermore, future efforts in this area should consider the development of integrated decision models under stochastic conditions, which are likely to be more realistic from an implementation standpoint. Nevertheless, the results obtained in this study are likely to be of some value to practitioners as broad guidelines for integrated pricing, recoverable product collection, production planning and inventory control decisions, as well as for designing more streamlined, well-coordinated supply
chains towards gaining competitive advantage. We also hope that our efforts will prove to be useful for researchers in shedding light on some of the intricate and inter-related aspects of product remanufacturing towards developing more effective decision making models for supply chain and reverse logistics management.

References


Minner, S., Lindner G. 2004. Lot sizing decisions in product recovery management, in


QED
Table 1: Summary of Results

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<th></th>
<th>$Q$ (units)</th>
<th>$p_s$ ($/unit$)</th>
<th>$r_c$ ($/unit$)</th>
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<th>$d$ (units/day)</th>
<th>$\chi$ (units/day)</th>
<th>$\Pi_r$ ($/day$)</th>
<th>$\Pi_m$ ($/day$)</th>
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+Allocated on the basis of proportional shares of total supply chain profit under retailer's optimal policy
Figure 1: The Recovery and Remanufacturing Process
a. Retailers Serviceable Inventory

b. Returns Product Inventory

c. Manufacturing Inventory

d. Remanufacturing Input Material Inventory

e. Manufacturing Input Material Inventory

Figure 2: Inventory - Time Plots